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TRANSFORMERS

THEIR PRINCIPLES AND DESIGN FOR LIGHT ELECTRICAL ENGINEERS

BY

F. C. CONNELLY

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Preface

It is the writer's belief that the preface is seldom read, at any rate until after the rest of the book, so he has included in Chapter I an outline of the remarks which it is more conventional to make here. To those logically minded readers who do, however, start to read at this point, an apology is offered for the repetition which they will find in the text.

This book is intended to meet the needs of laboratory workers, designers, and users of transformers in the light electrical industries. In such diverse fields as line and radio communication, radio-frequency heating, electronics, portable machine tools, electro-medical equipment, illumination, and measuring apparatus, many different types of transformers are required, and information about them has hitherto had to be obtained from a wide range of publications. It is hoped that this attempt to combine into one volume sufficient information for the usual needs of a designer will result in a considerable saving of time and effort.

In many textbooks on transformers the subject of leakage reactance is introduced at an early stage. Whilst this may be necessary in the study of large transformers where the reactance is a factor of major importance, it is in the writer's view wrong to introduce this rather involved subject before it is necessary to do so. If the usual methods of construction are followed, small 50-cycle power transformers can be designed quite satisfactorily without reference to magnetic leakage which has only a very small effect under the conditions normally applicable. Consideration of this aspect of the subject is accordingly postponed until Chapter VIII. On the other hand, the influence of the magnetizing current on the design is more important in small transformers than in large ones, and this matter is dealt with quite early; this is in contrast to the conventional treatment in which it is often only cursorily treated, and that only at a late stage. In the earlier chapters, attention is concentrated upon small power transformers; by Chapter VII it is considered that the reader should be able to deal with their design. That, of course, is not to say that at that stage he will have a complete grasp of all the factors, but he will gain confidence by tackling designing early on. So often, when taking up a new subject, the reader is obstructed by the completeness with which each aspect is treated before a general picture is obtained. In these pages it is intended to cover considerable ground in a quick sweep and afterwards to return to examine certain points in more detail. This principle has been adopted, even at the risk of an appearance of "patchiness," in order to avoid the discouragement which is so often felt by readers when each feature is exhaustively examined in turn.

Once the normal power-frequency type has been mastered special types are discussed, then audio-frequency transformers are considered. Here again a start is made with the easiest type—the output transformer—and only when these have become familiar are the effects of winding capacitance taken into account, as they must be, in the intervalve coupling types.

As regards the standard of knowledge of the reader, it is assumed that he has a general knowledge of Magnetism and
Electricity, including the use of the operator "j," and a workin
acquaintance with Calculus. For the sake of completeness
however, the early chapters give a rapid revision of the prin
ciples of electromagnetic induction, the meaning of self an
mutual inductance, and the magnetic properties of ferr
magnetic materials. Nevertheless it is by no means essenti
for the transformer designer to be capable of working out th
theoretical basis of all his methods. By accepting the result
developed in the mathematical sections he can produce reliab
designs by empirical methods—naturally, however, an und
standing of the underlying principles is a decided advantag

The author is greatly indebted to Murphy Radio, Ltd.,
permission to illustrate many sections with examples of tra
formers used by the Company, and to his colleagues in
Design Department, particularly Messrs. L. Driscoll
L. E. Salter, for providing test results and other data. T
Institute of Radio Engineers (U.S.A.) and the Institution
Electrical Engineers have kindly allowed data from t
papers to be extracted, and in all such cases the autho
quoted in the text. The Controller of H.M. Stationery Of
has also kindly agreed to the use of data on heat losses f

the report of one of the D.S.I.R. investigations. Messrs. Iliffe & Sons, Ltd., have generously allowed curves and other data to be taken from many of the articles in their publications, *The Wireless Engineer* and *The Wireless World*. The McGraw-Hill Publishing Co., Ltd., too, have kindly sanctioned the use of material from *Transient Electric Currents*, by H. H. Skilling.

Dr. B. Hague gladly gave his consent to the use of material from his excellent treatise on *Instrument Transformers*, and the author is very grateful to him.

Messrs. Johnson & Phillips, Ltd., have very kindly allowed many quotations to be made from their very comprehensive publication *The J. & P. Transformer Book*, and have provided illustrations on the heavy engineering side.

The author's grateful thanks are also due to the many other firms that have supplied information and illustrations, including Advance Components, Ltd., Richard Thomas & Baldwins, Ltd., British Thomson-Houston Co., Ltd., Ferranti, Ltd., London Electric Wire Co. and Smiths, Ltd., Magnetic & Electrical Alloys, Ltd., P. R. Mallory & Co., Inc., Newton & Wright, Ltd., Joseph Sankey & Sons, Ltd., Geo. L. Scott, Ltd., Standard Telephones & Cables, Ltd., the Telegraph Construction and Maintenance Co., Ltd., and Zenith Electric Co., Ltd.

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1. Power Transformers—2. Audio-transformers

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Table of Principal Symbols Used

SYMBOL	MEANING	FIRST USED ON PAGE
A	Constant	—
a	Area of cross-section	7
a_a	Cross-section of air-gap	413
B	Flux-density (Constant or r.m.s. value)	8
B_{max}	Flux-density (Peak value)	53
b	Radial width of a winding	173
C_p	Capacitance of primary	377
C_{ps}	Interwinding capacitance	377
C_s	Capacitance of secondary	377
d	Thickness	68
$\mathbf{E}$	Electromotive force vector	14
E	Electromotive force (r.m.s. value)	14
E_{max}	Electromotive force (Peak value)	14
E_e	Eddy-current e.m.f.	68
E_p	Induced e.m.f. in primary	19
E_s	Induced e.m.f. in secondary	42
e	Instantaneous value of e.m.f.	14
e	Thermal emissivity	117
f	Frequency	14
H	Magnetizing force	8
$H_{D.C.}$	Polarizing force	393
$H_{r.m.s.}$	r.m.s. value of magnetizing force	60
h	Height	68
$\mathbf{I}$	Current vector	14
I	Current (Constant or r.m.s. value)	14
I_{max}	Current (Peak value)	14
I_{mean}	Mean value of rectified current	250
I_B	Current in burden	344
I_D	Primary current at maximum efficiency	75
I_F	Primary current at full load	75
I_L	Current in load	41
I_m	Magnetizing component of exciting current	43
I_0	Exciting current	43
I_p	Primary current	45
I_s	Secondary current	45
I_w	Loss component of exciting current	43
i	Instantaneous value of current	14
j	$\sqrt{-1}$	157
K	Constant	—
k	Coefficient of coupling	158
k	Fractional contact time of vibrator	334

SYMBOL	MEANING	FIRST USED ON PAGE
k	Dielectric constant	437
k	Thermal conductivity	124
L	Self-inductance	20
L_{lp}	Leakage inductance of primary with respect to secondary	162
L_{Lp}	Leakage inductance of transformer referred to primary	383
L_{Ls}	Leakage inductance of transformer referred to secondary	436
L_{ls}	Leakage inductance of secondary with respect to primary	162
L_m	Equivalent shunt inductance (primary)	42
L_p	Primary inductance	156
L_s	Secondary inductance	156
L_{sc}	Short-circuit inductance of transformer	158
l	Magnetic length	7
l	Length of coil	173
l_a	Length of air-gap	413
M	Mutual inductance	156
m	Mean turn length of winding	173
n	Turns ratio (Primary/Secondary)	46
R	Resistance	7
R_a	Anode resistance of valve	373
R_B	Resistance of burden	345
R_L	Resistance of load	46
R_p	Resistance of primary winding	17
R_s	Resistance of secondary winding	43
R_w	No-load loss resistance	42
S	Reluctance	6
r	Radius	—
T	Turns on coil	9
T_p	Turns on primary	40
T_s	Turns on secondary	40
t	Time	14
V	Volume	411
V	Potential difference vector	15
V	Potential difference (r.m.s. value)	15
V_{max}	Potential difference (Peak value)	15
V_{mean}	Mean value of rectified voltage	250
V_p	Primary terminal P.D.	15
V_s	Secondary terminal P.D.	15
W	Energy loss	67
W_C	Heat loss by convection	118
W_c	Copper loss of transformer	73
$W_{D.C.}$	D.C. output of rectifier circuit	251
W_f	Iron loss of transformer	73

Symbol	Meaning	First Used on Page
W_r	Heat loss by radiation	117
X	Reactance	20
X_{eqp}	Equivalent reactance of transformer referred to primary	161
X_{eqs}	Equivalent reactance of transformer referred to secondary	161
X_{lp}	Leakage reactance of primary with respect to secondary	161
X_{ls}	Leakage reactance of secondary with respect to primary	161
X_p	Reactance of primary winding	157
X_s	Reactance of secondary winding	157
X_{scp}	Short-circuit reactance referred to primary	162
x	Distance	68
x	Enlargement factor	56
Z	Impedance	21
Z_{scp}	Short-circuit impedance referred to primary	157

GREEK LETTERS

Symbol	Meaning	First Used on Page
β	Function of fractional contact time	334
Δ	A finite change in . . .	—
δ	An infinitesimal change in . . .	—
ε	Base of Napierian logarithms	—
θ	Angle between two vectors	14
θ	Absolute temperature	117
μ	Permeability	7
μ	Amplification factor of a thermionic valve	377
μ_i	Incremental permeability	390
μ_0	Initial permeability	428
π	3·14159	—
ρ	Specific resistivity	68
σ	Stephan's constant of radiation	117
σ	Leakage coefficient	162
Φ	Total flux (Constant or r.m.s. value)	7
Φ_l	Leakage flux	165
Φ_M	Mutual flux	165
Φ_{max}	Total flux (Peak value)	14
ϕ	Instantaneous value of flux	14
ψ	Phase angle between voltage and current vectors	21
ω	Pulsatance ($= 2\pi f$)	15

CHAPTER I

Fundamental Principles

ALTERNATING-CURRENT supply systems are now so rapidly replacing direct-current schemes that in a few years there will be no direct-current distribution systems of any size left in Great Britain. This changeover is, of course, due fundamentally to the ease with which A.C. can be stepped up or down in voltage, so greatly facilitating the transmission of high powers over considerable lengths of line. The piece of equipment which makes this possible and in fact contributes more than any other to the success of the distribution system is the static transformer, a device with no moving parts and capable of far higher efficiencies than any other piece of electrical machinery.

A static transformer, or, as it is normally called simply a transformer, is a device by which power supplied at one pressure or voltage is converted to another pressure. **Transformers can operate only on an alternating or, at least, a fluctuating supply; an attempt to work a transformer from a steady d.c. supply would be unsuccessful and would damage the transformer.**

Transformers are made in sizes ranging from the tiny coupling types used in deaf-aid and portable radio equipments to the enormous ones seen in power distribution schemes. In the home small transformers are used for bell-ringing, low-voltage night-light operation, running model railways, etc., where a supply at 4–6 volts is required. Radio equipment often contains several types of transformers which are required to perform widely different tasks; a *power* transformer has to supply appreciable currents at several voltages for valve heaters, anode current, etc.; an *intervalve* transformer working from a high impedance source has to maintain a practically constant output to input ratio at all frequencies in the audible range or higher without, however, usually delivering any power, whilst an *output* transformer also working from a source of appreciable internal resistance has to supply constant power to a loudspeaker or similar load. Telephone systems utilize large numbers of transformers for purposes similar to those in radio sets, but they are known as induction coils, repeating coils, etc., and their construction often follows methods established some time ago; standardization of mountings and such details with apparatus already existing in large quantities has prevented the adoption of newer technique.

Transformers are required for the operation of most luminous discharge tubes such as the neon signs so popular for advertising purposes, and for X-ray equipments of all kinds. The advantages of easy transformation from one voltage to another are well illustrated in these applications, for the production and maintenance of a d.c. generator for these purposes would be wellnigh impossible. In heavy electrical engineering many transformers are used in addition to the obvious requirements for *power* transformers to step up voltages to transmission lines and for *distribution* transformers to step down at the far end to safe voltages for consumers. The system uses large numbers of *instrument* transformers which enable high voltages and heavy currents to be measured by instruments at a distant point without taking the main cables there and without exposing the attendants to risk of shock from the high voltages. The protection circuits of power systems also use many instrument type transformers for relay operation. In the laboratory, transformers are frequently required for feeding rectifiers for small d.c. supplies, for regulating and adjusting voltages, and for special applications, as, for example, the production of peaky or flattened wave-forms.

Apart from the many uses to which they are put for voltage-changing, transformers are sometimes needed for isolating purposes, as, for example, in electro-medical work for the safety of the patient in the event of his accidentally touching an earthed point, and in communications equipment.

These transformers take many different forms according to the work they have to do, but the basic features are always recognizable, namely a *primary* or input winding which is connected to the supply, a *core* of magnetic material around which the primary is wound, and a *secondary* winding from which the output is taken. also wound on the core. There is no metallic connexion between the input and output sides—

power is transferred from one to the other solely by electromagnetic induction. The two *electrical* circuits on the input and output sides are linked by the *magnetic* field in the core. In theory a core of magnetic material is not essential, and indeed at high frequencies is omitted in so-called "air-cored" transformers, but at low frequencies the omission of an iron core would so increase the dimensions and reduce the efficiency that its use can for practical purposes be regarded as essential.

The engineer engaged in the light electrical industries has frequently to employ transformers in his work, and a clear understanding of the basic principles of their operation is most important if he is to obtain the desired results from them. The actual design of suitable transformers is often considered a job for the specialist; whilst this is true of some of the more involved types, it is often a great time-saver in experimental work to be able to run off a design oneself, and have it made on the spot. It is hoped in these pages to build up from first principles a sound working knowledge of the small transformers (up to a few kVA) in frequent use in the communications and in electronic and allied industries. At the same time all mention of larger transformers has not been excluded, because it is felt that to do so would convey the impression that they were a distinct species, if not in theory at least in practice. Many electrical engineers who are quite prepared to undertake the production of transformers rated at a few hundreds of volt-amperes feel at a loss as soon as the more ambitious letters kVA appear. To employ exactly the same figures for working densities of iron and copper and for cooling calculations in the larger sizes would, of course, be unwise, but this aspect has been covered by noting always the trend of such figures with increasing size. For example, the influence of the magnitude of the magnetizing current on the permissible flux-density is emphasized in several places, and again the necessity for different forms of cooling is brought to the reader's notice although it is unlikely that he will ever have occasion to digress beyond the simpler forms of oil cooling. By this means it is hoped that the reader will be in the position of being able to *interpolate* rather than having to *extrapolate* the data given.

It is not only to give the designer confidence in extending his activities, however, that references are made to types of transformers which might be considered outside the scope of light electrical engineering. Communications and electronic engineering workers often have to face problems of interference due to heavy electrical machinery, and unless they can appreciate the difficulties and understand the reasons for the troubles, they cannot hope to collaborate successfully with their colleagues on the power distribution side to find a solution of the problem. For that reason the subjects of third harmonic in three-phase transformers, lighting surges, and on-load tap-changing have been dealt with in more detail than might at first appear necessary.

As it is felt that information cannot be properly assimilated unless it is put to some use, the object has been to put the reader at an early stage in the position of being able to design small power transformers. These are the simplest variety, because the frequency and input voltage are both fixed, and the frequency is low enough to allow the effects of capacitance and magnetic leakage to be neglected. A knowledge of leakage reactances or harmonic distortion is not necessary at this stage, and no better transformers would be produced by taking such factors into account. The design of these types is accordingly dealt with in Chapter VII. When the simplest types have been mastered, the subject of leakage reactance is introduced and so on.

An essential requirement for a proper understanding of transformers is a working knowledge of the properties of a magnetic field, and this accordingly forms the starting point of our explanation.

1. The Magnetic Field

It has been known for a very long time indeed that if a piece of a certain iron ore known as magnetite is suspended freely by a thread, it sets itself so that it always lies in the same direction. An arrangement of this kind was known as a "lodestone" or leading stone. Examination of such a piece of magnetite shows that there are two small areas at opposite ends, which have the property of attracting small pieces of iron or steel. These areas are known as *poles* and the stone sets itself so that the line joining the poles lies in a North and South direction. The two poles are not identical in their behaviour in that a particular one always moves northwards whilst the other goes to the south. They are accordingly known as North-seeking and South-seeking poles or simply N- and S-poles.

In later years it was found that a strip of steel could be so treated or magnetized as to possess the properties of a lodestone in a much more intense form, and a magnetized strip of steel suspended on a pivot is now known as a compass needle. Experiments with two compass needles show that whilst both N- and S-poles attract iron filings, only a N-pole will attract a S-pole or vice versa; if two N-poles or two S-poles are brought together, repulsion takes place.

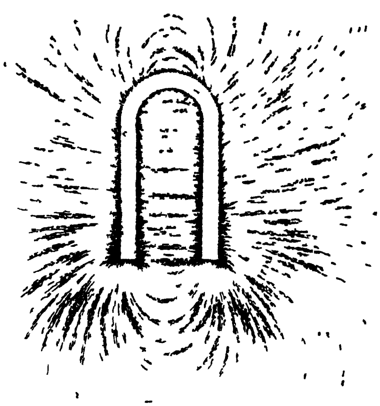

FIG. 1

The Shape of the Magnetic Field Surrounding a Magnet can be Indicated by Sprinkling Iron-filings on a Sheet of Card Placed over it

To explain these and other facts, it has been found convenient to postulate that every magnet is surrounded by a magnetic field of its own. The movements of a compass needle when a magnet is placed near it are accordingly attributed to the action of the field of the magnet upon it, and its directional properties to the action of the earth's magnetic field. If a permanent magnet, such as the horseshoe-shaped magnet used in a car magneto, is placed on the table, a piece of paper laid on top, and fine iron filings dusted on to the paper, it will be found that the filings take up positions in curved lines between the two poles near the ends of the magnet (Fig. 1). The chains of filings indicate the directions of the *lines of force* which make up the magnetic field near the magnet. Theoretically the field is of infinite extent but in practice it rapidly falls off as the distance from the poles increases and can for most purposes be regarded as negligible at a distance of a few feet from any small magnet.

The lines of force shown up by the filings are actually the paths along which an isolated N-pole would move if free to do so. An isolated magnetic pole is a theoretical conception which cannot be realized in practice, but the N-pole of a long thin magnet, such as a magnetized knitting-needle, approaches this ideal. The isolated N-pole placed in a field experiences a force driving it away from the N-pole of the magnet and towards the S-pole. Lines of force accordingly have *sense* as well as direction and this is usually indicated by arrows in a map of a magnetic field.

The relative strength of the field at different points is shown in a map by the closeness of the lines which crowd together at the strongest parts of the field and separate widely when it is weak. Lines of force are not, of course, confined to any one plane but permeate all space. The field is thus three-dimensional although only two dimensions are shown in a normal map. If a section is taken through the field (i.e. at right angles to the map) at any point, a greater or smaller number of lines will pass through the area according to the field strength which is expressed in "lines per square centimetre" or in "gauss."

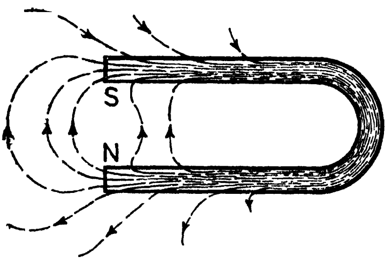

FIG. 2

Although the Lines of Force are only Evident in the Air-space Outside the Magnet, they are Considered to Complete their Circuit from S. to N. Inside the Steel.

Magnetic Circuits

Lines of force are not confined to the air but actually form complete loops returning through the material of the magnet to their starting points (Fig. 2). There is thus a "Magnetic Circuit" corresponding in many ways to an Electrical Circuit, and certain materials, notably iron and steel, are said to offer a low *reluctance* to the passage of the magnetic flux. Reluctance (S) thus corresponds to resistance in an electric circuit. Equivalent to an Electromotive Force (e.m.f. or voltage) there is the Magnetomotive Force or m.m.f. which can be regarded as the driving force which causes a magnetic *flux* (corresponding to the electric current) round the magnetic path. The value of the magnetic flux, usually denoted by Φ, is calculated by a law analogous to Ohm's Law—

$$\Phi = \frac{\text{m.m.f.}}{S} \quad . \quad . \quad . \quad . \quad (1)$$

Just as electrical resistance depends on the length, cross-section and nature of the material of which a body is made so the reluctance S is given by—

$$S = \frac{\text{length}}{\text{area}} \times \frac{1}{\text{permeability}} = \frac{l}{a} \times \frac{1}{\mu} \quad . \quad (2)$$

$$\left(\text{cf. } R = \frac{\text{length}}{\text{area}} \times \frac{1}{\text{conductivity}}\right)$$

The magnetic *permeability* and electrical conductivity are both properties of a material but, whereas the specific conductivity of copper is about 10^{20} times greater than that of an insulator such as ebonite, there are much smaller variations between the highest and lowest permeabilities. There is accordingly no such thing as a "flux-insulator." The permeability of a vacuum is taken to be unity but that of any other "non-magnetic" material such as air, wood, copper, etc., is so nearly the same that for practical purposes $\mu = 1$ for them too. Only iron, steel, and certain recently discovered alloys have values of permeability much greater than 1; under favourable conditions a figure as high as 100 000 can be reached for specially treated nickel alloys such as Mu-metal. If the magnetic circuit could be entirely restricted to a path in this material, the flux for a given m.m.f. would then be 100 000 times as great as it would be in air.

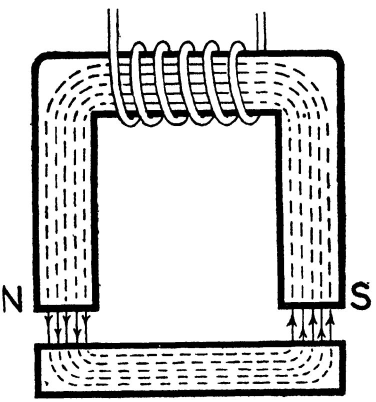

FIG. 3

A Magnetic Circuit May Pass through a Number of Different Materials of Varying Cross-section. The m.m.f. for the Whole Circuit is Equal to the Sum of the m.m.f.s for the Separate Portions.

A magnetic circuit need not be uniform in section or material throughout (Fig. 3). If it can be regarded as being composed of a number of sections each homogeneous, then the total reluctance is equal to the sum of the separate reluctance of each section.

$$S = \frac{\text{m.m.f.}}{\Phi} = \frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3} \quad . \quad . \quad (3)$$

The flux-density (i.e. number of lines per cm.^{2} of cross-section) in a material is a quantity of special importance in transformer work and the symbol B is always used to represent it. The total flux Φ is of course equal to $B \times a$.

For reference, the magnetic quantities with their symbols and units of measurement are given in Table I, together with the corresponding electrical quantities.

TABLE I

CORRESPONDING MAGNETIC AND ELECTRICAL UNITS

MAGNETIC		ELECTRICAL	
Quantity	*C.G.S. Unit*	*Quantity*	*Practical Unit*
Magnetomotive Force	Gilbert	Electromotive Force (E)	Volt
Reluctance (S)		Resistance (R)	Ohm
Flux	Maxwell	Current (I)	Ampere
Flux-density (B)	Gauss	Current-density	
Magnetizing Force (H)	Oersted	Potential Gradient	

When comparing the magnetic units with the corresponding electrical ones it is well to bear in mind that whereas the electrical quantities are nearly always expressed in practical units, the magnetic units in general use are the c.g.s. ones. Practical magnetic units have been specified but they have not been generally accepted by engineers. In this book practical *electrical* units will always be used, but magnetic quantities will be expressed in theoretical (c.g.s.) terms. The calculation of magnetic quantities from electrical data and vice versa will accordingly involve conversion factors between the practical and c.g.s. electrical units. Fortunately these conversions are required only in the case of two quantities, namely current and potential.

The factors in these cases are—

1 ampere = 0·1 c.g.s. units of current

1 volt = 10^8 c.g.s. units of potential (on the electromagnetic system)

2. THE FIELD PRODUCED BY AN ELECTRIC CURRENT

The m.m.f. has so far been regarded as a driving force residing in the permanent magnet being examined, but a m.m.f. is also frequently produced by an electric current. This happens whenever a magnetic circuit is linked to an electric circuit carrying a current, as, for example, when current is passed through a turn of wire wound round an iron core.

The m.m.f. in any magnetic system is defined as the work done in taking a unit pole round the magnetic circuit, and the units are called *Gilberts.* It thus corresponds to the e.m.f., which is the work done in taking a unit charge round an electric circuit. A unit pole in the c.g.s. system gives rise to 4π lines, that is to say, if the pole is imagined to be placed within a sphere of 1 cm radius (and therefore having a surface area of 4π cm^2) there will be 1 line per cm^2 at the surface of the sphere. In the case of a current circulating round an iron core, i.e. linked to a magnetic circuit, the 4π lines from the unit pole will cut the electric circuit, and if the current in the circuit is I c.g.s. units, the work done will be $4\pi I$ ergs, so that the m.m.f. is $4\pi I$ gilberts. This is a most important result, because it is the basis of all electromagnetic calculations. If there are T turns linking the magnetic circuit, and the current is expressed in amperes instead of c.g.s. units the formula becomes—

$$\text{m.m.f.} = \frac{4\pi IT}{10} \text{ gilberts}$$

Hence
$$\Phi = \frac{\text{m.m.f.}}{S} = \frac{4\pi IT\mu a}{10l}$$

i.e.
$$B = \frac{4\pi IT}{10l} \times \mu \quad . \quad . \quad . \quad . \quad (4)$$

The quantity $\frac{4\pi IT}{10l}$ (m.m.f. per unit length) is given a special name, the *Magnetizing Force,* and it is always denoted by the symbol H. The units in which it is expressed are *Oersteds.*

Hence
$$B = \mu \times H \text{ or } \mu = \frac{B}{H}$$

In air and non-magnetic materials generally, where $\mu = 1$, it follows that $B = H$. In iron and steel μ is much higher, but unfortunately it is not constant, a fact which makes most calculations on iron-cored apparatus difficult. In practice, data for most design work are obtained from curves published by manufacturers of the magnetic materials.

Applying the expressions for the values of B, H, I, etc., to the transformer, it will be evident that if a core of soft iron is wound with a given number of turns of wire and a known current is passed through it, the field in the core can be calculated. In order to do so it is, of course, necessary to know the permeability of the iron under the particular conditions of operation, and the dimensions of the core.

3. Electromagnetic Induction

The transformer has been described in an earlier paragraph as a device in which two electrical circuits are linked by a magnetic one. The first half of the magnetic link has now been forged—currents in the input circuit have been shown to produce a magnetic effect of calculable strength in the core—it now remains to show how the magnetic field produces electric currents in the output circuit. It might have been expected that this process would have been simply a reversal of the first one, that is to say when a magnetic field linked a conductor, a current would be produced in it, but in fact there is a very important distinction between the two processes. *An e.m.f. is induced in a conductor by a magnetic field only when the lines of force cut through the conductor*, or regarding the process slightly more generally, when the number of linkages of the field with the circuit is changing. The magnitude of the e.m.f. is proportional to the rate of change of these linkages. This phenomenon, known as electromagnetic induction, was first observed by Faraday in 1831. The effects of his discovery have been so far-reaching that it can be said that the whole structure of electrical engineering as we know it to-day is dependent on it, for not only is the present object of our studies, the static transformer, intimately connected with Faraday's work but the generation of electricity by mechanical means follows directly from it. Previously all electrical power had been generated by primary cells and the cost was so high that it could not be put to any practical use. Even to-day electrical power from a dry battery costs about £4 per unit as compared with ½d. from steam-generated supplies.

The fact that an e.m.f. is only generated when the field linkages are changing, means that a transformer is necessarily an alternating or intermittent current device. It cannot operate (and will usually be damaged) if connected to a d.c. supply. In passing, it should be noted that Faraday's Law states that an e.m.f. is induced; whether or not a *current* flows depends on the completion of the output circuit. A current in this case is only an unessential result, whereas it was the primary and essential cause of the production of the field from the input side. The e.m.f. there was only incidental, as a means of forcing a current through the primary coils.

It will be apparent from Faraday's Law that if an output or secondary winding is also wound on the core of the electro-magnet, as in Fig. 4, an e.m.f. will be generated in it whenever the current in the primary, or input coil (and hence the magnetic field in the core) is varied. Other factors being equal, the e.m.f. generated will be proportional to the number of turns in the secondary, so that the transformer may be either step-up or step-down. The direction of the e.m.f. depends on the direction of movement of the lines of force; when the field is increasing it will be in one direction, when the field is decreasing it will be in the other. The secondary e.m.f. is thus always an alternating one, even if the primary current although intermittent is still unidirectional.

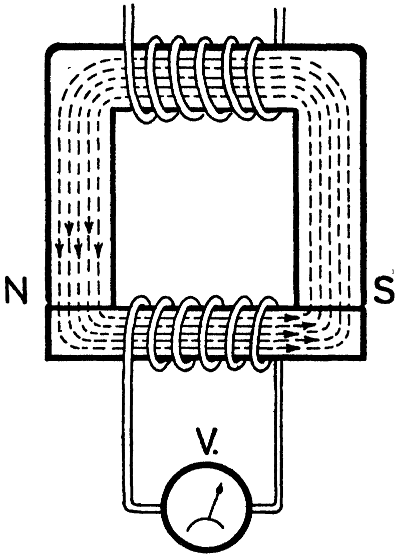

Fig. 4

The Essential Parts of a Transformer. Whenever the Current in the Upper (Primary) Winding Changes, an e.m.f. is Induced in the Lower (Secondary) Winding.

The direction of the induced e.m.f. relative to the primary current can be obtained by the application of Lenz's Law. Any change that takes place in the linkages between primary and secondary tends, according to that law, to be followed by the flow of current in the secondary in such a direction as to restore the linkages to the original state. If, for example, the primary current is *decreased*, an e.m.f. is induced in such a direction that it would, if the circuit were complete, cause a current to flow in the same direction as that in the primary. This would, of course, produce a flux in the *same* direction as the primary flux and so tend to restore the number of linkages to the original value. This direction is taken to be the positive direction of the induced e.m.f. The fact that a *decrease* in flux linkages produces an e.m.f. in the positive direction, introduces a negative sign into the equation relating these quantities.

An example of a simple transformer working on these lines is the spark coil formerly used in wireless transmitters and still in everyday use as an ignition coil for petrol engines. Here the primary circuit is interrupted at regular intervals by the contact-breaker and on each occasion the magnetic field of the primary coil collapses, cutting through the many turns of the secondary and inducing a very high voltage. In practice the two coils are wound concentrically on a straight core and not on opposite limbs of a closed core.

When appreciable powers are being handled, a contact-breaker cannot interrupt the current satisfactorily and so practically all transformers are operated from alternating current instead of from intermittent direct current.

Alternating Currents

An alternating current is one that starts from zero at a given instant, increases to a maximum, decreases again to zero, rises to a maximum again in the reverse direction, falls to zero again and then repeats the same behaviour again. The interval required to complete one such operation is known as a *cycle* or *period* of the alternating current, and the number of cycles per second is called the frequency. The type of alternating-current wave that can be most easily dealt with theoretically and the one that is as far as possible always used for supply purposes, has the shape which is generated in a loop of wire rotating

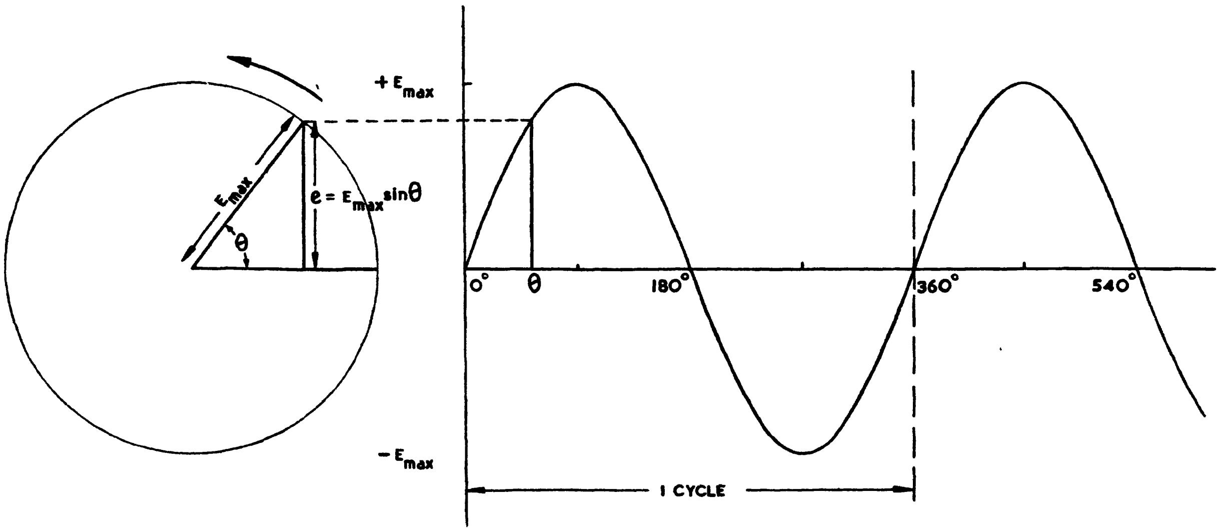

FIG. 5
Vector and Wave-diagrams of a Sinusoidal Alternating Voltage. The Magnitude (e) of the e.m.f. at Any Instant is Given by $E_{max} \sin \theta$

at a constant angular velocity in a uniform magnetic field (Fig. 5). This shape is known as a sine wave, and such currents are said to be *sinusoidal*. Quantities such as current or voltage that vary in a sinusoidal manner can be represented by a *vector* rotating at a constant velocity such that one revolution represents one cycle. The length of the vector is chosen to represent the maximum or peak value of the current (or voltage). The value of the quantity at any instant is then represented by the length of the projection of the vector on the vertical axis; this will have a value equal to the peak value for the quantity multiplied by the sine of the angle the vector makes with the zero reference direction. The instantaneous value will thus vary between zero and the peak value (positive or negative), since the limits of the sine function are 0 and $\pm$ 1. The value of the angle θ at any instant t will be $2\pi ft$ radians, since the vector rotates f revolutions per second and so moves through $2\pi ft$ radians. Vectors representing complex quantities (which require to be specified both in magnitude and phase) such as Current, Voltage, etc., are written as Roman capitals, I, E, whilst italic capitals E, I, indicate their magnitudes only (r.m.s. values, see below). Instantaneous values of quantities are denoted by small letters, e.g. i, e, ϕ, etc., peak values are shown with a suffix *max* thus: I_{max}, E_{max}, Φ_{max}. Since an alternating current or voltage is varying all the time, it is not at once evident how to describe its magnitude—a possible way would be to quote the peak value, but an alternating current having a peak value of say 1 ampere would obviously be less effective than 1 ampere D.C. The normal method of describing the magnitude of A.C. is to say that it has a *virtual* (i.e. effective) value of 1 ampere when its heating effect is equal to that of 1 ampere D.C. The peak value is then considerably more than 1 ampere, the actual value depending on the wave-form; for a sinusoidal current the peak value would be $\sqrt{2} = 1{\cdot}414$ amperes. The virtual value of any shape of wave can be calculated by finding first of all the average heating effect over one complete cycle when passed through a resistance of 1 ohm. This is done graphically by dividing the cycle into a number of equal intervals, squaring the current ordinate at the middle of each interval, adding together the $(\text{current})^2$ values and dividing by the number of intervals. The virtual value of the current is then obtained by taking the square root of this average value. The virtual value is often called the r.m.s. (root-mean-square) value from the method of its derivation. R.m.s. values are denoted by capital letters thus, I, E, etc.

$$\text{By definition } I = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 \ldots i_n^2}{n}}$$

In this book V is used to represent a terminal voltage, with suffixes p or s for primary or secondary, as distinct from E, which is used for the *induced e.m.f.* If I_{max} is the peak value of an alternating current, the instantaneous value i at a time t is therefore given by

$$\begin{aligned} i &= I_{max} \sin 2\pi f t \\ &= I_{max} \sin \omega t \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

where ω is the angular velocity in radians per second and is equal to $2\pi f$.

If an alternating current is passed through the primary of a transformer, it is accompanied by corresponding flux changes in the iron, and an alternating voltage is induced in the secondary, *of the same frequency as that of the supply to the primary.* The output voltage can, however, be varied at will by altering the number of turns on the secondary as compared with the primary. This result can be obtained more precisely by mathematical methods. The value of a sinusoidal alternating current at any instant can be represented by the expression $I_{max} \sin \omega t$.

If it is assumed that the permeability of the iron is constant (i.e. there is no tendency to saturation), the flux Φ will also follow a similar manner of fluctuation so that the instantaneous value of the flux is given by

$$\phi = \Phi_{max} \sin \omega t$$

Faraday's Law states that the induced e.m.f. e is proportional to the rate of change of flux linkages. The number of linkages will be equal to the product of the total flux with the number of turns on the coil so that, remembering that a positive e.m.f.

is produced by a decrease in the linkages, Faraday's Law can be stated thus—

$$\begin{aligned} e &= -\frac{d}{dt}(T\phi) \\ &= -T\frac{d}{dt}(\Phi_{max}\sin\omega t) \text{ c.g.s. units} \\ &= -T\frac{d}{dt}(\Phi_{max}\sin\omega t) \times 10^{-8} \text{ volts} \\ &= -T\omega\Phi_{max}\cos\omega t \times 10^{-8} \text{ volts} \\ &= T\omega\Phi_{max}\sin\left(\omega t - \frac{\pi}{2}\right) \times 10^{-8} \text{ volts} \quad . \quad . \quad (6) \end{aligned}$$

i.e. $$e = E_{max}\sin\left(\omega t - \frac{\pi}{2}\right) \quad . \quad . \quad . \quad . \quad (7)$$

where E_{max} is the peak value of the e.m.f.

The induced e.m.f. is thus shown to be sinusoidal in form, and lagging 90° behind the flux wave produced by the magnetizing current.

The Primary Back E.M.F.

A most important feature in the working of a transformer is the reaction of the magnetic field back on to the *primary.* When the field is collapsing and the lines of force are cutting through the secondary turns, so inducing the secondary voltage, the primary turns are also affected in the same way, so that a back e.m.f. is set up in the primary and it is in opposition to the mains voltage.

Readers may find it a little difficult to understand how this back e.m.f. can appear, so let us imagine that the primary winding has a "shadow" winding close beside it, and wound on with it, as in Fig. 6.

Regarding this shadow winding as a secondary, it will have an e.m.f. induced in it according to the laws described above. Since the shadow follows the same path as the primary throughout, we could (neglecting the small voltage drop due to its resistance) join the "shadow" to its "substance" at every point along its length without altering the voltages set up.

The primary can thus be seen to have an e.m.f. induced in it, in opposition to the applied voltage, and the consequence is that the primary current is not determined by the expression *Applied voltage/Primary resistance*, but is much smaller than this. This fact is expressed in symbol form by the following equation representing the relationship between the instantaneous applied voltage, resistance voltage drop, and induced back e.m.f.—

$$v = R_p i + (-e_p). \qquad (8)$$

Put into words, this means that the voltage v, which must be applied to the primary at any instant in order to produce a current i, is equal to the voltage drop due to the resistance of the winding added to the back e.m.f. induced in the primary by the varying flux. In practice, of course, the applied voltage is nearly always a given quantity, so that the real significance of equation 8 is that the back e.m.f. always assumes such values that it is at every instant equal and opposite to that of the supply voltage, less a small quantity due to the voltage drop in the winding resistance. As this drop can usually be neglected, the primary back e.m.f. is thus seen to be equal and opposite to the applied voltage at every instant, and therefore to be sinusoidal in shape (if the supply is sinusoidal). It is most important to realize at an early stage this fundamental feature in the working of a transformer, that the induced or back e.m.f. is for all practical purposes fixed by the applied voltage and that it is therefore unaffected by changes in the

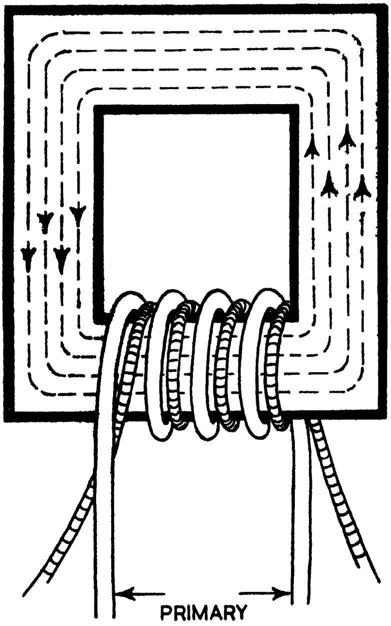

Fig. 6

A "Shadow" Winding Beside the Primary Helps to Introduce the Rather Difficult Idea of an e.m.f. Being Induced in the Primary Whilst it is Actually Connected to the Mains.

shape, size, or material of the core or by the secondary load (if any). Going a step farther, since the flux in conjunction with the number of turns, decides the back e.m.f., and that is fixed by the applied voltage, so the flux too is independent of the core characteristics and the load. If the applied voltage is sinusoidal, the flux variation is also sinusoidal. The quantity which does, however, vary with changes in the core conditions, etc., is the current; this varies in such a way that the required flux is generated whatever happens to the internal or external conditions. If, for instance, a core material of lower permeability is substituted for that in use, the flux would remain unaltered since it still produces a back e.m.f. equal and opposite to the applied voltage, but the current necessary to produce this flux would increase. Since the permeability of any ferromagnetic material varies with the degree of magnetization, it follows that the permeability will change over the cycle—the flux will nevertheless remain sinusoidal, but the current producing the flux will be distorted, becoming "peaky" as a disproportionate increase is required near the flux maxima.

The assumption made at an earlier stage in the argument (p. 15) that the flux is at every instant proportional to the current, although seldom true in practice, does not in fact invalidate the final results because in spite of the variations in permeability of the core over the cycle, the *flux* wave-form is sinusoidal. If the previous discussion is taken up at the point where a sinusoidal flux variation is written down, it will be seen to be perfectly rigid.

Throughout the theory of transformer operation it will be found that the subject is treated in such a way as to reduce to a minimum the need for taking into account in the calculations the effect of the non-linearity of the core material. The use of the principle of back e.m.f. is an important feature in this treatment.

Inductance

Although the conception of a back e.m.f. in the primary circuit limiting the current to low values is of fundamental importance, it is sometimes convenient to regard the matter in a rather different way. It must not, however, be forgotten that the effects of non-linearity in the core manifest themselves strongly in this latter method of treatment—the instantaneous value of the inductance of an iron-cored coil varies over the cycle of applied voltage. From this other point of view, the current in a transformer primary is considered to be reduced below its expected value not by a reduction in the effective voltage round the circuit, but by an increase in the apparent "resistance."

Faraday's Law indicates that e_p is proportional to the rate of change of linkages of the field with the primary winding,

$$e_p \propto -\frac{d\phi}{dt}$$

If it is assumed that the permeability of the iron is constant, ϕ is proportional to i, so that

$$e_p \propto -\frac{di}{dt}$$

$$\therefore \qquad e_p = -L\frac{di}{dt} \text{ (where } L \text{ is a constant)} \qquad . \quad (9)$$

$$= -L\frac{d}{dt}(I_{max} \sin \omega t)$$

$$= -\omega L I_{max} \cos \omega t = \omega L I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Hence $E_{p\,max} = \omega L I_{max}$ (the current maximum occurring $\frac{\pi}{2}$ earlier than the voltage maximum)

$$\therefore \qquad I_{max} = \frac{E_{p\,max}}{\omega L}$$

or, using r.m.s. values,

$$I = \frac{E}{\omega L} \qquad . \quad . \quad . \quad . \quad (10)$$

Since the resistance has been neglected, the applied voltage V is equal to E numerically, although opposite in phase. Equation 10 therefore gives the r.m.s. value of the current in the circuit when a voltage V is applied.

The primary no-load current can thus be calculated from Ohm's Law provided that the resistance is replaced by a quantity known as the *reactance* of the circuit. The reactance, like the resistance, is measured in ohms and is denoted by the symbol X. The constant L has a special name—the *Coefficient of Self Inductance*, or more frequently simply the *Inductance.* In the practical system of units the inductance is measured in henrys, and the definition is based on equation 9: "a circuit is said to have an inductance of 1 henry if an e.m.f. of 1 volt is generated in it when the current through it is changing at the rate of 1 ampere per second." Since

$$e_p = -L\frac{di}{dt} \text{ and } e_p = -\frac{d}{dt}(T\phi) \text{ also,}$$

it follows that

$$L\frac{d}{dt}(i) = \frac{d}{dt}(T\phi) \qquad . \quad . \quad . \ (11)$$

Hence a circuit will have an inductance of 1 henry if the passage of 1 ampere through it produces 10^8 linkages between the flux and the circuit. (The factor of 10^8 arises in the conversion from c.g.s. units of potential to the practical units of volts. 1 volt $= 10^8$ c.g.s. units.) The inductance is thus a function only of the shape of the circuit and the permeability of the medium surrounding it; it is independent of the current flowing (except in so far as the permeability may be altered by the current).

The inductance of a coil with a closed magnetic circuit of known dimensions, such as the primary of a transformer, can be calculated on two assumptions—

(*a*) That there is no flux leakage from the core, i.e. that all the lines of force link all the turns of the coil.

(*b*) That the permeability of the core material is constant.

It has been shown that the inductance in henrys is numerically equal to the number of linkages between the coil turns and the flux when a current of 1 ampere is flowing through the coil. Now,

$$\begin{aligned} \Phi &= B \times a = H \times \mu \times a \\ &= \frac{4\pi IT}{10l} \times \mu a \text{ lines} \\ \therefore \quad L &= \Phi \times T \times 10^{-8} \text{ when } I = 1 \\ &= \frac{4\pi T^2 \mu a}{l} \times 10^{-9} \text{ henrys (dimensions being in cm)} \quad (12) \end{aligned}$$

The resistance drop in the primary has so far been neglected, but, if this has to be taken into account, it must not be forgotten that the virtual values of induced e.m.f. and resistance voltage drop cannot be added arithmetically because they are not in step or, as it is said, *in phase*. The voltage across the resistance is in phase with the current through it, but the induced e.m.f. lags 90° behind the current because the current is in phase with the flux and the e.m.f. lags 90° on the flux. In order to add together the two voltages it is necessary to use vectorial addition so that the conditions are thus shown in Fig. 7. The resultant voltage across the primary is represented by the hypotenuse of the right-angled triangle.

If each side of the triangle is divided by the same quantity I, the shape will remain unchanged and the two sides enclosing the right angle are then $R_p I/I = R_p$ and $E_p/I = \omega L$ respectively. The hypotenuse is V/I and this quantity is given the name of *impedance*. The impedance is denoted by the symbol Z, and like the resistance R and reactance X is measured in ohms. Since the R and X are always at right angles in a vector diagram, Z can be calculated from the relationship

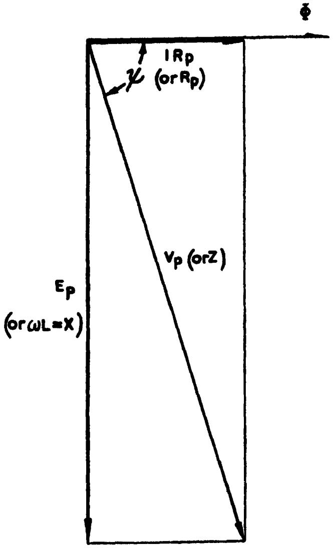

FIG. 7
Vector Diagram showing the Relations Between Induced Voltage, IR Drop, and Applied Voltage.

$$Z = \sqrt{R^2 + X^2} \qquad . \quad . \quad . \quad (13)$$

The *phase angle*, i.e. the angle by which the current vector lags or leads the voltage vector is denoted by ψ, and is evidently the angle between Z and R. Cos ψ is clearly R/Z and this quantity is known as the *power-factor*.

In a purely resistive circuit where the current is in phase with applied voltage ($\psi = 0$), the power dissipated is equal to $I^2R = VI$ watts. No power is dissipated in a pure inductance, however, because it returns to the circuit during one half of *the cycle all the energy it absorbed during the other half.* If, therefore, an inductance is connected in series with the resistance in the circuit, the power is thus only dissipated in the resistance so that it is still I^2R. This is no longer equal to the product of current and applied voltage, however, since the applied voltage must have been raised to maintain the original current in spite of the volts dropped across the inductance. The fraction by which the volt-ampere product must be multiplied to obtain the true watts is, as noted above, the *power-factor*, cos ψ.

i.e. $$VI \times \text{P.F.} = I^2R$$

$$\therefore \quad \text{P.F.} = \frac{IR}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$$= \cos \psi \quad . \quad . \quad . \quad . \quad (14)$$

Summarizing the facts of this section, the primary current of a transformer working without any load connected to its secondary, is much lower than would have been expected from its d.c. resistance. Of this small input, only a fraction represents a power consumption in watts.

To illustrate the magnitudes of the various quantities in a practical case, the following figures are given for a small transformer such as that used in a typical radio receiver operating from 230 volts 50 cycles.

Primary resistance = 28 ohms
No-load current = 0·05 amp
Power input = 3 watts
Hence Impedance (Z) = 4 600 ohms (cf. $R = 28$)
Volt-amperes = 11·5 (cf. watts = 3)

4. Operation on Load

We must now see what happens when a load, such as a resistance, is connected across the secondary winding, which we have so far assumed to be operating on open circuit. As would be expected, a current flows through the load, the magnitude of the current being given as usual by Ohm's Law.

Now, however, a very interesting reaction occurs. The secondary current flows not only through the external current but also through the secondary winding, and hence round the core in which it produces an additional magnetic field. The direction of the current in the secondary is such that the secondary field is in opposition to that set up by the primary, with the result that the field tends to be partially cancelled. But it has already been made clear that the flux is determined by the back e.m.f., which is in turn fixed by the applied voltage, so that the demagnetizing ampere-turns of the secondary cannot reduce the flux. What happens is that the primary current increases to such an extent that the resultant ampere-turns remain unaltered and the flux is accordingly unaffected.

For reasons which will be evident later on, it is often convenient to look at this flux cancellation effect of the secondary current on the main field in a rather different way, although the two pictures are really only different ways of describing the same result. Instead of saying that the primary current increases when the secondary load comes on, it is convenient to regard the magnetizing component of the primary current as continuing to flow unchanged, producing the normal flux in the iron core, and, superimposed on this current, is the "load" component of the primary current.

This latter component is of such a value that the ampere-turns it produces are exactly equal and opposite to the ampere-turns due to the current in the secondary winding. The loading up of the transformer output thus leaves the magnetic state of the iron unaffected. This is an important fact, as will be seen when the losses are analysed.

The purpose of this chapter has been to give a general picture of the principles on which a transformer depends for its operation, and some insight has been obtained into the mechanism by which it regulates the intake from the supply to meet the outgoings. In this way a relatively high efficiency is maintained even at light loads.

CHAPTER II

The Magnetic Properties of Iron and Steel

THE function of the core of a transformer as the link between the output and input circuits is so important a one that a detailed knowledge of its behaviour is necessary in order to choose the best operating conditions.

Of all the vast range of materials known to civilized man, very few exhibit magnetic properties to a sufficiently pronounced extent to enable them to be detected without instruments. Amongst the few materials which do show obvious magnetic effects, iron and its alloys assume a position of unchallenged supremacy. Iron and materials which resemble iron in its magnetic properties are said to be *ferromagnetic*; nickel and cobalt fall into this category.

The exceptional properties of iron are now believed to be due to a particular combination of electron-spin orientations in the atoms composing the crystal lattice of the material.(1, 2, 3) The theory of the origin of magnetic properties has not, however, so far played a very useful part in the development of magnetic materials, which have been produced entirely by empirical methods. Apart from the variations in magnetic properties due to changes in the chemical composition of a material, the previous metallurgical history of a specimen has a profound effect on its behaviour in a magnetic field. The most significant difference between annealed and quenched ferrous bodies lies in the ease or otherwise with which they can be magnetized or caused to give up their magnetism. Thus, steel hardened by quenching is difficult to magnetize, i.e. requires a high value of H to produce a given intensity of magnetization, but once magnetized retains its magnetism in spite of vibration, etc. Soft iron, on the other hand, acquires a considerable intensity of magnetization at low values of H, but very easily loses its magnetization when the magnetizing force is removed. The magnetic "hardness" is thus seen to be related to physical hardness, and theory attributes both to a distortion or state of strain in the crystal lattice. As the strain is relieved by annealing, so the material becomes softer mechanically and more easily magnetizable.

The phenomenon of magnetization of ferromagnetic materials can be broadly explained by assuming that the particles of which a magnetic substance such as iron is composed are themselves tiny magnets. The magnetic properties of these particles as individuals remain unaffected by outside magnetic influences, but their effect as a whole depends on their positions relative to each other. When a piece of iron or steel is in an "unmagnetized" state, the magnetic particles form closed groups or chains so that there is little external magnetic effect because each N-pole has a similar S-pole of another particle close to it, so neutralizing any effect at a small distance away. The process of magnetizing the bar by stroking with a permanent magnet always in the same direction has the effect of aligning the particles so that many of them lie with their axes along the axis of the bar. Although cancellation of the magnetic effect still takes place in the middle of the length of the bar, owing to N- and S-poles of adjacent particles neutralizing each other, there will be N and S magnetic properties apparent near the ends of the bar, where there are "free" N- or S-poles of the particles. The softer the iron, the more easily can the particles be turned into alignment, but they can also more easily turn back into closed chains as soon as the magnetizing influence is removed. This tendency to re-form internal chains is very marked, particularly in bars which are short compared with their diameter and such samples of iron retain very little magnetic strength. Hardened steel, on the other hand, owes its physical hardness to the great difficulty with which its particles can be moved relatively to each other, so that it is difficult to rearrange them into straight lines to secure external magnetic properties. Equally, if such an alignment is carried out, it tends to remain instead of collapsing into short chains if subjected to mechanical shock or disturbing magnetic fields.

Both soft and hard magnetic materials have important uses in electrical work, but for alternating current purposes a *soft* material is nearly always required. Now, as already mentioned, softness is associated with freedom from internal strains in the crystal lattice of which the material is built up. These strains appear as a displacement of the boundary atoms of the individual crystals so that the greatest degree of magnetic "softness" can be secured by increasing the individual crystal size (or grain size) as far as possible. It is not, however, practicable to increase the grain size above that corresponding to the existence of less than two or three crystals in the sheet thickness because it would then be very weak mechanically. (Fractures of cold materials always occur across the crystals and not along the inter-crystalline boundaries.)

Magnetization is usually accompanied by changes in the physical shape of the body; an iron rod when magnetized increases in length, an effect known as magneto-striction.

1. *B-H* CURVES

The magnetic behaviour of a particular grade of iron, described in general terms above, is expressed quantitatively by curves showing the relationship between the magnetizing force H and the resulting flux-density B in the specimen. It will be remembered that H is the magnetomotive force *per unit length* of path, whilst B is the number of lines per unit area of cross-section (usually per cm^2).

When taking measurements of magnetic properties, the magnetizing force H is always produced by passing a known electric current through a coil of known dimensions surrounding the specimen. In this way H can be calculated with a reasonably high degree of accuracy. One method of carrying out the plotting of a B-H curve, known as the Magnetometer method, is described here to illustrate the behaviour of "ferromagnetic" substances such as iron and steel.

The magnetizing coil or solenoid is a closely-wound coil having an axial length great compared with its diameter. It is set up with its axis magnetic E. and W. and a compass needle is placed on the axis at a distance large compared with its own length. The field is then assumed to be equal at the two ends of the needle, which will take up a position along the resultant of the earth's field and that due to the magnetic specimen which will be placed inside the solenoid. Before doing this, however, it is necessary so to adjust the position of a small coil connected in series with the magnetizing coil that it at all times cancels the field at the compass needle due to the direct action of the solenoid. Any disturbance of the needle is then due only to the specimen and not to the coil. The compensating arrangement is for simplicity omitted from Fig. 8, which shows the general set-up. Another precaution which must be taken in order to secure satisfactory results when using this method is to have the specimen under examination in the form of a long thin rod, and to have the magnetizing coil extending well beyond the ends of the rod. In this way the material is kept in a uniform field and also the uncertainty as to the position of the poles is kept small.

When the various compensating and zero adjustments have been made, the sample to be tested is placed inside the coil. Its dimensions, i.e. length and cross-sectional area, have already been carefully measured. Starting from the unmagnetized sample, which does not deflect the needle, we slowly increase the current through the coil, noting the deflexion of the needle at different values of current.

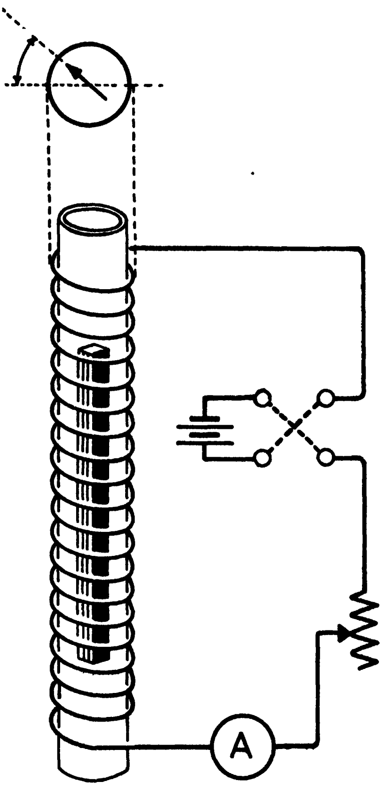

Fig. 8
The Magnetometer Method of Taking B-H Curves.

The deflexion will increase rapidly at first, then more slowly, until eventually no further increase in deflexion is observed, however much the current is increased. The iron is then said to be saturated. This is the end of stage 1.

Saturation is simply explained on the theory of magnetized particles by saying that it is the state in which all the particles have been turned into the line of the axis of the specimen. When this state is reached, no further increase in magnetizing force will produce any greater magnetization of the iron.

If the current is now decreased slowly to zero (stage 2), it will be found that the magnetic state of the iron does not correspond at any given current to that attained during the period of current increase; in fact, when zero is reached there is still an appreciable amount of residual magnetism.

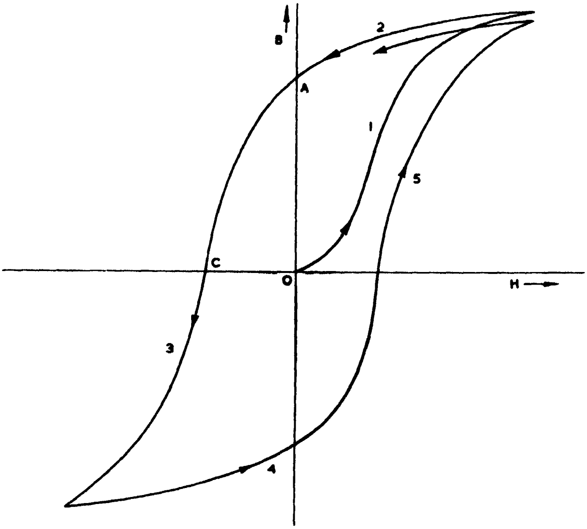

FIG. 9

The Five Stages in Taking a B-H Loop of an Initially Unmagnetized Specimen.

In order to demagnetize the specimen it is necessary to apply a magnetizing force in the reverse direction which can be done by reversing the current through the coil. A larger current in this reverse direction will succeed in overcoming and reversing the magnetic flux in the iron, as indicated by the deflexion of the compass needle in the opposite direction (stage 3).

Saturation will eventually be reached in the reverse direction, and the current is then reduced again to zero (stage 4), leaving a residual magnetization in the reverse direction. Once more reversing the current (i.e. allowing it to flow in the same direction as at first) will demagnetize and then magnetize in the original direction (stage 5).

The results of this experiment can be summarized in a *B*-*H* curve, values of *B* (the flux-density, i.e. lines per cm² cross-section) in the iron being calculated from the compass deflexions and *H* (magnetizing force) from the coil data and current flowing. The five stages described above are indicated by numbers 1 to 5 in Fig. 9. A repetition of these current variations will take the iron round practically the same curve again, although the first few loops obtained in this way are not quite closed ones, the maximum values of *B* becoming slightly smaller at each cycle. After a few repetitions, however, the iron settles down and closed cycles are obtained—the iron is then said to be in a cyclic state.

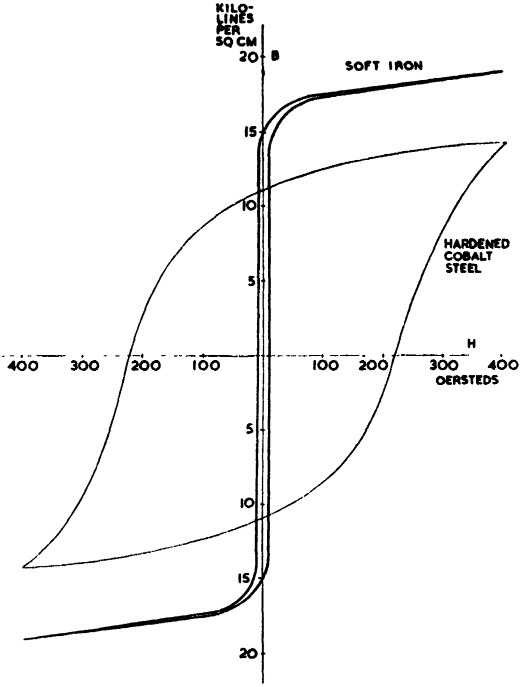

FIG. 10

The Extremes of Magnetic Softness and Hardness; Soft-iron and Hardened Steel as Used for Permanent Magnets.

The magnetometer method of measuring *B*-*H* curves has been selected for simplicity and ease of description. It does not, however, lend itself to particularly accurate results and in practice other methods are used.

The *B*-*H* curves (Fig. 10) for soft iron and hard steel show strikingly the difference between the properties of the two materials which can be taken as representatives of the extremes of magnetic substances. The curve for iron is narrow, whilst that for steel is wide and also lower, i.e. at saturation the flux-density is not so high as in iron. The value of H at which the curve crosses the horizontal axis (OC in Fig. 9) is known as the *coercive force*, i.e. the magnetizing force which must be applied in a reverse direction to a magnetized specimen in order just to destroy its magnetism. The value of B remaining after the magnetizing force has been reduced to zero (OA in Fig. 9) is known as the *residual induction.* In the particular case in which the sample has previously subjected to a magnetizing force sufficiently large to cause saturation the terms *coercivity* and *retentivity* are used.

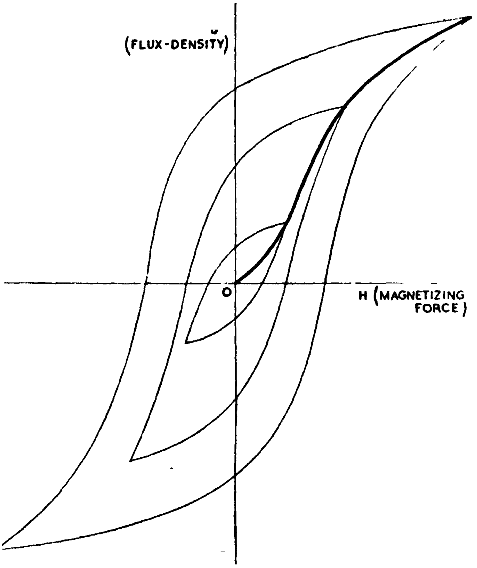

Fig. 11
The Normal Magnetization Curve Through the Tips of B-H Loops of Increasing Size.

In many magnetic calculations the fact that there may be two values of B for any value of H according to whether it is increasing or decreasing causes considerable difficulty and for many purposes a single-valued curve called the *Normal Magnetization* curve is used. This is the curve which can be drawn through the tips of a series of hysteresis loops of progressively increasing size (Fig. 11).

The *B-H* curve plays a very important part in the theory of A.C. magnets and transformers because it presents in a convenient and compact form a large amount of information about a magnetic specimen. The ability to explain at least qualitatively the shape of the *B-H* curve is therefore an essential feature of any adequate theory of ferromagnetism. Ewing's elementary theory described at the beginning of this chapter assumed that the material was composed of minute permanent magnets which could actually be turned into line by external magnetic fields. Apart from the improbability of an actual motion of material particles, no explanation is given of the original source of the magnetic properties. It is accordingly desirable at this stage to record briefly the present state of the theory.

The basic source of the magnetism in a material is the magnetic moment of the electrons due to their orbital motions around the nuclei and to their spinning motions on their own axes. The atoms do not, however, behave as individual magnets as Ampère imagined; they operate together in relatively large blocks. Owing to inter-atomic forces the magnetic moments of the atoms in a particular region are locked parallel to each other; one atom cannot have its magnetic moment realigned unless all the others change theirs too. These spontaeously magnetized regions or "domains" are quite large compared with atomic dimensions, as they may be up to 0·001 cm across and contain some 10^{15} atoms (4). The borders of the domains seem to follow strains in the crystal which form local distortions of the lattice. The existence of differentiated areas within a single crystal is not confined to ferromagnetic substances, but it is only in these that each area has a resultant magnetic moment. Across the border in adjacent domains the directions of parallel locking are mutually perpendicular so that the resultant moment of a whole crystal is zero. The domains can be regarded as the minute magnetic particles of our original theory, but as the moment of a domain can be changed in direction simply by the reorientation of electron spins, the difficulty about a physical motion of the material is avoided.

Magnetization of a ferromagnetic material is thought to take place in three separate stages corresponding with the three portions of different slope in a magnetization curve.(5, 6) The three processes overlap somewhat, so producing a smooth *B-H* curve. A piece of iron, as normally produced, consists of many small crystals with their axes distributed in all directions at random. Each crystal is built up from a "body-centred cube" lattice of atoms (Fig. 12), and within the crystal are domains all forming part of one cube lattice, but with spin orientations in the various domains along the six different

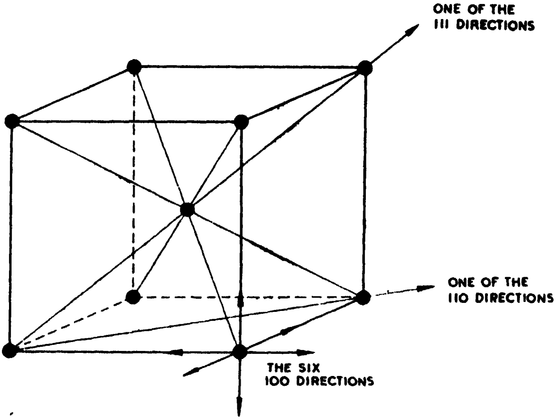

FIG. 12

A Body-centred Cube Forming Part of the Crystal Lattice of α-iron. The 100 Directions are Parallel to the Edges of the Cube, the 110 Directions are Parallel to the Face Diagonals, and the 111 Directions are Parallel to the Cube Diagonals. The 100 are the Easiest and the 111 are the Hardest Directions of Magnetization.

directions parallel to the edges of the cube lattice, known as 100 directions. Within each crystal the domains are always fully magnetized, but with a random distribution of orientations along the six possible directions (Fig. 13). A crystal as a whole is thus normally in a non-magnetic state. The first effect, when an external field is applied, is the growth of those domains which are magnetized in the direction of the field, at the expense of adjacent ones less favourably directed. This is a smooth, progressive change represented by the initial portion of the *B-H* curve. At higher values of applied field, whole domains realign their electron spins so that the magnetic moment of the domain is along the particular 100 direction most nearly parallel to the field. These changes occur discontinuously so that the *B-H* curve over this range, where the slope is highest, is really a series of minute steps. This phenomenon, known as the Barkhausen effect, can be detected as a sound in telephones connected to a coil surrounding a sample of iron being magnetized to this stage. The discontinuous changes which occur during this second stage are non-reversible, being associated with inelastic strains; they are thought to be the cause of hysteresis losses. The third step occurs in strong fields which are capable of aligning the moments of the domains along the direction of the applied field instead of along the nearest crystal axis. This process is continuous and occurs in the region above the "knee" of the *B-H* curve.

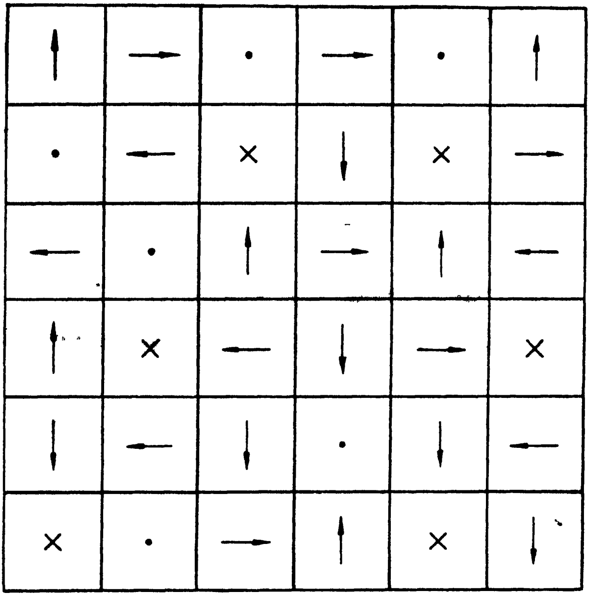

FIG. 13
The Directions of Magnetization of the Domains are Equally Divided amongst the Six 100 Directions of the Lattice.

2. THE HYSTERESIS LOSS

When the sample is being taken round the cycle described above, the fact that a reversed magnetizing force has to be applied in order to annul the previous magnetization means that work has to be done in realigning the magnetic moments of the domains, a fact which is demonstrated by the heating up of the steel. The harder the steel the wider the *B-H* curve, and the greater the power loss and consequent heating. This loss of energy in the sample is called the hysteresis loss, and since the loss in one cycle is proportional to the area of the *B-H* (or hysteresis) loop, it follows that the iron loss in an a.c. magnet increases as the flux-density is increased, and also that the loss is proportional to the frequency of the supply.

The term hysteresis (meaning a lagging behind), although frequently used without qualification to denote *magnetic hysteresis*, is actually a general one applying to any condition in which the state of a body under a given force depends on whether that force was decreasing or increasing previously. Thus a copper wire twisted about its axis past its elastic limit undergoes *torsional hysteresis* and a loop is found when a curve is plotted showing angle of twist against twisting couple. In this book, however, the word is applied only to magnetic hysteresis.

The hysteresis loss is lowest in very pure soft iron, but it is very difficult to prepare the iron in sufficiently pure form, and comparable results can be obtained by incorporating small quantities of other elements in the iron. Silicon is an element which is very widely used for this purpose, and practically all the electrical sheet steel used for power transformer manufacture contains a percentage of silicon.(7) It should not be thought that the use of the word "steel" in this connexion means that a magnetically "hard" material is implied. Steel is used in this sense as meaning an alloy of iron with other elements. The percentage of silicon used is generally of the order of 2½–4, although, for high supply frequencies, up to 4½ per cent may be used. The introduction of silicon into electrical sheet steels was a development of the highest importance. In the early transformers serious difficulties were experienced owing to "ageing" of the cores, i.e. a gradual deterioration of the iron due to the repeated heating and cooling which showed up as a serious increase in hysteresis loss. It was often necessary to take the core apart and re-anneal at intervals of a few years or even months. This trouble has been completely prevented by the use of small percentages of silicon

—even $\frac{1}{2}$ per cent is sufficient, but more is usually added, as already noted, to reduce the hysteresis loss as far as possible.

It is now known that ageing is due to the effects of nitrogen which forms nitrides and to a smaller extent to carbon.(8) The nitrides and carbides are initially in an unstable solid solution in the iron, and their precipitation in a very finely divided state is accelerated by prolonged operation at temperatures over 50°C. The presence of these particles causes strains in the crystal lattice with consequent "hardening" in a magnetic sense. Silicon and also aluminium are able to prevent this precipitation.

The previous physical treatment of a specimen of ferromagnetic material has a profound influence on its magnetic properties. Annealing, for example, greatly reduces the retentivity of a sample of iron; again, the magnetic properties may not be equal in two directions in a sheet owing to the influence of rolling, and advantage is taken of this in the "clock-spring" type of core used for current transformers, in which the material is used so that the direction of the field is always along the direction of rolling and never across it, as in parts of a normal stamping.

The anisotropic properties of silicon steel can be considerably accentuated by a special technique of cold-rolling and subsequent annealing.(9) By these means it is possible to obtain a considerable degree of grain alignment along the preferred direction of "easy magnetization." The non-reversible changes in potential energy which make themselves apparent in the Barkhausen effect during the second stage of magnetization (p. 33) are reduced by the cold-rolling processes and considerably lower hysteresis losses result. The permeability at low fields may be as much as three times greater in the direction of rolling than across it, whilst, at high intensities, a ratio of 10 to 1 has been found.(10) When applying material of this kind for transformer cores it is, of course, essential to use it in the correct direction.

Owing to mechanical difficulties in cold rolling high silicon-content steels, the percentage of silicon in these materials is at present limited to about 3 per cent.

Comparatively recently a number of nickel alloys have been produced having remarkably low hysteresis losses, but, as they saturate at much lower values of B than the silicon-steels and are much more expensive, they have only been used in power transformers for special purposes, such as in aircraft, where weight is of supreme importance. Consideration of the properties and applications of the nickel alloys is therefore deferred until the sections on audio-frequency transformers are reached.

3. Eddy-current Losses

Another cause of energy loss in transformer cores is that known as "eddy-current loss." If the core were a solid bar of iron (Fig. 14a), there would be circular paths in the body of the iron, round which induced currents would flow, just as though closed copper rings were fitted round it, and these eddy-currents would, of course, result in a serious loss. The core of a transformer or any other a.c. magnetic device is therefore split up into flat plates (Fig. 14b) or round wires, insulated from each other by paper, varnish, or some such layer of non-conductor. Often a film of oxide on the core plates is sufficient.

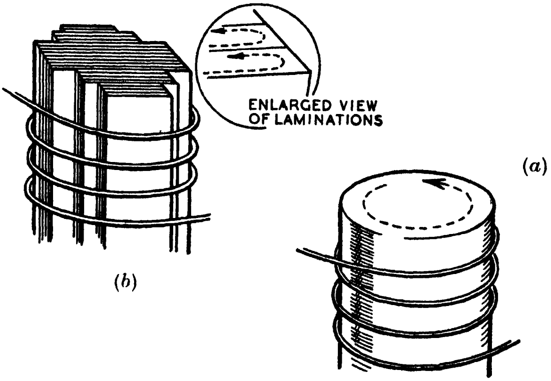

Fig. 14

(a) Eddy-Currents Circulate in the Core of Magnet Energized by Alternating Current. (b) The Paths of the Eddy-Currents are Restricted by Laminating the Core.

The eddy currents are also reduced by increasing the electrical resistance of the core material. Pure iron, although capable of yielding extremely high permeability values ($\mu = 280\,000$) when suitably heat treated by annealing in an atmosphere of hydrogen,(11) is unsuitable for use in a.c. fields because of its low electrical resistance. The silicon steels, however, are of relatively high resistance, so that as well as having low hysteresis losses, they are also good from the eddy-current point of view.

The eddy-current loss, like the hysteresis loss, increases with frequency, but it can be reduced by reducing the thickness of the individual laminations. Dust cores, used for radio-frequency coils, are an example of the extreme lengths to which it is necessary to go in order to keep eddy-current losses down to reasonable proportions when the frequency is measured in megacycles. The individual iron particles in the best dust cores are reduced to about 0·005 mm in size. This degree of subdivision is not necessary even at audio-frequencies and certainly not at power frequencies. In small transformers working at 50 cycles, it is sufficient to split the core up into laminations 0·020 in. thick, whilst 0·014 in. is a suitable thickness for most audio-frequency transformers, although for special purposes stampings down to 0·004 in. thickness are sometimes used.

4. TOTAL IRON LOSS

For most calculations of core loss in transformers it is not necessary to know the values of hysteresis and eddy-current losses separately. Manufacturers of electrical sheet-steel accordingly supply curves showing the total loss in their materials under specified conditions of frequency, flux-density, etc. A series of curves for steels having various silicon contents are given on p. 465. These curves have been compiled from information kindly supplied by Messrs. Joseph Sankey, Ltd.

It will be seen that when a particular stack of stampings is under consideration (so that the total weight, thickness of laminations, and grade of iron are known) it is possible, by reference to curves or tables, to calculate quickly what iron loss may be expected at the particular frequency and flux-density adopted. Generally, the problem is to decide the flux-density to be used, since the permissible iron loss is known from considerations of heat dissipation from the core, and the supply frequency is also known. The curves then enable the flux-density to be decided upon.

Taking as an example the type of transformer fitted in the usual four-valve radio set, the weight of core will be about 3 lb., and the iron loss which can be tolerated without overheating would be about 3 watts; the loss is, therefore, 1·0 watt per lb. Referring to the curves in Fig. 167, it will be seen that the flux-density at which this loss occurs in 0·014 in. stampings is approximately 12 000 lines per cm^2.

This is perhaps an opportune moment to remind readers of the fact, explained at the end of the first chapter, that (apart from small changes due to the resistance of the primary) the state of magnetization of the core, and hence the iron losses, remain unaltered whatever the current (if any) drawn from the secondary windings. The iron loss is thus conveniently tested under no-load conditions, when it is for practical purposes the only loss. When other losses appear, as the secondary supplies current to the load, the iron loss can safely be assumed to continue practically unchanged.

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CHAPTER III

Theory: Simplified Treatment

A TRANSFORMER has hitherto been regarded as an arrangement of two windings which are coupled together magnetically. One of these, the primary, is connected to a source of power, and the other, the secondary, delivers power to the load at a different voltage. It would appear to be quite possible to calculate the performance of a transformer under any specified conditions by means of the coupled-circuit theory but, owing to the variation in permeability of the iron-core material over the cycle of the applied voltage, the calculations would be extremely laborious. It is accordingly better to start from the assumption of an *ideal transformer* in which the core is assumed to have a very high and constant permeability, and to be free from the various causes of power loss, and to add externally circuit elements representing departures of the actual transformer from the ideal.

1. THE IDEAL TRANSFORMER

It is assumed that in the ideal transformer—

(*a*) The resistances of the windings are negligible.

(*b*) The capacitances of the windings can be neglected.

(*c*) All the magnetic flux links all the turns of both primary and secondary.

(*d*) The permeability of the core material is extremely high and constant.

(*e*) The hysteresis loss in the core is negligible.

(*f*) There are no eddy current losses in the core, fittings, or windings.

These assumptions are equivalent to the statement that the transformer has no winding resistances, no magnetizing current, no iron losses, and no magnetic leakage.

In the brief survey of the operating principles of the transformer in Chapter I it was indicated that a back e.m.f. is generated in the primary winding, just as an e.m.f. is induced in the secondary. It is accordingly a general practice to refer to the induced pressure in the primary as in the secondary without distinction, for of course the directions and magnitudes would be identical for coils of the same number of turns. In an ideal transformer, since all the flux links both coils (assumption (c)), the instantaneous induced e.m.f.s in the various windings must be proportional to their numbers of turns,

i.e.
$$\frac{e_p}{e_s} = \frac{T_p}{T_s} \quad . \quad . \quad . \quad . \quad (15)$$

Assumption (*d*) is equivalent to saying that the m.m.f. required to produce the working flux is negligibly small. This m.m.f. is the resultant of the m.m.f. due to the primary current and that due to the secondary current so that

$$T_p i_p + T_s i_s = 0$$

$$\therefore \quad \frac{i_p}{i_s} = -\frac{T_s}{T_p} \quad . \quad . \quad . \quad . \quad (16)$$

The negative sign is a recognition of the fact noted in the first chapter, that the current in the secondary circulates in such a direction as to tend to neutralize the field created by the primary, so tending to reduce the primary back e.m.f. and allowing the primary current to increase.

Multiplying together equations 15 and 16, we get

$$\frac{e_p i_p}{e_s i_s} = -1 \quad . \quad . \quad . \quad (17)$$

The primary and secondary powers at any instant are thus equal. The negative sign shows that whereas the primary absorbs power from the source, the secondary delivers power. In the ideal transformer no power is lost internally so that the two quantities are equal.

It is perhaps desirable at this stage to explain that the appearance of the negative sign in equation 16 follows from assumption that the positive direction of current in the secondary winding is such as to produce a flux in the core in the same direction as that produced by the primary current when the circulation around the core is in the same direction as that in the primary. On the grounds of consistency, this is clearly a sound definition to make. At any *instant* the current in the secondary is, however, in the opposite direction to that in the primary, notwithstanding the fact that e.m.f.s are induced in the two windings in the same direction. In the primary the current is in the opposite direction to that of the induced voltage since the applied voltage overcomes and exceeds the induced (or back) e.m.f. by the resistance drop in the winding. Power is accordingly supplied to the primary. At the same instant, current in the secondary is flowing in the same direction as the induced e.m.f. and so power is delivered by the winding. The secondary current is thus in the opposite phase to that in the primary and is negative according to the original convention. To avoid the negative sign it is convenient to consider a *load current* i_L which is at any instant flowing in the direction of the applied voltage. Hence $i_L = - i_s$.

2. DEPARTURES FROM THE IDEAL

If the operation of an actual transformer is examined, it will be evident that in fact it does depart from the ideal. In small sizes the most obvious defect is the existence of a current in the primary, even when the secondary is on open circuit. This current (known as the **no-load or exciting current**) results from the imperfections of the core which is of finite permeability and so requires an appreciable magnetizing force to produce the operating flux. Since the maintenance of a magnetic field does not absorb power, however, this current should be a watt-less one, i.e. 90° lagging on the applied voltage; in other words, the primary should be represented as including a shunt inductance path. This is not quite the whole story, because in practice the iron has hysteresis and eddy-current losses so that in fact some power is absorbed at no-load; this is represented by the addition of a shunt resistance as well as the inductance.

The extent to which the imperfections of a real transformer show up in practice depends probably more on the operating frequency than on any other single factor. The **optimum frequency** for most applications is in the neighbourhood of **500 cycles per second.** Audio-frequency transformers at this point in their range closely resemble the ideal, whilst power transformers for a given output can be made smaller for this frequency than for higher or lower frequencies. At the normal power frequencies of 25–100 cycles it is necessary to introduce some of those factors the effects of which have been neglected in ideal transformers. These are that—

(*a*) The windings have resistance.

(*d*) The core permeability is not infinitely high, i.e. a magnetizing current is required to produce the flux.

(*e*) There is an appreciable hysteresis loss in the core.

(*f*) There are eddy-current losses.

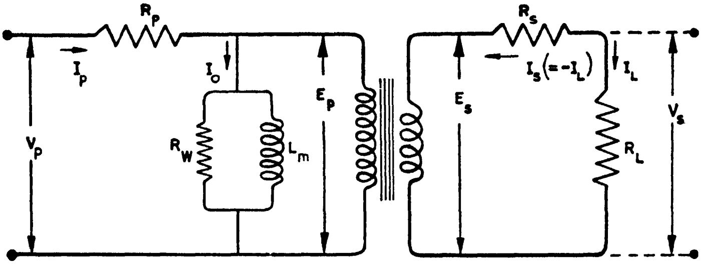

FIG. 15

A Practical Transformer Represented as an Ideal Transformer to which Components have been Added to Simulate Winding Resistances, etc.

The practical transformer accordingly takes the form shown in Fig. 15. In large sizes assumption (*c*) has also to be considered, i.e. there is some flux leakage, but this will not be taken into account for the present. In Fig. 15 the resistance R_w is of such a value that the power dissipated in it is equal to the combined hysteresis and eddy-current losses. The inductance L_m appears because an appreciable m.m.f. is now needed to produce the required flux since the permeability is assumed to be finite. To produce this m.m.f. a magnetizing current is necessary and its magnitude determines the value of L_m, which is assumed to pass this value of current. *It should be carefully noted that the values of L_m and R_w change if the frequency is altered*, owing to the variation of the iron characteristics with the degree of magnetization. This does not apply to anything like the same extent to the other components the values of which are largely independent of the frequency.

The introduction of the shunt paths through R_w and L_m illustrates clearly the conception of the primary current being composed of several components, the magnetizing component, the iron-loss component, and the load component. The first two are present, practically unchanged in magnitude, whether the transformer is supplying a load or not, and together they make up the *exciting current*, but the load component is proportional to the secondary current. The effect of the winding resistances R_p and R_s is evidently to cause a fall in output voltage with increasing secondary current. (R_p contributes to this fall because the load component of the primary current increases as the secondary current increases.)

A concise statement of the mode of operation of a transformer can very conveniently be made in the form of a vector diagram of currents and voltages. The vector diagram shows not only the relative magnitudes of the various quantities but also the phase angles between them. It thus provides a very easy way of adding together components of current flowing in the same circuit, as for example load and exciting components of the primary current. At the present stage of explanation it is desirable to draw separate diagrams for primary and secondary because generally the voltage and current scales are widely different, but in a later section it will be shown that it is possible to combine them.

No-load Conditions (Fig. 16)

If the flux Φ is taken as the starting point from which to build up a vector diagram of an unloaded transformer, the magnetizing current I_m will be in phase with Φ. In (*a*) the loss component I_w of the no-load current will be 90° ahead of the flux, so that the no-load primary current I_0 will be the resultant of I_m and I_w. (This is only strictly true if I_m is a sinusoidal quantity; if it is not, it cannot be represented by a vector.) The induced voltages lag 90° behind the flux, and so the back e.m.f. induced in the primary (not shown) would be 90° behind the flux. The primary voltage $-E_p$ will accordingly be 180° ahead of the back e.m.f., i.e. 90° ahead of the flux. The small component I_0R_p, representing the voltage drop in the primary due to the flow of I_0 through the primary resistance, will be in phase with I_0. The resultant of $-E_p$ and I_0R_p will give V_p

the applied voltage from the supply. The angle ψ between V_p and I_0 is the no-load phase angle. Cos ψ is the no-load power-factor.

In (*b*) the secondary induced voltage E_s lags 90° behind the flux. There is of course no secondary current.

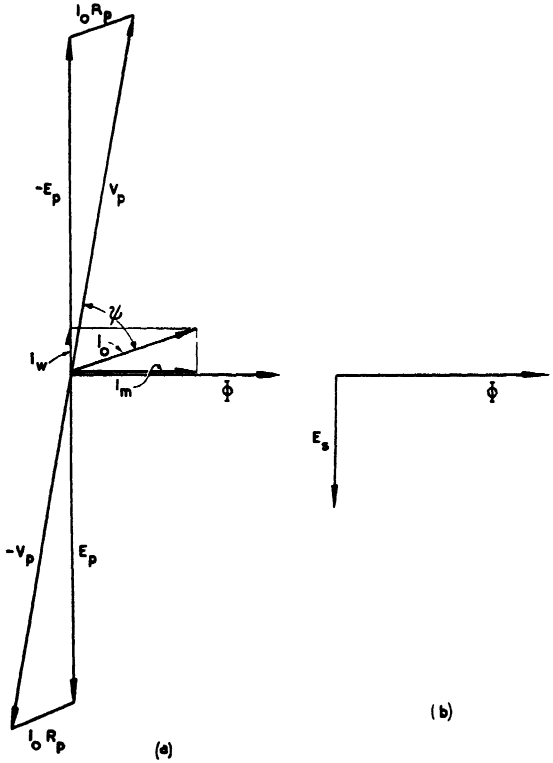

FIG. 16

Vector Diagrams for a Transformer Operating on Open Circuit: (*a*) Primary, (*b*) Secondary.

Resistance Loading (Fig. 17)

The effect of connecting a unity power-factor load to the secondary is to cause a secondary current I_s to flow; this will be in phase with E_s. The load component of the primary current I_L' will be in phase opposition to the secondary current I_s. The total primary current I_p is the resultant of I_L' and I_0. The phase angle ψ is now considerably less than before, giving a power-factor nearer unity. Owing to the increase in the primary current the resistance drop in the primary is now greater, and in addition its phase angle relative to the applied voltage has become smaller. The vector difference giving E_p, the primary induced voltage is now less, and the secondary induced voltage is reduced.

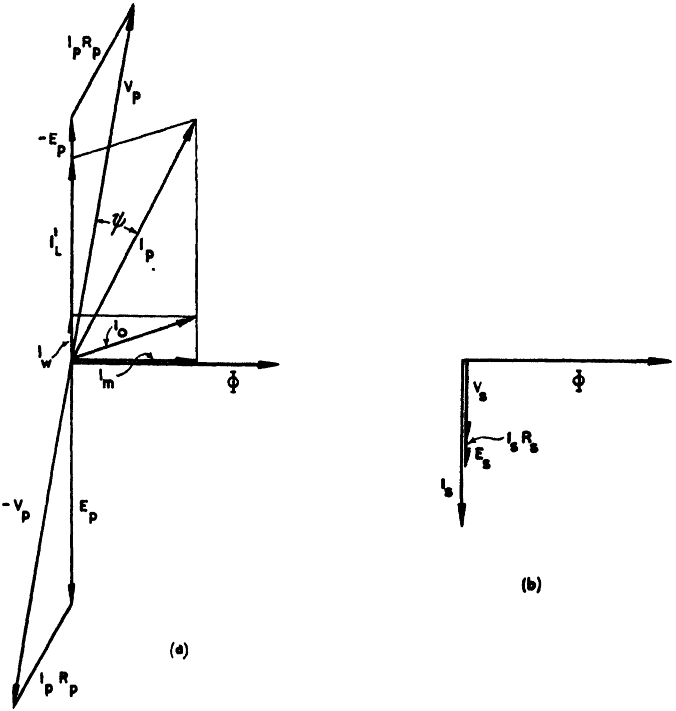

FIG. 17

Vector Diagrams for a Resistive Load.

The output voltage V_s will be less than the secondary induced e.m.f. E_s by the in-phase voltage drop in the secondary windings.

The voltage drops have been exaggerated for clearness; in practice these components would be quite small relative to the induced voltages and the output voltage change between no-load and full load would be much less than is indicated by the diagrams given.

3. THE EQUIVALENT CIRCUIT

The primary and secondary circuits have hitherto been considered as being entirely separate except, of course, for the effects of magnetic coupling. It is often very convenient to be able to regard the two circuits as being joined into one common network which represents the whole transformer. This is possible because an equivalent value can be derived on the primary side for each element which is actually connected to the secondary, or vice versa. It is then quite easy to transfer all the elements to one side or the other, depending on the particular problem under examination. Take, for example, a resistance R_L representing a load connected across the secondary winding of an ideal transformer of ratio $n\left(=\frac{T_p}{T_s}\right)$.

From equations 15 and 16

$$\frac{e_p}{i_p} = -\left(\frac{T_p}{T_s}\right)^2 \times \frac{e_s}{i_s} \quad . \quad . \quad . \quad (18)$$

Writing $i_L = -i_s$ and remembering that $\frac{e_s}{i_L} = R_L$ (secondary winding resistance assumed to be zero) then

$$\frac{e_p}{i_p} = n^2 \times R_L \quad . \quad . \quad . \quad (19)$$

In other words, the effect of connecting R_L across the secondary is to make the primary behave as though a resistance $R_L \times n^2$ is connected across it. The quantity $R_L \times n^2$ is described as the secondary load resistance "referred to the primary," and it is denoted thus, R_L', so that

$$\frac{e_p}{i_p} = R_L' = R_L \times n^2 \quad . \quad . \quad . \quad (20)$$

The quantity R_L' is the resistance which connected across the primary would take from the supply a current equal to that which would flow through the primary of the ideal transformer when R_L is connected to its secondary. If the load contains a reactive component it may be denoted by Z_L, and the corresponding value of the transferred impedance is $n^2 Z_L$ as seen from the primary side. Similarly in a practical transformer the windings resistances can be transferred from secondary to primary (or vice versa) by the same transformation. Thus R_s is seen from the primary side as $R_s' = R_s \times n^2$. When all the circuit elements have been transferred from the secondary to the primary by replacing them by their referred values as in Fig. 18, an ideal transformer connected at the output end would transform the primary voltage to the secondary value according to the rule $\frac{e_p}{e_s} = \frac{T_p}{T_s} = n$. The network obtained in this way will behave in exactly the same manner as that shown in Fig. 15, and it is known as the equivalent circuit of the transformer. In practice the ideal transformer is usually omitted, and the secondary voltage is also referred to the primary as $V_s' = V_s \times n$. The load current I_L when referred to the primary becomes I_L'.

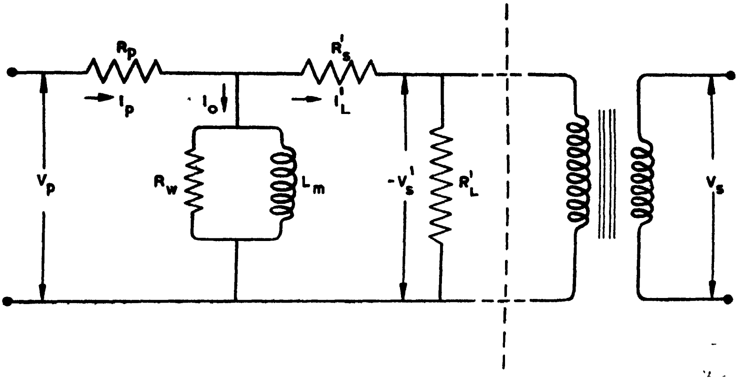

FIG. 18

The Equivalent Diagram of a Transformer Showing All Quantities Referred to the Primary. The Ideal Transformer on the Right is Usually Omitted.

The voltage-changing aspect of the apparatus has now no longer to be taken into account, all the quantities involved having been altered in magnitude so that they produce a result at the source when operating at the primary voltage, exactly the same as that which they would have produced in the original arrangement. The equivalent load current i_L' that flows is, for example, exactly equal to the load component *of the primary current which would have been taken from the supply in the actual arrangement*, because from equation 16 for the ideal transformer

$$\dot{i}_p = \dot{i}_s \times -\frac{T_s}{T_p} = -\dot{i}_s/n = \dot{i}_L/n$$

Hence, $\dot{i}_L' = \dot{i}_L/n.$

In this case the secondary loads and winding resistances have been referred to the primary, i.e. replaced by corresponding loads, etc., on the primary of such values that the primary current remains unchanged. It may be more convenient in any particular case, however, to refer the primary impedances to the secondary, i.e. replace primary resistance, inductance, and loss resistance by corresponding factors on the secondary side of such values that the input and output are unaltered. The circuit then commences with an ideal transformer on the left supplying the transferred resistances, etc., from its secondary. In practice the ideal transformer is omitted and the input voltage is assumed to be $V_p' = V_p/n$.

Vector Diagram

The separate diagrams of Figs. 17 (*a*) and (*b*) can now be combined into one without difficulties arising from the different scales of primary and secondary voltage, provided that the quantities on one side are replaced by their values referred to the other side. By that artifice the scales of current and voltage become the same for primary and referred secondary quantities. The case of a transformer with a resistance load is shown in Fig. 19, which should be compared with Fig. 17. If the secondary load is inductive, the diagram becomes modified to that shown in Fig. 20.

It is advisable at this stage to recapitulate the statements which have already been made regarding the positive directions of voltage and current in the various circuits of a transformer. Unfortunately several different conventions have grown up and confusion is likely to result unless it is clearly understood which system has been adopted. Throughout this book the starting point is always the supply pressure. This is a logical basis from which to start since none of the other parameters has any significance without an applied pressure. The only cases in which the application is difficult are those in which the direction of power flow is not known, but the user of small transformers is not likely to become involved in such difficulties. Taking the applied pressure at any instant as being the positive

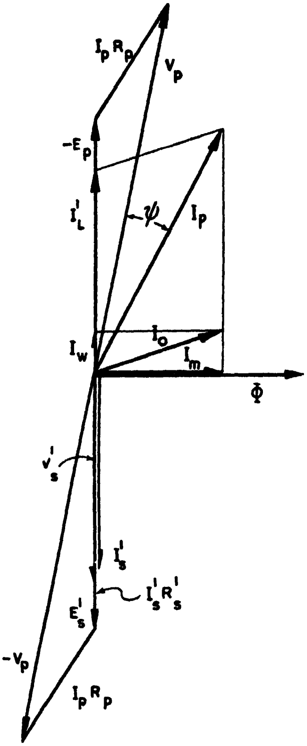

Fig. 19

The Separate Primary and Secondary Diagrams of Fig. 16 Combined into One by the Device of "Referring" all Quantities to the Primary.

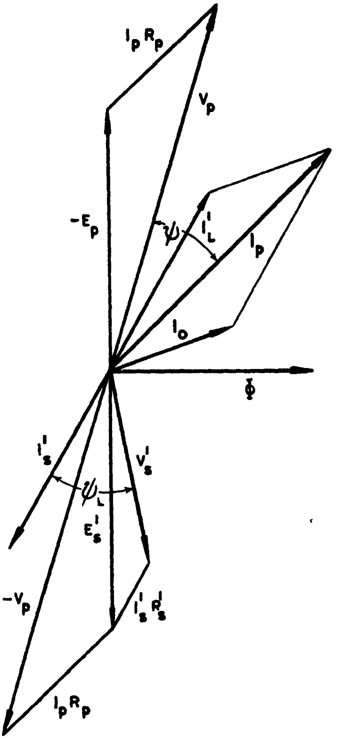

Fig. 20

The Complete Vector Diagram for an Inductive Load of Phase Angle ψ.

direction of pressure, it follows that the positive direction of *current* flow is that which results from the application of the supply pressure to a resistive circuit. This corresponds to the convention adopted in direct-current circuits that the positive direction of current flow is that of the current *coming out* of a battery or other source and that of a current *entering* a motor or other consuming device. The primary current I_p of a transformer is thus considered to flow in the positive direction. The flux is, of course, related to the primary current by the universally accepted convention that the positive direction of flux is that in which a right-handed screw would progress if rotated in the same direction as that in which the current circulates around the core.

The positive direction of the secondary current I_s is such that it produces a flux in the same direction as that produced by the primary current when circulating around the core in the same direction of rotation; in other words, it follows the right-handed screw convention too. This is the point at which confusion may arise because owing to the 180° phase difference, the primary and secondary currents are *at any instant* flowing in opposite directions round the core. This does not, however, affect the fact that the positive directions of current are the same in the two circuits.

The adoption of the same sign convention for the primary and secondary currents simplifies the writing of circuit equations since they then take the same form for both windings, but it is not so convenient for use in equivalent circuit methods. It is preferable for such purposes to use a load current I_L ($= - I_s$) and so secure a current which at any instant flows through the external circuit in the direction of the applied voltage. In the equivalent circuit (Fig. 18) the current I_p from the source then divides into two portions I_L' and I_0. The former flows straight through the winding resistances and out to the load, whilst the latter passes through the excitation impedances R_w and L_m. As many of the explanations here are based on equivalent circuit technique, the quantity I_L frequently appears in vector diagrams. It is, of course, drawn 180° out of phase with I_s and is of equal magnitude. By using I_L in place of I_s in this way the arrows showing the positive directions of current flow can also be taken to indicate the relative directions of the currents in the circuits, at any rate over the greater part of the cycle.

As regards the positive directions of the induced e.m.f.s, since the flux links both windings it induces e.m.f.s in the same direction in each. The e.m.f.s lag 90° behind the flux which produces them, and as the flux vector itself is 90° behind the applied voltage vector the induced e.m.f.s are in the opposite direction to the applied voltage. In the case of the secondary this induced voltage E_s is the open-circuit output voltage V_s, but in the primary circuit the induced voltage E_p acts in opposition to the applied voltage forming a back e.m.f. Following this convention for the positive directions of the induced pressures, V_s is seen to be in the opposite direction to V_p. Whilst this is convenient for physical explanations of the working of the transformer, it does mean that the voltage equations for the primary and secondary differ in regard to signs. In coupled-circuit analysis, where similarity between the equations is helpful, the reversed secondary voltage $(-V_s)$ is accordingly used. A similar device has already been used when introducing the primary back e.m.f. Equation 8 was written as

$$v_p = i_p r_p + (-e_p)$$

in order to bring home that the back e.m.f. is at every instant equal and opposite to the applied voltage, provided that the resistance drop $i_p r_p$ can be neglected.

When drawing vector diagrams E_s is always shown 90° lagging on the flux, but E_p can be drawn either in a similar position or as $(-E_p)$ in a direction 180° out of phase with E_s. The latter position has usually been adopted here because by the addition of the primary voltage drop vectors it enables the applied voltage vector to be drawn. The first alternative, that of showing E_s and E_p in the same angular direction, is useful when illustrating the working of the equivalent circuit. When the various quantities have all been referred to one side of the transformer to produce this equivalent circuit, E_p and E_s are identical, being in fact the e.m.f. induced in the windings of an ideal one-to-one transformer. The induced e.m.f. which can then be written simply as E (without a suffix) is the voltage across the shunt components in the centre of the equivalent circuit. The input voltage V_p is clearly $(-E)$ plus the primary voltage drop, and the output voltage $(-V_s)$ is $(-E)$ minus the secondary voltage drop, as will be seen from Fig. 18.

4. WINDING TURNS

Although it has been noted that the ratio of the voltages on the primary and secondary sides of an ideal transformer is the same as that of the turns ratio of the two windings, nothing has been said about the *actual number of turns* in each. This is decided by the magnetic properties of the core material; in an ideal transformer having a perfect core, the windings could be the smallest which would give the desired ratio with integral numbers of turns. In practice the turns of the primary are arranged to be just sufficient to keep the iron losses and magnetizing current down to reasonable values. To increase the turns beyond that figure would only add unnecessarily to the resistance of the windings without effecting a corresponding improvement in the core performance. When the primary turns have been decided by flux considerations it is possible to calculate the secondary from the required voltage ratio, making small corrections for resistance drops in the windings.

Two factors may influence the number of turns used for the primary winding of a transformer; firstly, the permissible iron loss and, secondly, the maximum no-load current which can be tolerated. In large transformers the first of these factors comes into operation earlier, setting a minimum number of turns below which the flux-density and, therefore, the iron losses become excessive. At this point the no-load current is sufficiently small to have practically no effect at all on the full-load current, since it is nearly in quadrature with the load component of a transformer working into a resistance load. In very small transformers it is not usually possible to work the iron at the maximum flux-density allowable from iron-loss considerations, because the no-load current becomes excessive. If a value of about 30 per cent of the load-component is exceeded, as it would often be, the resultant full-load current begins to go up, requiring a larger section winding or increased copper losses. In general, however, the iron loss is the more important factor in determining the primary turns, and in fact the turns of all windings, because the secondary turns are derived from the primary figures. It is accordingly necessary to derive an expression relating the number of turns on the coil with the flux-density in the core material, its cross-sectional area, and the frequency of the supply. The iron loss for a core of any particular material can be calculated using the maker's published curves of total iron loss against flux-density at different frequencies. The figure is dependent on the thickness of the individual laminations, and the appropriate set of curves must be used.

The e.m.f. derived in the following paragraph is actually the *induced e.m.f.* The applied voltage (i.e. the input from the supply) is slightly greater by an amount depending on the voltage drop in the resistance of the winding. At no-load, however, this difference is quite small and can usually be neglected, so that the applied voltage is taken to be equal and opposite to the induced e.m.f. Now the induced e.m.f. is due to the primary winding being cut by the field, and the magnitude of the e.m.f. at any instant is proportional to the rate of change of linkages between the winding and the field. A rate of change of 10^8 linkages per sec results in the production of an e.m.f. of 1 volt. This rate of change will evidently be greatest at the moment when the flux is changing direction from positive to negative or vice versa, so that the peaks of the back e.m.f. will occur at moments of zero flux, i.e. the back e.m.f. will lag behind the flux by 90° (Fig. 21).

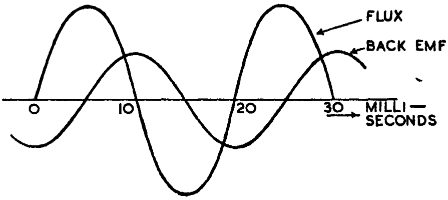

FIG. 21

The Back e.m.f. (or Induced e.m.f.) Lags a Quarter-cycle Behind the Flux.

The rise of back e.m.f. from zero to maximum and back (i.e. one half-cycle) will thus correspond to the change of flux from a maximum in one direction ($+ \Phi_{max}$) to a maximum in the other ($- \Phi_{max}$). This is a change of $2\Phi_{max}$, and it occurs in a period of time equal to one half-cycle, so that the rate of change is $2\Phi_{max} \div 1/2f = 4\Phi_{max}f$. If the primary has T_p turns, then each line of flux will make T_p linkages, and since $\Phi_{max} = B_{max} \times a$ then the rate of change of linkages will be $4B_{max}T_p af$. This means that the average voltage produced over a half-cycle is $4B_{max}T_p af/10^8$.

Now for a sine wave the r.m.s. value of the voltage (which is the usual method of expressing a.c. voltages), is 1·11 times as great as the average value, so that the back e.m.f. is $4{\cdot}44B_{max}T_p af \times 10^{-8}$ volts (r.m.s.). Neglecting the primary resistance, this back e.m.f. will be numerically equal to the applied mains voltage (V_p), so that—

$$V_p = 4{\cdot}44 B_{max} T_p a f \times 10^{-8} \text{ volts} \quad . \quad . \quad (21)$$

where B_{max} = maximum flux-density in the core,
a = cross-sectional area of core,
f = supply frequency,
T_p = turns (primary).

This expression is a very important one, for it is the starting point for the design of all transformers. There are a number of points which should be noted—

(1) Although the magnetic flux is produced by the current in the primary, the value of this current does not appear in the formula, which enables the flux-density and hence (from curves) the iron loss to be determined, without knowing the magnetizing current. This arises from the fundamental statement (made in Chapter I) that the back e.m.f. must at every instant be equal and opposite to the applied voltage, less a small term representing the voltage drop in the primary-winding resistance. If, as is usually the case, the drop can be neglected, then the back e.m.f. is always equal to the applied voltage, *and this is true whatever the value of the flux, the core dimensions, or material.* The back e.m.f. is always equal to the input voltage, and the current adjusts itself to whatever value is necessary to establish the required flux. Variations in the material of the core do not therefore cause a change in B; the only change is in the value of the magnetizing current which is required to produce the particular flux-density. Again, if the length of the magnetic path were increased by a change in the core shape, the induced voltage would still remain unaltered (since it is determined by the applied voltage), but the magnetizing current would go up because a larger m.m.f. would be required to keep the same value of flux through a magnetic circuit of higher reluctance.

(2) For a core of a given cross-section, wound with a fixed number of turns and supplied with a fixed voltage, the flux-density goes down as the frequency is increased. Conversely, for a given flux-density the turns can be reduced as the frequency is increased. A 25-cycle transformer can thus be used quite safely at 50 or 100 cycles, but not vice versa.

(3) The primary turns are proportional to the input volts, if other factors remain unchanged.

In equation 21, the cross-section and flux-density figures are of course required to be in the same units, i.e. both in cm² or both in in.², but in practical calculations it is convenient to use the dimensions of the core in inches whilst the flux-density is best quoted in kilolines per cm². Combining the constant $4{\cdot}44 \times 10^{-8}$ with the conversion constant, the equation becomes

$$V_p = \frac{4{\cdot}44 \times 2{\cdot}54^2 \times 1\,000}{10^8} B_{max} T_p a f \text{ volts}$$

$$= \frac{B_{max} T_p a f}{3\,490} \text{ volts (r.m.s.)}$$

or
$$B_{max} = \frac{3\,490 V_p}{T_p a f} \text{ (kilolines per cm}^2) \quad . \quad . \quad . \quad (22)$$

EXAMPLE

A small transformer for 230-volt 50-cycle operation is to be made up on a core 1¼ in. × 1¼ in. in section, built up from 0·014 in. thick laminations. The flux-density required is 12 000 lines per cm². Calculate the primary turns.

Owing to the insulation, etc., between laminations, the effective cross-section will be less than the external dimensions indicate (see Stacking Factor, p. 90). The effective value of cross-section is $1\frac{1}{4} \times 1\frac{1}{4} \times 0{\cdot}91 = 1{\cdot}42$ in.²

$$T_p = \frac{3\,490 \times 230}{12 \times 1{\cdot}42 \times 50} = 941 \text{ turns}$$

This figure is, of course, arrived at on the assumption that the primary has a negligible resistance. In the chapter on winding calculations it will be seen that the figure may be slightly modified when the voltage drop in the primary winding is allowed for. This will reduce the effective input voltage to slightly below the 230 volts used in this calculation.

In arriving at the figure of 941 turns, no account has been taken of the possibility of the magnetizing current being excessive. This can only be determined when other particulars such as magnetic length and material are specified.

5. NO-LOAD CURRENT

Among the ways in which an actual transformer departs from the ideal the most prominent in the case of a normal power-frequency type is the existence of a primary current when the secondary is on open circuit. This state of affairs is, of course, represented in the equivalent circuit by the inductance L_m and the resistance R_w. The no-load primary current which lags behind the applied voltage by a phase angle of some 70°–80° is thus regarded as having a reactive or watt-less component which flows through L_m and an in-phase or loss component flowing through R_w. The reactive component is strictly the "magnetizing current," although the term is often loosely applied to the whole no-load current. The true magnetizing current is watt-less because no work is done in maintaining a magnetic field; energy is required to establish a field and this energy is stored in the field. When the field collapses, the energy is completely returned to the supply, so that over a complete cycle no energy is taken from the mains. The loss-component supplies the energy which is dissipated in hysteresis and eddy-current losses.

The ratio of no-load current to full-load current decreases progressively as the size of a transformer increases, assuming that the same general conditions apply. such as that the flux-density in the iron remains unchanged and that the window proportions remain unaltered.

The reason why the no-load current is proportionately greater in small transformers than large ones working at the same flux-density in the core can best be seen by considering a large and a small transformer both built to the same proportions, i.e. every dimension of the large one is, say, x times as great as the corresponding dimensions of the small one. Consider first the kVA ratings of the two; since the cross-sectional area of the core is x^2 that of the small transformer, the large one will on that account require only $1/x^2$ turns for the same induced voltage, and the voltage which can be produced at a given current in the winding is x^2 times as great. But since both dimensions of the window have been increased x times, the area of the window is x^2 times greater; the voltage producing ability has therefore been increased x^4 times, i.e. the kVA rating is proportional to the fourth power of the linear dimensions. Turning now to the magnetizing current, this can be calculated from the expression for H, viz.

$$H = \frac{4\pi I_m T_p}{10l} \times \sqrt{2} \quad . \quad . \quad (23)$$

on the assumption that the current is sinusoidal. This is not strictly true, but the relationship between current and dimensions which we are seeking is not affected by this assumption, since any other wave-form would be covered by substituting another constant in place of $\sqrt{2}$, which represents the ratio of peak to r.m.s. value for a sinusoidal wave-form. The value of H is the peak value reached during the cycle and in the particular magnetic material used for the core it corresponds to the value of B_{max} chosen for the transformer. The value of B reached for a given value of H is, of course, a property of the material and is not dependent on the shape or size of the specimen.

Transposing equation 23,

$$I_m = \frac{10lH}{4\sqrt{2}\pi T_p} \quad . \quad . \quad . \quad . \quad (24)$$

Now, if the full-load primary current is I_p,

$$I_p = \frac{\text{kVA} \times 1\ 000}{V}$$

The ratio of magnetizing to full-load current is, therefore,

$$\frac{I_m}{I_p} = \frac{10lH}{4\sqrt{2}\pi T_p} \times \frac{V}{\text{kVA} \times 1\ 000}$$

$$\frac{lHV}{565\pi T_p \times \text{kVA}}$$

In this expression, l, V, T_p and kVA vary with the linear dimensions whilst H has been assumed constant. In the larger of the two hypothetical transformers, one of which is x times greater in linear dimensions than the other, the quantities become xl, x^4V, x^2T_p, and x^4 kVA, respectively, if l, V, T_p, and kVA are the quantities for the smaller one.

Hence, $\frac{I_m}{I_p} \propto \frac{x \times x^4}{x^2 \times x^4}$ i.e. inversely proportional to x.

The ratio of magnetizing to full-load current is thus inversely proportional to the linear dimensions, i.e. varies inversely as the fourth root of the kVA.

In practice a proportion of no-load to full-load current of 10 per cent is a generally acceptable figure for transformers of fair size. Large ones rated in thousands of kVA may reach a figure as low as 3 or 4 per cent even at the high values of B of 14 000 to 15 000 lines per cm^2 which are used at 50 cycles. In sizes rated only in tens of kVA a rather lower flux-density becomes necessary in order to keep down to 10 per cent, and in fractional kVA sizes the figure is allowed to rise to some 20 to 30 per cent.

Calculation of No-load Current

Although it might appear desirable to estimate the loss-component and magnetizing current component separately, afterwards adding them vectorially to obtain the total no-load current, in actual practice the variations due to manufacturing tolerances in the magnetic characteristics of the core material and the differences in length of unintentional air-gaps where laminations are butted together, result in quite considerable uncertainties in the value to be expected. It is usually, therefore, sufficiently accurate to work out the magnetizing component, which is likely to be much the larger, and neglect the loss-component, which being in quadrature would in any case contribute little to the resultant figure.

If necessary the loss component is easily obtained from the iron loss (Section 4) and the primary voltage.

One method of calculating the magnetizing current I_m has already been mentioned. Equation 24, based on the fundamental relationship between the current and the magnetizing force in a coil of a given number of turns, enables I_m to be calculated for a given value of H. By referring to the a.c. magnetization curve for the core material in use, the value of H can be read off against the known value of B. The a.c. magnetization curve for a core material is the a.c. counterpart of the B-H curve used for calculations in d.c. circuits. It shows the peak value of B reached during each cycle when the specified r.m.s. values of magnetizing force are applied. When the value of B_{max} has been decided, the value of H can be read off from the curve.

Since $H = \frac{4\pi I_m T_p}{10l}$ the value of I_m can readily be obtained.

Owing to the curvature of the magnetization curves the variation of B over the cycle will be sinusoidal only if H is not sinusoidal and vice versa. As the most usual case is that of a

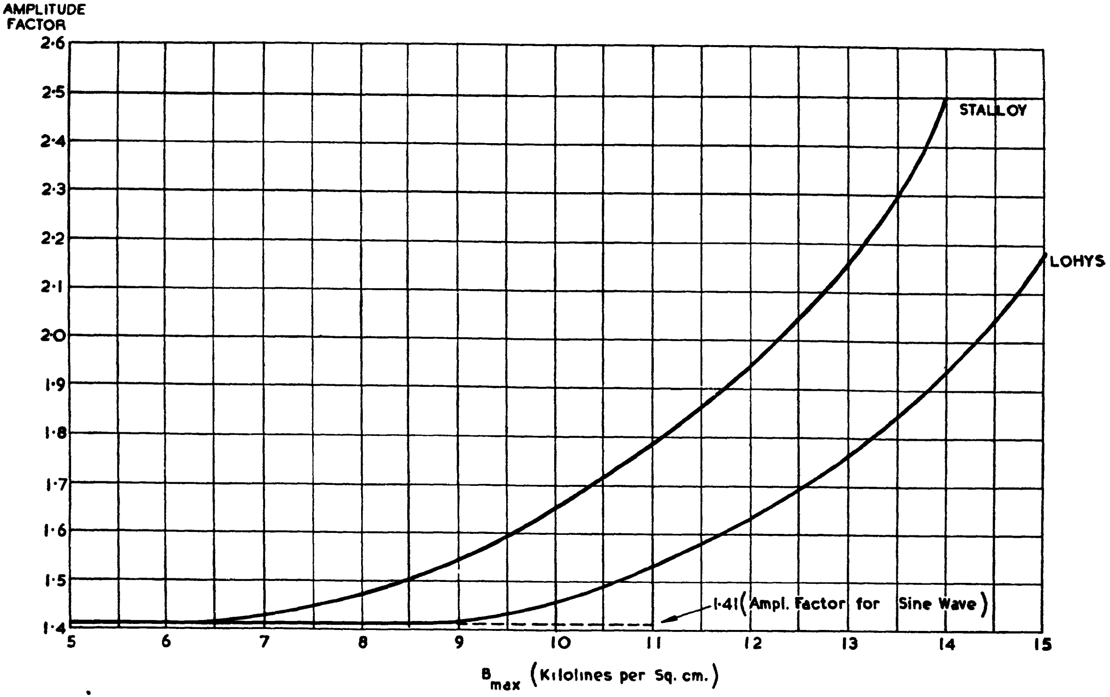

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FIG. 22

Values of the Amplitude Factor (i.e. Ratio of Peak to r.m.s. Value of Magnetizing Current Wave) for Two Grades of Silicon-Steel at Various Flux-densities.

sinusoidal input voltage (and therefore a sinusoidal flux) curves are drawn for these conditions; they are not really applicable to sinusoidal H conditions. Since the curves are usually plotted for B_{max} against $H_{r.m.s.}$, the value of I_m is obtained directly as an r.m.s. value. If, however, the value of H is given as a peak value, the current will also be a peak current and it will need conversion to r.m.s. In deriving equation 24 it was assumed that the magnetizing current was sinusoidal, so that to obtain the r.m.s. value of the current it was necessary to divide the peak current (which produces the peak H and so B_{max}) by $\sqrt{2}$. If, as is usually the case, I_m is not sinusoidal, another factor must be used in place of $\sqrt{2}$. The figure to be substituted depends on the material and the working conditions; it can be read from a curve of *amplitude factor* against flux-density published by the makers (Fig. 22).

An alternative method of obtaining the value of the magnetizing current is to make use of curves of *reactive volt-amperes* against flux-density provided by the manufacturers. These curves correspond to those of total loss (watts) against flux-density, but whereas the latter curves show the true power loss, the former show the reactive "power." From the known weight of the core and the primary voltage the magnetizing current can easily be deduced.

The reactive VA figure for a particular grade of iron, like the total loss, is a characteristic of the material and is independent of the shape and size of the core. To show that this is true, consider two alternative designs of transformers having cores of the same total volume of iron, but of different shapes, so that the cross-sectional area of one is x times that of the other. The magnetic lengths will be in the inverse ratio since the volumes are equal,

i.e. $$a_1 \times l_1 = a_2 \times l_2, \text{ where } \frac{a_1}{a_2} = x \text{ and } \frac{l_1}{l_2} = \frac{1}{x}.$$

In order to work at the same flux-density, the primary turns of Design 2 will be x times those in Design 1, so that the inductance of Design 1 will be

$$L_1 = \frac{4\pi T_p{}^2 a_1 \mu}{10^9 l_1}$$

and that of Design 2 will be

$$L_2 = \frac{4\pi T_p{}^2 x^2 a_2 \mu}{10^9 l_2}$$

(μ is the same for both transformers, since B_{max} is equal in the two cases.)

$$= \frac{4\pi T_p{}^2 x^2 a_1 \mu}{10^9 l_1 x^2} = L_1$$

Since the inductances of the two designs are equal, the magnetizing currents and therefore the reactive VA will also be equal.

To illustrate the method of calculating the magnetizing current, an example is worked out by two methods.

EXAMPLE

Determine the magnetizing current of the small power transformer for which the primary turns have been calculated on p. 55.

The data already given are—

Input . . .	. 230 volts 50 cycles
Core section . .	. $1\frac{1}{4}$ in. × $1\frac{1}{4}$ in.
Stacking factor .	. 0·91
B_{max} . . .	. 12 000 lines per cm^2

The primary turns have been calculated to be 941.

The magnetic length is now disclosed as 7·5 in. and the material is 4 per cent silicon steel, 0·014 in. thick.

Method 1

From the a.c. magnetization curve for 4 per cent silicon steel (Fig. 171), the r.m.s. magnetizing force for $B_{max} = 12\ 000$ lines per cm^2 is 5·5 gilberts per cm.

But $$H = \frac{4\pi I_m T_p}{10 l} \quad \text{so that}$$

$$I_m(\text{r.m.s.}) = \frac{10Hl}{4\pi T_p} = \frac{10 \times 5{\cdot}5 \times 7{\cdot}5 \times 2{\cdot}54}{4\pi \times 941}$$
$$= 0{\cdot}088 \text{ amp.}$$

Method 2

The curve of VA per lb (Fig. 170) gives a figure of 7·0 VA per lb at $B_{max} = 12\ 000$ lines per cm^2 and a frequency of 50 cycles per sec. The volume of iron is 10·7 in.^{3} (i.e. $1\frac{1}{4} \times 1\frac{1}{4} \times 0{\cdot}91 \times 7{\cdot}5$ in.^{3}). Taking the density as 0·27 lb per in.^{3} the total weight of the core is 2·88 lb.

The total VA = 20·2.

Hence $$I_m(\text{r.m.s.}) = \frac{20{\cdot}2}{230} = 0{\cdot}088 \text{ amp.}$$

The reader should not be misled by the close agreement of the values of I_m obtained by the two methods of calculation; they are not representative of the accuracy which can be expected in the calculation of magnetic quantities, for the basic data are the same for both. The agreement of the figures thus indicates nothing more than that the arithmetic is correct. Calculations of magnetic quantities should not normally be expected to agree with measurements on any particular batch of material to better than ± 20 per cent, since variations of this order of magnitude occur between different samples of a given grade of laminations.

Effect of Joints

The calculation of magnetizing current by either of the methods described assumes that the magnetic path is uniform and entirely in iron. In practice this is true only of certain special transformers wound on ring-shaped laminations and used for measuring or telephone purposes; all normal power transformers have cores which are split in one or more places to allow easier assembly. The joints in one layer of laminations are arranged not to coincide with those in the next layer, so that the effect is not simply that of an air-gap straight across the path of the flux, but rather to cause a greatly increased flux-density in that portion of the adjacent stampings above and below the gap. This result is shown in Fig. 23. In consequence of the high flux-density in the iron at these points, the permeability is considerably reduced and some flux also strays into the air around the joint. These effects are very difficult to calculate with any degree of accuracy, and they are of course dependent on the care taken in stacking, the pressure applied by the clamps, etc. The result is, however, to cause an increase in magnetizing current which is too large to neglect, and some assessment should be made. O. I. Butler and C. Y. Mang(1) have examined the effect in considerable detail, giving formulae and figures for both Mumetal and 4 per cent silicon steel. A. Boyajian and G. Camilli(2) give a curve for the relationship between flux-density in a typical specimen of iron and the ampere-turns per inch required for its magnetization, and include a curve showing the additional ampere-turns required per joint at these flux-densities. These curves indicate that at the normal values of B, the ampere-turns required per joint are several times greater than the ampere-turns per inch of core; in other words a length of a few inches per joint should be added to the actual magnetic length to obtain the effective value for the purposes of magnetizing current calculation.

E. B. Harrison⁽³⁾ states that a useful reduction can be obtained by interleaving the stampings two at a time rather than singly. Stampings are frequently inserted in twos or even threes to speed up assembly, and if this also reduces the magnetizing current, it is certainly well worth adopting. The only disadvantage is likely to be an increased tendency to chatter, but if the core as well as the windings are vacuum-impregnated in wax or other insulating compounds (Chapter V), no trouble will occur.

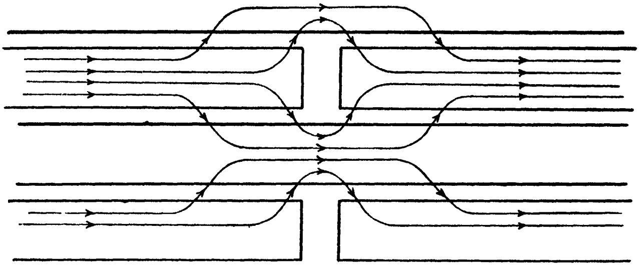

FIG. 23

The Path of the Flux at a Lapped Joint in the Core. The Flux-density in Strips Adjacent to the Gaps is Considerably Increased.

Tests made on a stack of E- and I-laminations for a 60-VA transformer at 50 cycles gave the following results—

TABLE II

No. of Stampings Facing Same Way	Average Magnetizing Current (mA)
1	79
2	78
3	78
4	80
5	83
6	89

The laminations were carefully pressed together to close the gaps as much as possible, and they were assembled a number of times at each grouping.

6. POLARITY

When two sources of e.m.f. have to be connected together, say, in series, it is obvious that a knowledge of the direction of action of each e.m.f. round the circuit is essential. In a d.c. circuit the meaning of this *polarity*, as it is called, is quite obvious, and a pair of terminals can be marked as having positive and negative polarity without difficulty. In an a.c. circuit, however, it might not appear possible to give any meaning to polarity, but the *relative* polarity of two circuits can still be stated, and it is of the greatest importance, when connecting two separate sources of e.m.f., that the *instantaneous* directions of the pressures should be suitably related.

The polarity of transformer terminals is determined in practice by the direction of winding of the coil to which they are connected and the method of placing it on the core. If a single turn of wire is considered, the direction of winding has no significance, for the result is the same either way. As soon as two or more turns are made, however, the direction of rotation of the spindle of the winding machine upon which the coils are wound and the direction of feed become important. For convenience in handling, it is usual to arrange that the top of the former moves away from the operator, i.e. the rotation is opposite to that in the normal operation of a lathe. When a single-layer helical coil is started at the left-hand end, a helix similar to a right-hand screw thread is produced. If, on the other hand, a coil is started at the right-hand end of the former, it corresponds to a left-handed screw thread. If now these two coils are placed on a common core, so that they are cut simultaneously by the same alternating flux, the potentials of the upper ends will be opposite at every instant. If the coils are to form part of the same winding, the top of one would be joined to the top end of the other, so adding together the e.m.f.s generated in the two portions. On the other hand, if the two coils had both been wound in the same direction, the top of one would have had to have been joined to the bottom of the other in order to cause their e.m.f.s to add. When coils of the flat spiral (i.e. pancake or disc) type are considered the ends are described as "inner" or "outer." If the two coils are wound similarly, the inner end of the first must be joined to the outer end of the second, whereas coils of opposite direction of winding must have two inner ends connected together. Pancake coils of this kind are often wound in pairs from one continuous strip, so as to avoid a join at the inner ends. The two halves are then wound in opposite directions. A flat spiral coil differs from the helical coils previously considered in that the apparent direction of winding can be reversed by turning the coil over, top to bottom, whereas a right-handed helix still remains right-handed when turned over in that way. Flat spiral coils must accordingly be assembled on the core the right way up relative to the winding direction.

When connecting together coils forming parts of one winding, the sections must of course be arranged so that their e.m.f.s are added. The relative directions of the primary and secondary can on the other hand be arranged in either direction. If the primary and secondary of a transformer are both wound in the same direction round a particular limb, the induced voltages at the upper ends (in case of concentric windings) will at every instant be of the same polarity. The corresponding ends of primary and secondary are normally taken out at the same end of the case, and the polarity is then said to be *subtractive.* This term is derived from the fact that if two adjacent ends are connected together and a voltmeter applied between the two remaining terminals, it will show a voltage equal to the *difference* between the separate winding voltages if the polarity is subtractive. This is the normal method of connexion. The *additive* arrangement is of course the reverse of this. If corresponding terminals of primary and secondary are joined, the voltage between the remaining terminals is then the sum of the primary and secondary voltages.

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(2) A. BOYAJIAN AND G. CAMILLI. *Transformer Engineering* (Ed. L. F. Blume). John Wiley & Sons, 1938.

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CHAPTER IV

Losses, Efficiency, Regulation

THE hypothetical conception of an ideal transformer has been introduced in Chapter III, and some of the effects of the departures from the ideal which occur in practice have been noted. In Chapter III the defects which appear in practice were examined from the point of view of their effect on the output voltage, and in some cases, e.g. the iron losses which were represented by a shunt resistance, their effect was seen to be quite small. It is the purpose of this chapter, however, to look at the matter from the slightly different viewpoint of power loss—it will then be seen that the size of the iron loss has a very considerable influence on the design of a transformer.

1. LOSSES

For convenience in calculation and testing it is usual to divide losses into two groups—

(*a*) those which are independent of the load,
(*b*) those which increase with the load.

Since the major item in (*a*) is the core loss and in (*b*) the copper loss, it is usual to speak of the two groups loosely as iron and copper losses respectively, although certain other small losses are in practice lumped together with the principal ones.

Iron Losses

The assumption that the iron losses are independent of the load follows the reasoning of Chapter III in which it is assumed that there is no magnetic leakage. This assumption may have to be modified somewhat when leakage cannot be neglected, but even in these conditions it is nearly always substantially valid. This aspect of core losses is referred to again in Chapter VIII.

The reasons for the losses in the core with their attendant heating have already been considered in some detail. The core losses are due to two different causes; hysteresis and eddy-currents and their relative magnitudes vary with the frequency of the supply.

Hysteresis losses are proportional to frequency if other factors remain unchanged since the loss is related to the area of the B-H loop for the material; the loss in a given time period is thus proportional to the number of occasions on which the loop is traversed. At a given frequency the hysteresis loss increases as the peak flux-density increases, but the quantitative relationship varies in form according to the material. Steinmetz(1) formulated a law bearing his name which indicated that the area of the hysteresis loop for a given material would be proportional to the 1·6th power of the peak flux-density reached during the loop. This was reasonably true of the older magnetic materials for the flux-densities at which they were likely to be employed, i.e. up to 13 000 lines per cm^2, but excluding the very low values. The recently introduced nickel-iron alloys, however, give losses which increase more rapidly than the 1·6th power and indeed often the behaviour cannot be represented by an expression of that form at all over a wide range of flux-densities. Over limited ranges an expression of the form

$$W = \mathrm{K}B^n$$

can be used, the values of the constants K and n being determined experimentally for the specimen in question.

The hysteresis loss is the major iron loss in power transformers of normal design for use at 25–100 cycles, but the eddy-current loss is not negligible even at these frequencies(2) and it becomes of greater importance at higher frequencies. This of course follows from the shapes of the loss-versus-frequency relationships in the two cases.

The eddy-current loss for a material, unlike the hysteresis loss, cannot be quoted without relation to the thickness of the laminations upon which it is dependent to a major extent, although it depends also on the resistivity of the substance.

The eddy-current loss in laminations of the normal thicknesses can be calculated from the laws of electromagnetic induction. Consider a section through one lamination and assume (as is usually the case) that the flux enters the section at right angles (Fig. 24). The thickness of the stamping is taken to be d cm and its height, which is assumed to be large compared with d, is h cm. The eddy currents will follow a path within the section and except at the extreme top and bottom the sides of this loop will be parallel. Let two elements δx wide and situated x cm from the centre-line of the section form the sides of the current loop which will then have a total length of $2h$ cm. If the flux-density is uniform over the section, the e.m.f. induced in the loop will be equal to that induced in one turn of the transformer winding multiplied by the ratio of the area $2xh$ of the loop to that of the whole core section. If an e.m.f. of E_s volts is induced in T_s turns of the secondary, the e.m.f. E_e in the eddy-current loop will be given by

$$E_e = \frac{E_s}{T_s} \times \frac{2xh}{a}$$

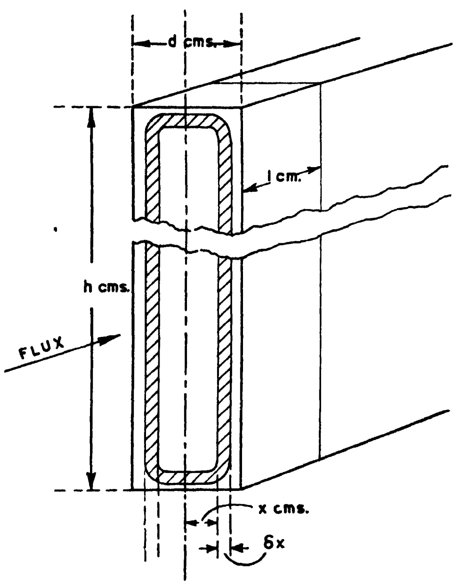

FIG. 24

The Eddy-current Loss in a Thin Strip Perpendicular to the Field is Calculated by Considering the Current in a Narrow Element, δx wide Situated at x cm from the Centre of the Strip.

If the path of the current is assumed to be non-inductive the value of the current will be E_e/r and the eddy-current loss will be E_e^2/r. If the specific resistance of the core material is ρ ohms per cm^3, the resistance r of the path will be $\rho \dfrac{2h}{\delta x}$ ohms per cm depth of stamping (i.e. in the direction along the flux). The loss in the loop through a 1 cm deep slice will be

$$\left(\frac{E_s}{T_s} \times \frac{2xh}{a}\right)^2 \frac{\delta x}{2h\rho}$$

Over the whole cross-section this will be

$$\int_0^{\frac{d}{2}} \left(\frac{2xh}{a} \times \frac{E_s}{T_s}\right)^2 \frac{dx}{2h\rho} = 2\int_0^{\frac{d}{2}} \left(\frac{E_s}{T_s}\right)^2 \frac{h}{\rho a^2}\, x^2 dx$$

The volume of the 1-cm deep slice will be *hd* cm³, so that the eddy-current loss per cm³ will be

$$W_e = \frac{2}{hd}\left[\left(\frac{E_s}{T_s}\right)^2 \frac{h}{\rho a^2}\,\frac{x^3}{3}\right]_0^{\frac{d}{2}}$$

$$= \left(\frac{E_s}{T_s}\right)^2 \frac{d^2}{12\rho a^2} \text{ watts}$$

But the volts per turn $\frac{E_s}{T_s} = \sqrt{2}\pi B_{max} a f \times 10^{-8}$, so that

$$W_e = \frac{\pi^2 B_{max}^2 f^2 d^2}{6\rho} \times 10^{-16} \text{ watts per cm}^3 \quad . \quad (25)$$

This expression is more useful for indicating the manner in which the eddy-current loss varies with the various factors concerned than as a means of determining the absolute value of the loss in a material. This is because some of the assumptions made are not completely valid in practical cases. If the laminations are thin and the frequency low, the eddy-current loss agrees reasonably well with that given by equation 25, especially if the constant term $\pi^2/6\rho$ is replaced by a constant obtained empirically. The maximum thickness for which the relationship is valid is of the order of 0·2 in. at 50 cycles and 0·014 in. at 500 cycles. If the thickness is much greater than this an effect begins to be apparent that is well known in radio work as the "skin-effect." This results from the screening of the centre of the lamination from the field, by the currents flowing in the outer layers of the material. The direction of the induced eddy currents is such as to produce as econdary flux in opposition to the main flux in just the same way as the currents in the secondary windings produce an opposing flux. The effect of this screening is to *reduce* the eddy-current loss in the core material, but it *increases* the hysteresis loss by reducing the effective iron section and so putting up the flux-density in the outer portions of each lamination. Since the hysteresis loss increases with the flux-density at a higher power than unity, the hysteresis loss over the whole section is increased.

A corresponding skin-effect occurs in large-section copper conductors in a magnetic field. (See copper loss later in this chapter.) In that case the eddy-currents induced in the conductor have the effect of causing an apparent increase in resistance due to the unequal distribution of current over its cross-section.

Copper Losses

It is evident from well-known principles that the secondary copper loss in a given transformer is proportional to the square of the secondary current. The load component of the primary current can be referred to the secondary, so that the primary copper loss due to this component will also vary in the same way. The effect of the no-load component on the total copper loss is small in transformers of any size but in small transformers for a few hundred watts or less, it may be appreciable. In those cases where account is taken of this loss it must of course be reckoned as a no-load loss, but to do so complicates the picture because the loss depends upon the power-factor of the secondary load and the subject will therefore be dealt with on the assumption that the copper loss increases in proportion to the square of the output current.

Another loss which varies with the load current is that arising from eddy-currents in the conductors due to the fields produced by the alternating currents flowing in them. These losses are also proportional to the square of the current and can be conveniently added to those due to the normal copper loss. If this is done the total copper loss is equivalent to that which would occur if the winding were assumed to have a higher resistance than its true value, and it is very often convenient to regard the effect in this light. From this point of view the winding is said to have a higher resistance to A.C. than to D.C., an effect familiar to radio engineers as the "skin-effect"; at radio frequencies the effect is of course much more pronounced. The ratio of a.c. resistance to d.c. resistance is greater for large section conductors than for thin ones and it is also greater in coils than straight conductors; consequently it is appreciable even at power-frequencies in the very large conductors used in the low-voltage windings of large transformers. The skin-effect can be reduced by using a number of separately insulated small section conductors instead of a solid bar, and, as will be seen in Chapter V, the helical type coils used for these high-current windings are usually wound from stranded conductor in this way. In order that the current shall be equally distributed amongst the separate strands, however, it is necessary to ensure that in their passage from one end of the coil to the other the various strands each occupy a given position in the bundle for the same distance. For example, each separate strand must be on the inside of the coil for the same distance as the others, and so on. Not only does this prevent overheating of some strands relative to others, but it also results in a lower overall a.c. resistance and, therefore, a lower copper loss. In order to bring about this state of symmetry the separate strips are "transposed" at regular intervals down the length of the winding. This method corresponds to that used for radio-frequency inductance coils in which "Litzendraht" (i.e. Litz wire) is employed. The separate strands of Litz are so laid together that each occupies the same position in the bundle in turn. True Litz is made up from a number of separate wires which is a power of three, e.g. 3, 9, 27, 81, etc. Three wires are twisted symmetrically in the required manner, then three groups of these twisted cables are themselves twisted, and so on. The "transposition" is in this case continuous, but that would not be a practicable method for heavy transformer conductors.

Although the eddy-current losses in the copper can conveniently be grouped with the resistance losses as an apparent increase in resistance to A.C., it should not be forgotten that they are really distinct. One instance of when this should be borne in mind is in considering the effect of temperature on the resistance. The true ohmic resistance of copper conductors increases with temperature at the rate of 0·38 per cent per 1°C but the eddy-current loss *decreases* at the same rate with rising temperature, because the magnitude of the circulating current is reduced by the higher resistance of the path round which it flows, and the loss, being proportional to the square of the current, decreases more rapidly on this account than it increases on account of the high resistance of the path to which it is only proportional to the first power. The ratio of a.c.resistance to d.c. resistance thus decreases at higher temperatures because of the lower contribution of eddy-current losses to the total copper loss, and the apparent temperature coefficient of a.c. resistance is lower than the normal figure. The extent of this reduction is, of course, dependent on the proportion of the eddy-current to the true ohmic loss.

2. EFFICIENCY

The efficiency of a transformer, like that of any other device, is defined as the ratio of power output to power input. The difference between the power input and power output must equal the losses, so that the efficiency can also be given in terms of the losses, thus—

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

$$= \frac{\text{input} - \text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{input}} \quad . \quad . \quad (26)$$

This expression is often more convenient for calculation than the simple ratio of output to input, and in cases where the efficiency is very high, leads to more accurate results.

Other ways of expressing the efficiency are—

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{1}{1 + \dfrac{\text{losses}}{\text{output}}} \quad . \quad . \quad (27)$$

$$\text{Efficiency} = 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{output} + \text{losses}} \quad . \quad (28)$$

The losses referred to in the expressions above are, of course, those occurring under the particular conditions obtaining at the moment. They are made up of two groups; those which are independent of the load on the transformer (usually called iron losses) and those which increase with load (called copper losses). It will be evident that the efficiency will vary considerably according to the output taken from the transformer. If it is always used fully loaded, the manner of the variation with load does not matter, but it is of the highest importance in many applications, such as a distribution transformer which is always left connected on the primary side but which may be only very lightly loaded at certain periods of the twenty-four hours.

Condition for Maximum Efficiency

A particular transformer reaches its maximum efficiency when the copper losses have become equal to the iron losses; if the load is constant, this equality of losses should obviously be arranged to occur at that load. This condition for maximum efficiency can be derived as follows—

Let W_f and W_c represent the iron and copper losses respectively.

W_f is constant,

$W_c = I_L^2R$,

where R is the equivalent resistance of the whole of the windings referred to the secondary.

It is desired to find the relationship between W_f and W_c at which the curve of efficiency against load current reaches its highest value.

From equation 27, Efficiency $= \dfrac{1}{1 + \dfrac{\text{losses}}{\text{output}}}$.

This expression will be a maximum when $\dfrac{\text{losses}}{\text{output}}$ is a minimum.

At this point

$$\frac{d}{dI_L}\left(\frac{\text{losses}}{\text{output}}\right) = 0$$

i.e.

$$\frac{d}{dI_L}\left(\frac{W_f + I_L^2R}{VI_L}\right) = 0$$

i.e.

$$\frac{d}{dI_L}\left(\frac{W_f}{V}I_L^{-1} + \frac{R}{V}I_L\right) = 0$$

$\therefore$

$$-\frac{W_f}{V}I_L^{-2} + \frac{R}{V} = 0$$

$\therefore$

$$W_f = I_L^2R = W_c \quad . \quad . \quad (29)$$

A transformer is thus shown to reach its highest efficiency at the particular load at which the copper loss has reached equality with the iron loss. This load is not necessarily the maximum load which the transformer can carry even on a continuous rating, and it is seldom as great as the overload capacity. The fact is that the best proportion between the two has to be settled for each set of operating conditions. As already indicated, some transformers are left continuously in circuit, but are usually only quite lightly loaded and seldom if ever give their full output. It is important in such cases to keep the iron loss (which continues at the same level as long as the primary is energized) as low as possible. Naturally this means a corresponding increase in copper losses unless the size and first cost are to be increased greatly, but even so the total losses over a period of time will be lower, in spite of the increase at peak-load conditions.

There are, however, disadvantages in making the iron loss very low at the expense of the copper loss. A transformer designed on these lines would have a very small core section —this follows from the fact that the iron loss *per lb* remains the same at a given flux-density and frequency—and a large weight of copper. The mechanical properties of such an arrangement are likely to be unsatisfactory, since the core forms the main framework upon which the apparatus is built, and a small core and large coils are likely to form a weak structure. Again, the large radial depth of winding is likely to make cooling the inside of the coil difficult, and the interior temperature for a given surface temperature is likely to be high. If the losses reach equality at a small proportion of full-load, the copper losses will become very high indeed at overload outputs, since the copper loss increases in proportion to the square of the current. The overload capacity of such a transformer will therefore be low. Finally the regulation, i.e. the variation of output voltage with load, will be greater. As will be seen in section 3, good regulation requires low-resistance windings and low magnetic leakage, neither of which is true of the low iron-loss design. The feature of poor regulation found in the arrangement can be expected on general grounds since the series losses (i.e. copper losses) have been increased by decreasing the shunt losses (i.e. iron losses) which do not have any effect on the output voltage.

In most distribution transformers of normal design the ratio of full-load copper to iron loss is between 2 and 3 to 1. The difficulties already mentioned set a limit to the ratio of between 4 and 5 to 1.

Other transformers, such as those feeding motors working at constant load, are never worked light so that the iron losses are not so important. Others again may work over a regular load cycle and the best proportions can be worked out to enable the highest efficiency to be attained over the cycle.

In order to provide a basis for comparison between different designs, it will now be assumed that the total losses at full load are the same for all designs. This means in effect that the comparison is made between transformers of equal external dimensions, since it is the surface area which ultimately determines how much heat can be dissipated. If the full-load losses are the same for all designs, the area required to dispose of the heat represented by these losses will also be the same. It is, of course, assumed that the same methods of cooling are used for each transformer, i.e. natural air, forced air, natural oil circulation, or water-cooled oil. Changing from natural ventilation to forced air blast would increase the heat which could be dissipated from a transformer of a given size and so increase its possible kVA rating. It is not intended to deal here with the relative sizes achieved by different cooling systems, but to examine the effect on the efficiency of apportioning the total full-load loss in various proportions between iron and copper.

Let the ratio of copper to iron loss at full-load be x.

Then $x \times W_f = I_F{}^2R$, where I_F = full-load current.

But at the point of maximum efficiency, the copper loss is equal to the iron loss.

$$\therefore \qquad W_f = I_D{}^2R$$

where I_D = current at maximum efficiency.

$$\therefore \qquad \frac{I_D}{I_F} = \sqrt{\frac{1}{x}}$$

$$= \sqrt{\frac{\text{iron loss}}{\text{full-load copper loss}}} \quad . \quad . \quad . \quad (30)$$

For example, if the full-load copper loss is four times the iron loss, the maximum efficiency will occur at half-load.

Fig. 25 shows graphically the effect of varying the ratio of full-load copper loss to iron loss, keeping the total full-load loss constant at 4 per cent. These curves show clearly the advantage of departing from the equality ratio when a transformer is working with a variable load; since the load will not be allowed to exceed appreciably the full-load figure, any variations must be downwards. The 1 : 1 loss-ratio transformer begins to drop in efficiency whilst the 4 : 1 rises to a figure *higher than the* 1 : 1 *type ever reaches*. As the *average* load is less than full load, the 4 : 1 loss-ratio type is bound to have a better average performance.

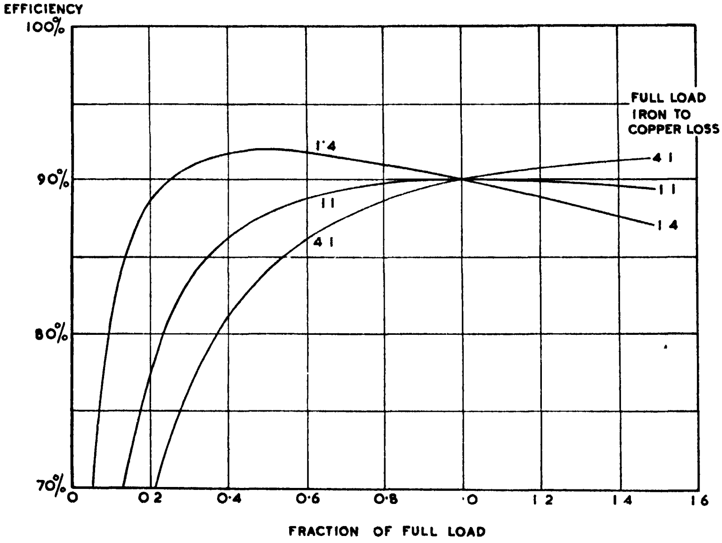

FIG. 25

The Efficiency of Any Transformer is a Maximum when its Copper Losses Become Equal to the Iron Loss. When the Ratio of Iron to Copper Loss is Low, the Efficiency Reaches a High Value at a Low Load.

The considerably improved efficiency that results from working a 4 : 1 loss-ratio transformer at half-load is another way of stating that the full-load efficiency of any type of machine can always be increased by making it larger; this is in effect what has been done by working a transformer of given rating at half-load. At this operating point the 4 : 1 transformer has equal copper and iron losses and so may be regarded as a larger design of transformer having maximum efficiency at a full-load of half the original figure. The efficiency of any transformer of a given kVA rating can always be increased by increasing its dimensions. The iron loss can be *reduced* by using a larger core notwithstanding the increase in weight of iron, because the flux-density and hence the loss per pound can be reduced at a faster rate than the weight goes up. Similarly by increasing the weight of copper the copper loss is *reduced* because the resistance falls more rapidly on account of the larger cross-section than it goes up on account of the greater length of turn in a larger coil.

These points have been mentioned to emphasize that the efficiency of a transformer can be made as high as the purchasing abilities of the customer allow. It is too often assumed that the efficiency is a quantity which comes out at a certain percentage for a particular rating and method of construction, and has to be accepted. This idea arises from the basis of rating the permissible kVA output. The normal rating is derived from a determination of the highest output which can be delivered continuously without exceeding a stated temperature at any part of the windings. At this output the total losses will reach such a figure that the heat generated can only just be removed fast enough by the particular method of cooling which has been adopted. If the method of cooling is now improved by some means, such as the substitution of forced air circulation for natural circulation, the rating can be increased without altering the electrical design at all. The *efficiency* of the device will in all probability *decrease*, because it would already have reached or passed equality of iron and copper loss at its previous full-load rating. The only way to increase the efficiency again would be to attempt to decrease the copper losses at the expense of iron losses by reducing the turns per volt and so putting up the flux-density in the core. Very often, however, this would result in excessive magnetizing current or harmonic content and no improvement can be made. The introduction of improved cooling, whilst it has enabled a larger output to be obtained from a given frame size, and so probably reduced cost, has produced a less efficient transformer. On the other hand, by reducing iron and copper working densities a more efficient but larger design could be produced. Efficiency is thus closely related to cost; *the normal* *design based on maximum permissible heating considerations gives the low limit of efficiency.*

Measuring Efficiency

The most straightforward method of measuring the efficiency of a transformer would appear to be to connect it to the correct supply voltage and frequency on the input side and apply the desired load to the secondary, taking measurements of input and output power. In practice, however, this method has serious disadvantages and is employed only for very small transformers. First, it is not likely to be very accurate, and, secondly, in testing large transformers it would require very large power supplies which might not be available; in any case the waste of energy would involve considerable cost. The efficiency is, therefore, nearly always calculated from the losses, which are measured as iron and copper losses by separate tests.

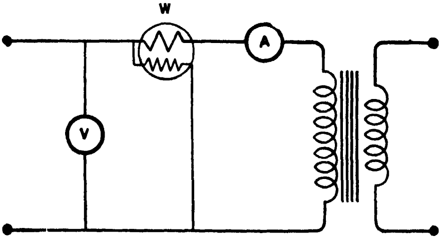

FIG. 26
Circuit Arrangements for the Open-circuit Test Used for Measuring the Iron Loss.

The *iron loss* is first measured by connecting the primary to a supply of the correct voltage and frequency, and measuring the power input by means of a suitable wattmeter (Fig. 26). The no-load current can also conveniently be measured at the same time. The whole power input in this test is being dissipated as losses in the transformer, since the secondary is on open circuit. These losses are almost entirely the iron loss, although there is a small loss in the primary due to the flow of no-load current through its resistance. In small transformers it may be advisable to correct the loss figure obtained from the wattmeter reading by subtracting the small primary no-load copper loss. The iron-loss figure obtained from this open-circuit test is assumed to remain constant at all loads—an assumption which is amply justified. If the primary voltage is a high one, the transformer is frequently excited from the low-voltage side for convenience. The result is the same provided that the same flux-density is reached in the iron.

It sometimes happens that in testing a large transformer on a somewhat inadequate supply, the flow of non-sinusoidal magnetizing current causes some distortion of the voltage wave applied to the primary. This effect results in a different value of iron loss from that which would be obtained from the sinusoidal applied voltage on which the transformer would be rated. In order to correct the iron loss measured to that which would be obtained under sinusoidal conditions, the eddy-current and hysteresis components have to be separated from each other; the hysteresis component is then corrected and added to the eddy-current component which is independent of waveform. If a supply at a different frequency is available, the hysteresis and eddy-current components can be separated by measuring the total iron loss *at two frequencies at the same flux-density in each case.* This means, of course, altering the applied voltage in proportion to the frequency. Now the hysteresis loss is directly proportional to frequency, whereas the eddy-current loss varies as the square of the frequency. Put in another way, the hysteresis loss *per cycle* is independent of frequency but the eddy-current loss per cycle is proportional to frequency. A curve is, therefore, plotted of $\frac{\text{iron loss}}{\text{frequency}}$ against frequency, and extrapolated to zero frequency. The intercept on the $\frac{\text{iron loss}}{\text{frequency}}$ axis will evidently represent the hysteresis loss per cycle, and the hysteresis loss at the rated frequency can then be obtained. To correct the hysteresis loss for waveform, it is necessary to make use of Steinmetz's Law or a corresponding law using a higher power (p. 67). This law states that the hysteresis loss is proportional to the n^{th} power of the peak flux-density, n being between 1·6 and 2·0 according to the actual value of B. The peak flux-density under non-sinusoidal conditions is related to that under the desired sinusoidal ones by the ratio of the form factor of the actual voltage wave to that of a sine wave, i.e. 1·11. The corrected figure for the hysteresis loss is thus obtained from the measured hysteresis component by multiplying by—

$$\left(\frac{\text{Form factor of applied voltage}}{1{\cdot}11}\right)^n$$

Returning to the measurement of transformer losses, the *copper loss* is next determined by the short-circuit test. The secondary winding is completely short-circuited and a greatly reduced voltage is applied to the primary terminals (Fig. 27). The applied voltage is adjusted so that the primary ammeter reads the load current at which it is desired to find the copper loss. The power input to the primary is now measured. In this case practically all the power input is going into copper loss, the iron loss at the small fraction of normal primary volts being negligible. The wattmeter reading is thus taken to be the copper loss at the particular load in question.

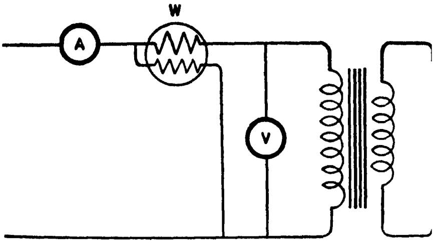

Fig. 27

Connexions for the Short-circuit Test for Measuring Copper Loss and Impedance.

The efficiency can now be calculated at any output by making use of equation 28,

i.e. Efficiency

$$= 1 - \frac{\text{losses}}{\text{output} + \text{losses}}$$

3. Voltage Regulation

It is an unfortunate fact that, like most electrical appliances, a transformer does not maintain an absolutely constant output voltage when supplying a varying load. As the current taken from the secondary is increased, the voltage at the terminals of the secondary tends to drop. This behaviour is referred to as the regulation of the transformer, a small variation of output voltage with load being spoken of as good regulation. This term has been commonly applied to various kinds of electrical apparatus for many years, but in view of the recent introduction of various forms of automatic regulating transformers it is rather a confusing one. Regulation in the present sense, meaning variation of output voltage with load, must not be taken to imply that the apparatus exercises any regulating (i.e. stabilizing) effect on the supply.

More precisely, the regulation expressed as a percentage is defined as the secondary voltage drop between no-load and full-load, divided by the voltage on load and multiplied by 100, i.e.

$$\text{Regulation} = \frac{E_s - V_s}{V_s} \times 100 \text{ per cent} \quad . \quad (31)$$

Just how troublesome the output voltage variation is, depends on the duty the transformer has to perform. A transformer always working at constant load can have quite a poor regulation, provided that the windings are calculated for the particular load expected. Good (i.e. low) regulation in distribution transformers is desirable from the point of view of the consumer, because it prevents low supply voltage during busy periods and excessive voltage at periods of light load. There are, however, limits to the goodness of regulation, partly for reasons of expense and partly for the protection of the transformer under fault conditions. If the regulation is very low, the currents flowing under short-circuit conditions are very high, and damage is likely to result. A high regulation figure means that the impedance is high and the fault currents are restricted to values which can be handled for a long enough period for the protecting gear to save the transformer. In a large transformer the worst effects of a short-circuit are mechanical. The forces exerted on the conductors are proportional to the square of the current flowing, and hence become so great that serious damage may be caused long before overheating troubles appear—only a high regulation can keep these forces down to manageable values.

In the elementary treatment of the theory of the transformer given in Chapter III, the only source of impedance (and, therefore, of regulation) has been the resistances of the windings. In some cases the impedance is due more to the effects of magnetic leakage between the windings than to resistance, at any rate in large transformers and at loads of low power-factor. The effect of this leakage is to give an apparent reactance to the windings as well as a resistance. The load current flowing through this reactance causes a voltage drop, but owing to the fact that it will be in quadrature with the resistance drop for a unity power-factor load, the effect of the reactance drop will be lower than when the load is inductive. For the present the effects of leakage reactance will be neglected; they will be examined in Chapter VIII.

As regards the effect of winding *resistances* on regulation, the effect of secondary resistance on output voltage is quite straightforward, and can be calculated directly by Ohm's Law, viz. the voltage drop at a current I_s amp is $I_s \times R_s$ volts, where R_s is the resistance of the secondary. The turns must, therefore, be calculated to produce an extra voltage on open circuit of I_sR_s volts to allow for a drop of this magnitude at the working load of I_s amp. This, however, is not the only allowance which must be made.

The reaction of the secondary current back on to the primary must not be forgotten. As a result of the demagnetizing ampere-turns of the secondary, an increased primary current flows, and a corresponding voltage drop takes place in that winding. Continuing for the moment to neglect the fact that the magnetic field between primary and secondary is not quite complete, the increase of primary current, due to I_s in the secondary is—

$$\frac{I_s}{T_p} \times T_s$$

where T_p and T_s are the numbers of turns on the two windings.

In addition to this load component, there is the magnetizing current which, in general, will not be in phase with the load component. We can, however, estimate the total primary current on load; and from this the voltage drop can be calculated. This drop can, as before, be assessed as a proportionate number of winding turns, but in this case the turns are deducted from the previously calculated primary turns. This follows from the fact that the primary drop prevents the full mains voltage from being available, and the primary turns have, therefore, to be arranged to be suitable for the nominal input voltage, less that dropped in the winding.

For calculations of regulation it is often convenient to obtain a figure for the added resistance which the primary appears to transfer to the secondary, by reason of the extra current flowing in the primary due to the load. Let n represent the ratio of primary to secondary turns (i.e. the transformation ratio): then the primary current increase due to the load is I_s/n, and the corresponding primary loss is $\frac{I_s^2}{n^2} \times R_p$. We could imagine this loss transferred to the secondary, by connecting a resistance of value R_p/n^2 in series with secondary. The same number of watts would then be lost in this resistance, owing to I_s flowing through it, as would be lost in the primary resistance by the load component I_s/n.

The transferred resistance of the primary, as it appears in the secondary, is thus R_p/n^2. Hence, adding this value to R_s, we obtain a figure for the total effective resistance of the transformer, and the voltage drop at any output current can then be estimated directly. This method is applicable to a transformer with several secondaries, provided that the appropriate turns ratio is used.

Although it has been stated that the results of magnetic leakage can be neglected in small transformers working at frequencies below 100 cycles on unity power-factor loads, there is one fairly common application in which the leakage may have some effect. This is in transformers supplying rectifiers with reservoir capacitors. The pulse of current taken by the rectifier over the short conductive period, has the effect of reducing the secondary terminal voltage appreciably and the output from the rectifier is lower than would have been expected from the equivalent resistance figure. This is not really an exception to the opening statement of this paragraph, because the sharp current pulse contains harmonic components (see Chapter X) having much higher frequencies than the supply. Since the leakage has the effect of adding an apparent inductance to the circuit, the voltage drop is proportional to frequency and may become appreciable in this arrangement.

4. THE PERCENTAGE SYSTEM OF QUANTITIES

It is often possible to simplify transformer problems, particularly those concerning efficiency and regulation, by the expression of the various quantities involved in terms of a percentage value rather than in volts, amperes, or watts. For instance, the iron loss can be expressed as a percentage of full-load output, the magnetizing current as a percentage of full-load primary current, the secondary voltage drop due to the resistance of the windings as a percentage of the secondary voltage, etc. Although there is a gradual change as the size of transformers increases, many of the quantities expressed as a percentage are similar in magnitude for quite widely different ratings. The magnetizing current for a small 230-volt 50-watt transformer may be 0·04 amp, whilst that for a 10-kVA transformer working on this voltage may be 4 amp, yet the percentage magnetizing currents are 17 per cent and 9 per cent respectively. Again, the percentage does not vary with the input voltage for which a given type of transformer is designed. If, for instance, the 10-kVA transformer had been arranged for 3 300 volts, its magnetizing current would have been the quite different figure of 0·28 amp, yet the percentage value would still have been 9 per cent.

A quantity which is more frequently expressed as a percentage than any other is the reactance. The basic quantity used for comparison is the impedance of the load, i.e. terminal voltage/full-load current. The percentage reactance is then

$$\frac{\text{reactance}}{\text{load impedance}} \times 100$$

The reactance drop at full-load as a percentage of normal voltage is numerically equal to the percentage reactance.

The percentage reactance varies from about 1 per cent or less in very small transformers up to about 10 per cent in very large sizes working at high voltages.

REFERENCES

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(2) G. C. RICHER: "Electrical Sheet Steel," *Physics in Industry—Magnetism*, Lecture II, Institute of Physics, 1938.

A stamping which is of special interest is the "no-waste" E and I, in which the dimensions are so arranged that the I's are produced from the spaces in the E's. Two E's are grouped together facing each other so that the spaces between the limbs are continuous in the pair (Fig. 30). It is evident that the width of the "window" must be equal to the width of the outer limbs in this particular stamping, and this proportion is not regarded with favour by some designers. In the U.S.A. the method is widely used with appreciable reductions in cost.

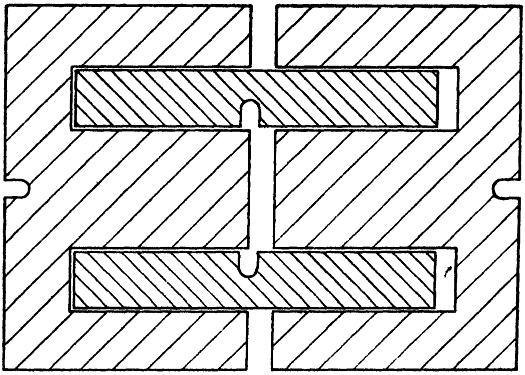


FIG. 30

The Method of Making "No-waste" E- and I-Stampings (Right-hand E Slightly Displaced for Clarity).

The maximum size generally produced in T- and U-form is about 10 in. × 12 in. overall. In larger transformers cores are built up from rectangular strips. This is partly because the waste of material in making T- and U-pressings of large size would be prohibitive and partly because the tools required to blank out such stampings would be very large and the laminations when produced would be difficult to handle.

The use of separate rectangular strips also facilitates the adoption of a "stepped" build-up of the lamination stack so as to produce an approximation to a circular cross-section. Small cores are usually square in section, or sometimes rectangular when it is desired to produce a transformer of a rather larger output than can be produced from a square section and the increase in output is insufficient to justify the use of a larger lamination. The use of a stack giving a centre limb (in the case of a T- and U-form) up to one and a half times as deep as it is wide is usually worth while, although the difficulties of winding at high speed may be appreciable. Larger departures than this from the square form result in an undue rise in the resistance of the windings with consequent increases in copper losses which greatly offset the effects of the greater core cross-section. They are justifiable only on grounds of expediency in commercial production on account of standardization of laminations with other transformers of smaller rating. The ideal cross-section is, of course, circular, since the length of a turn is then a minimum for a given cross-sectional area. The circular form is also best from the point of view of ease of winding, so that the larger transformers are always made with circular coils and cores built up from laminations of varying width arranged to fill up the circular space within the coil as completely as possible.

When the limbs of the core are stepped in this way, the yokes are also stepped in a similar manner. Care is taken that the section area of each step in the yoke is equivalent to that in the limbs so as to avoid any cross-movement of the flux. Unless this is done, the reluctance of the iron path is increased owing to the flux having to pass sideways through the laminations and hence through air-gaps between them, and also heavy local eddy-currents are likely to be set up.

Apart from the obvious loss of cross-sectional area which arises from the use of a rectangular section to fill a circular coil centre, another reduction in *effective* core section occurs as a result of the non-magnetic insulating material between the stampings. Between individual laminations there are always small air spaces due to irregularities in the sheets, and very often there are in addition layers of oxide scale adhering to the iron. These natural insulating layers are often supplemented by painting the stampings on one side or covering them with a thin layer of kaolin (china clay). More substantial insulation is sometimes provided by sheets of paper. As a result of the introduction of these materials, the useful cross-section is less than that indicated by the external dimensions of the stack of laminations. The ratio of the effective cross-section to the gross cross-section is known as the *stacking factor*.

The thinner the laminations, the lower will be the stacking factor, but the method of clamping also affects the figure to some extent. The following values are representative of those obtained in practice in small transformers—

TABLE III

STACKING FACTORS

THICKNESS OF STAMPINGS	STACKING FACTOR
in.	in.
0·025	0·95
0·020	0·94
0·014	0·91
0·007	0·80
0·004	0·70
0·002	0·60

Relative Dimensions of Limbs and Yoke

The general practice when designing a lamination is to make the effective cross-sectional area the same at all points of the magnetic path; that is to say, in a shell-type core the side limbs and the yokes are half the width of the central limb. Since the path divides into two at the ends of the central limb, the flux-density is the same at all points, apart from small effects due to leakage and at the corners.

It is possible, however, to reduce iron losses without at the same time adding to the copper losses by increasing the cross-section of the yokes and outer limbs. This does not of course apply to the centre limb because any increase there would immediately put up the mean turn length and so add to the copper loss. Stampings of the form described are quite often used for audio-frequency transformers, particularly when nickel-iron alloys are used (see Table XXIV, p. 483).

Improved Methods

The hysteresis loss in silicon-steel sheet falls as the size of the individual crystals in the material increases. This desirable state of affairs is brought about by cold-rolling and subsequently annealing. The magnetic properties of steel treated in this way are markedly improved along the direction of rolling(2) so that any construction attempting to make use of these improved materials must employ them in such a manner that the flux follows this direction. This leads to a method of assembly in which a long continuous strip of cold-rolled core material in the form of a clock-spring is threaded repeatedly through the wound coils to build up a solid cylindrical core(3) (Fig. 31). Originally developed for instrument transformers where the best characteristics are demanded, this method is now being applied to small power transformers where a reduction in size is required without any sacrifice in performance.(4, 5)

Part of the advantage of cold-rolled strip can be obtained in the stamping-type core by using rectangular strips (cut with their longer dimensions along the grain) and making mitred joints at the corners. It would also appear worth while to use grain-oriented strip for the "no-waste" E- and I-stampings since the I's are punched in a favourable direction. It is only in the comparatively short lengths of the yokes that the flux would cross the grain.

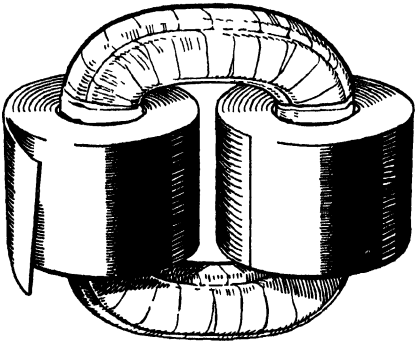


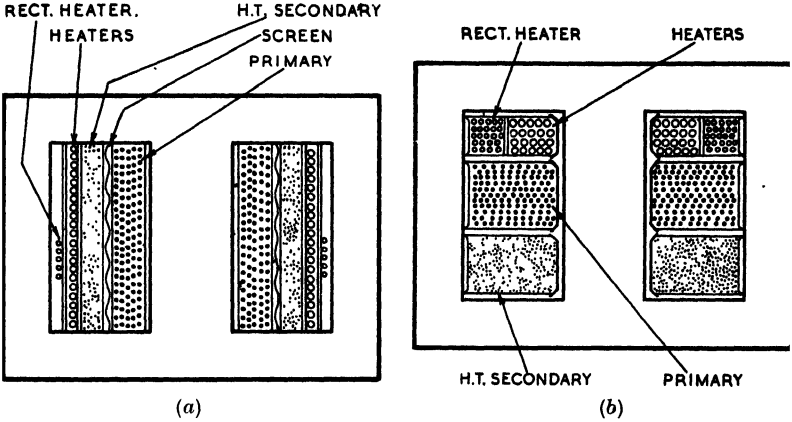
FIG. 31

A Small Power Transformer Employing the New Wound-core Technique.

2. THE WINDINGS

The conductors used in small transformers are nearly always round in section, but in larger sizes or for special applications, such as spot-welding types, rectangular strip is used for the heavy-current windings. The wire or strip may be covered with any of the insulating materials normally used for such purposes, such as enamel, silk, paper, or glass. The choice of the particular material is governed chiefly by the nature of the insulating fluid or impregnating substance which fills the spaces between the turns of the winding. Transformers used in the audio-frequency stages of communication apparatus are sometimes constructed without such impregnation, but almost all power transformers, even of the smallest sizes, are treated with liquid or solid impregnants to exclude moisture and prevent breakdowns between windings or between turns of the same winding.

The proportions of the primary and secondary coils vary according to the shape of the window in the core. It is important that a very high percentage of the flux produced by the primary should link the secondary and vice versa. Any flux which does not do so is known as *leakage flux*, and it gives rise to *leakage reactance*, a term explained in Chapter VIII. This effect is of greatest importance at higher frequencies, but some arrangements have to be made even at 50 cycles to minimize the leakage. For this reason, the coils are shaped so as to give the greatest magnetic coupling between primary and secondaries. If the window is long in the axial direction, it is best to make the coils extend the full length (along the axis) and be comparatively thin in a radial direction. On the other hand, if the window is of the short, wide type, the coils are best made short axially but of the full radial width of the window. The two types are shown in Fig. 32. If the voltages generated by (or applied to) a transformer are unusually high relative to the power rating, e.g. 10 000 volts from a 50-watt size, special arrangements have to be made which may not conform to the general rules for the reduction of magnetic leakage. It often happens, however, that a relatively large leakage reactance is not of great importance in these cases and no undue difficulties are encountered.



(*a*) (*b*)

FIG. 32

Two Methods of Winding a Small Transformer for a Radio Receiver. (*a*) Layer Type. (*b*) Side-by-side or Bobbin Type.

The method of supporting the windings is largely decided by the size of the conductors involved. A framework suitable for carrying 0·5 in. × 0·1 in. strip is naturally not ideal for winding SWG 44 wire, and vice versa. Since the size of the conductors increases in a general way as the rating of the transformer goes up, the constructional methods can be grouped with the power outputs, although of course there are considerable overlaps between the groups.

The first group comprises transformers having outputs up to 10 watts. This includes many types used in telephone, radio, and similar equipment that deliver negligible power, e.g. the input, intervalve, and output types. The method of winding adopted is almost always to use a single bobbin upon which the different windings are wound in turn with layers of paper or fabric between them to provide inter-winding insulation. The bobbin is best moulded from a plastic material such as phenol-formaldehyde, provided that precautions are taken to see that the moulding is fully cured during manufacture so as to avoid any risk of corrosive gases such as ammonia being liberated after the windings are in place. Built-up bobbins from fibre or other impregnated sheet material are often used, although their lack of rigidity as compared with the moulded type makes them less satisfactory, particularly when very fine wires are used. The stresses tending to collapse the central tube and to bulge out the cheeks are very large, as will be realized when it is remembered that each of some 10 000 turns may be applied under a tension of a pound or more. The wire covering adopted in these very low-power types is often enamel, or preferably one of the newer materials such as "Lewmex," which combine the advantages of small space possessed by the normal enamel with greatly improved abrasion-resisting properties. Enamel is quite satisfactory for many audio-frequency applications where the occurrence of a short-circuited turn would only cause unsatisfactory performance and not introduce any fire or other consequential risk. It is often not possible to impregnate coils of this type because the resulting increase in self-capacity has a serious effect on the performance of such transformers. Small-power transformers are, however, generally impregnated, and this process is dealt with in the next section.

Merging into the first group of ratings are the class of small-power transformers ranging up to 500 watts, and employing what is known as "paper-interleaving." Windings of this type are built up on a central tubular former of some rigid insulating material such as synthetic resin-impregnated paper board of a suitable section for the core (usually square). No cheeks are attached to the former in this construction, although loose discs of insulating material may be placed in position after the winding is complete. A foundation is first laid by taking a few turns of the interleaving paper round the former before the wire is applied. The paper is good quality kraft paper having a thickness of 0·001 in. to 0·006 in., according to the electrical and mechanical stresses to which it is subjected, and it is supplied in rolls of the correct width for the particular winding. Having prepared the starting end of the wire in a suitable manner for leading out, as, for example, attaching a section of flexible wire, a layer of wire is wound on, care being taken to keep the turns close together but without any crossing each other. When the layer is completed to within a reasonably close distance from the end, the required number of layers of paper are put on, and the next layer of winding continued. In this way the complete winding is built up with paper insulation between successive layers of wire. The inter-winding insulation is usually also formed from a number of turns of interleaving paper although, if the voltage between windings is high, it may be advantageous to use other materials.

Paper-interleaved windings must always be vacuum-impregnated with a suitable insulating compound to exclude moisture. Paper, although an excellent insulator when quite dry, readily absorbs moisture from damp atmospheres, if not treated with a wax-like material of some kind. Paper interleaving is sometimes used for transformers of the audio-frequency type (falling into the first group of types up to 10-watts output) but the risks of electrolytic corrosion are quite serious with fine wires when there is a d.c. potential between two windings or windings and frame unless great care is taken to use paper with a very low acid content. Unfortunately the nature of the circuits in which these transformers are used is such that the fine-wire winding is positive in potential relative to the rest of the assembly, so that the least trace of electrolysis results relatively quickly in the winding becoming open-circuited by anodic corrosion.

The paper-interleaved construction lends itself to the simultaneous winding of a number of coils on a common mandrel. Ten or more coils can in this way be wound in little more time than it takes to wind one on a single machine (Fig. 33). The time-saving factor is greatest when the winding is a straightforward one of many turns without taps, etc. Separate formers are threaded on to a long mandrel provided with suitable clips to hold them in position with a short space of approximately $\frac{1}{8}$ in. between each. The purpose of this separation will be apparent shortly. Each coil is, of course, supplied from a separate reel of wire which is complete with its own tensioning arrangements. The interleaving paper, however, is often used in one width equal to the width of the whole assembly and not in a number of narrow strips. This makes for even feeding-in of the paper, which is done in many machines automatically without appreciable slowing down. The length of paper wound in this way is also automatically increased progressively as the diameter of the winding gets greater as each layer is added. When completed, the multiple winding is divided into separate assemblies by a rotary knife which is applied at the correct points and passes between the central formers in the spaces which have been left for it. Since the ends of the windings cannot be brought out from the edges of the bobbin in the usual way, various techniques have been evolved for dealing with this difficulty. They all have this much in common that the wire which will eventually form the lead-out is housed in a compact form under the layers of paper at the end of a winding and is extracted with a pair of tweezers from the edge of the coil, after the cutting-apart process. The lead-out wire can be a piece of flexible conductor compressed into a zig-zag shape and soldered to the winding wire, but a commonly used method is to take several extra turns of the winding wire itself round the former suitably anchored to the underlying interleaving paper, after the correct number of turns for the coil itself (Fig. 34). These surplus turns are subsequently drawn out and form a multi-strand lead-out. Only one strand of this wire is, of course, in use electrically, the others being merely for mechanical support.

Transformers rated at 500 watts to 5 kilowatts vary in lay-out according to the method of cooling which is employed.

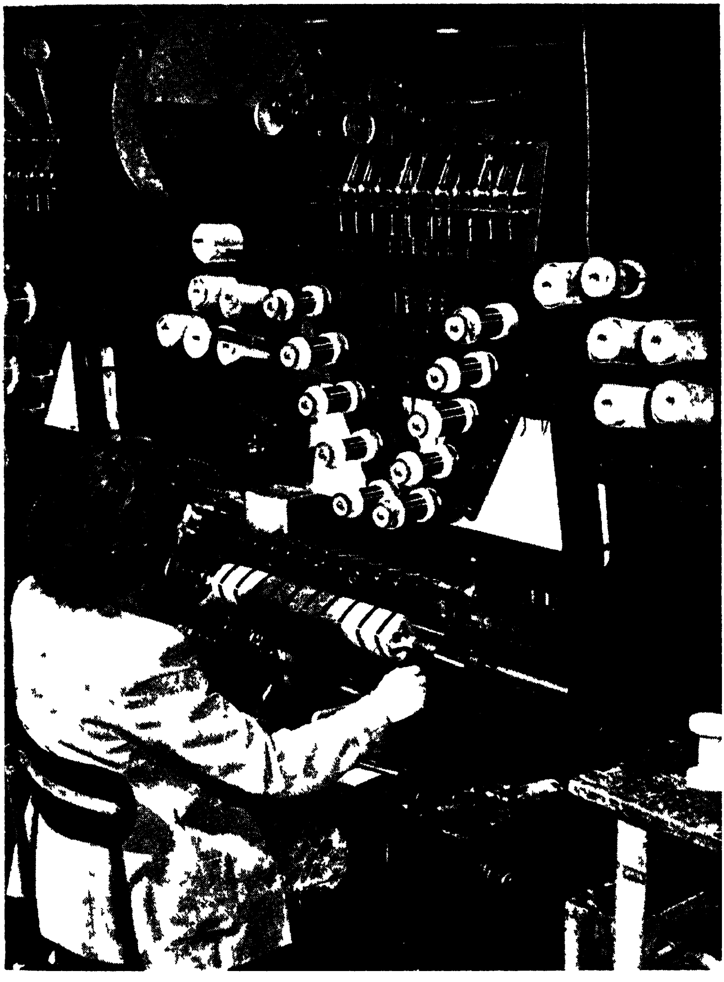


Fig 33

A Multiple winding Machine at Work Ten Coils are Being Wound Simultaneously.

Air-cooled types are often paper-interleaved in a manner similar to that adopted for the smaller types, although it may be advantageous to wind the primary and each secondary as separate parts and assemble them on to tubes of solid insulating material with discs of similar material to provide separators. This is particularly useful for high-voltage windings, which can be wound in sections and assembled so that any points between which there is a large potential difference are either kept well apart in air or have a barrier of solid insulation between them.

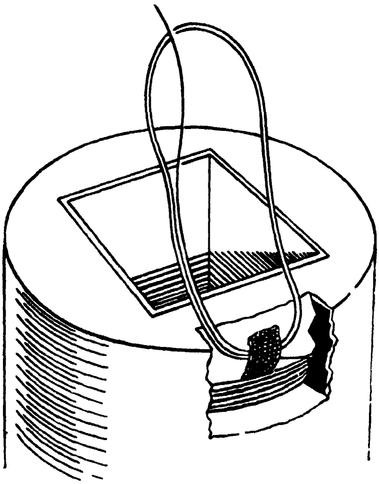


FIG. 34

The Method of Strengthening the Lead-out Wires when Multiple-winding Machines Are Used. (A Portion of the Coil is shown Sectioned to Uncover the End of the Winding.)

In practically all the smaller sizes, the central tubular former on which the winding is built up is left in place after completion, but it does become practicable in larger sizes to wind on a collapsible mandrel which is removed after the coil has been made self-supporting either by impregnating compounds or naturally, if of heavy-section conductor. If this is done, the individual coils are bound toroidally with tape for mechanical protection when placed on the core.

Oil-cooled types of this rating follow the lines described for larger transformers of this kind, although some simplifications may be possible.

To avoid uneven heating and to improve cooling generally practically all transformers having power outputs exceeding some 5–10 kW are oil-cooled. The windings are supported in open skeleton frameworks through which the oil can flow freely by either natural or forced circulation. The hot oil is cooled by radiator tubes connected to the tank housing the transformer and themselves cooled naturally or by fans.

The need for special cooling measures of this kind is much greater in the larger sizes of transformers because the ratio of surface area (which decides the rate at which heat can be dissipated) to the volume (which determines the rate of generation of heat) decreases in proportion to the size. This statement assumes, of course, that the same flux-density is used in the iron and the same current-density in the copper for all sizes. To use lower flux-densities and current-densities would be to make inefficient use of the materials, and this would increase the size of the transformer considerably. It might be thought that it would be possible to work transformers of lower ratings at higher flux-densities still by using artificial cooling methods, but this is not the case. Certainly as far as the iron goes this is an essential limitation because any attempt to raise the flux-density results in the generation of excessive harmonic percentages. It is also true that the current-density in the copper cannot be raised when natural cooling methods only are employed, because the temperature gradient across the insulation immediately surrounding the wires would be very high. Oil-cooling, by withdrawing the heat directly from the point at which it is produced, does in fact allow some increase in current-density, particularly during heavy overloads of short duration.

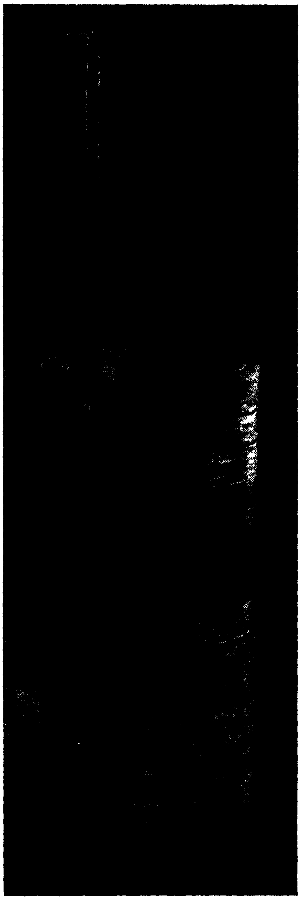


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FIG. 35

A "Helical" Type Coil for Heavy-current Windings.

As already mentioned, the coils used in oil-cooled transformers are of a skeleton type so as to allow free circulation of oil round the conductors. Since the coils must for this reason be largely self-supporting, it is necessary to use different methods according to the size of the conductor. The three types of coil in common use are: helical, disc (or pancake), and cross-over.

The helical type (Fig. 35) is used for currents of 100–200 amp and higher. The conductor is usually a bundle of square or rectangular copper strips taped together as they are wound on to a solid cylinder of insulating material to form a single-layer winding. The cotton covering used as insulation on the strips themselves is supplemented by the tape which binds the various strands together, but when special insulation of the end turns is required to withstand voltage surges which occur in distribution networks from lightning discharges and faults this can easily be provided by additional spacing of the turns.

Disc-coils used for moderately large currents are wound in the form of a flat spiral (Fig. 36), the conductor or bundle of conductors being wound on its wider dimension, as compared with the helical type, which are edge-wound. The inter-turn insulation is assisted, if necessary, by winding in a continuous strip of tape or presspahn with the conductor. Flat discs or washers are placed between the coils when they are assembled so as to provide insulation between sections.

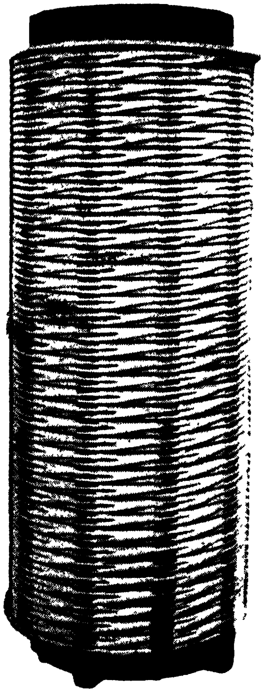

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FIG. 36

An 11-kV Continuous Disc Winding for a 5000 kVA Transformer. Three Conductors are Used in Parallel and the Transposition between each Disc can be Clearly Seen.

Cross-over coils (Fig. 37) are used for low-current windings and round-section wires are employed. The coils resemble narrow paper interleaved ones in which the paper is replaced by insulating board or varnished cloth, the whole assembly being wound on a former which is removed after winding and the coil bound in tape. As in the case of the sectional type, a number of coils are placed end to end with insulating discs between, to form the complete winding. All the coils are wound similarly, the inner end of each being left long and brought out through an insulating tube to join the outer end of the previous coil.

3. IMPREGNATION

Choice of Compounds

The chief requirements of an impregnating substance are that it shall be a first-class insulator and that it shall exclude moisture and corrosive vapours from the winding under all conditions of temperature. As a corollary it will be understood that it must be of a sufficiently elastic nature not to crack when roughly handled or under temperature strains, because this would at once allow moisture to penetrate. It is desirable that the impregnating agent should be easy to get into the middle of the windings and, of course, it must not attack copper or cause deterioration of the insulation material. The number of substances which completely meet all the requirements is very small, if indeed any can be said to have been found. There is nevertheless quite a range of materials each of which is satisfactory in a limited set of conditions.

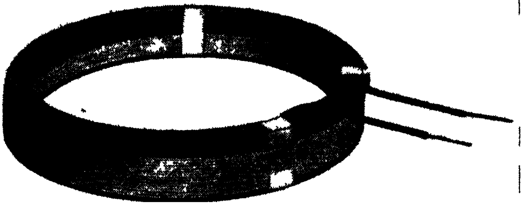

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FIG. 37

A "Cross-over" Coil for Low-current Windings of Large Transformers.

The compound can be introduced in two distinct ways: first in a molten state, and second in solution in a suitable solvent. The molten form is to be preferred on most grounds; extensive baking under vacuum is not required after impregnation in order to extract the solvent and there are fewer air spaces left inside the coil as a result of contraction. Waxes of many kinds are used for impregnation in a molten state and are probably the most generally satisfactory medium. Their rather low melting point is the chief limitation, because although various chlorinated-naphthalene compounds are available that do not melt at the highest temperatures that can safely be used for the insulating materials in the transformer, they are then apt to crack at low temperatures; they have a tendency to attack the enamel covering on wires when molten and they may also in the course of time decompose sufficiently to liberate traces of chlorine.

It has also been proposed that coils should be impregnated with some of the newer thermoplastic moulding materials. The inert nature of such materials as polythene certainly commends them for the purpose, but the difficulties in introducing the viscous fluid into the winding are very considerable.

Bitumen is frequently used as an impregnant and some manufacturers claim to be successful in getting it to enter the coil properly in a molten state. The difficulty is to choose a grade of bitumen which is sufficiently fluid at temperatures below the charring point of the paper or other insulation and yet not liable to slow flow at the working temperature of the equipment. Nevertheless bitumen is a valuable material because it is sufficiently flexible even at low temperatures to stand rough handling without cracking and it has consequently been used frequently in a solution of benzene or similar solvent. In this state it can easily be introduced into the coil, but there are, of course, the disadvantages of a long baking period to remove the solvent and the fact that the interstices are less completely filled.

Phenol-formaldehyde resins appear to offer advantages as impregnating compounds but have proved somewhat disappointing in practice because the completed coil after polymerization of the resin is extremely hard and rigid, so that differential temperature strains cause the breakage of fine wires and cracking of the moisture barriers also occurs. Once minute cracks or crazing of the resin on the wires has taken place, breakdowns quickly occur in damp conditions. This brittleness is not nearly as serious in the oil-base types of synthetic resin varnishes, but a baking time at least as long as that required to drive off solvents from other kinds is a serious disadvantage.

Oil-immersion (as opposed to the use of oil primarily for cooling purposes) is often adopted for the small high-voltage transformers used in X-ray equipment in order to prevent corona troubles. Solid impregnants are apt to leave small air spaces inside the windings, or internal cracks develop, and corona discharges may take place in these spaces where the potential gradient is of course higher than in the surrounding solid medium which has a greater dielectric constant. Oil, being always fluid, does not suffer from this trouble, and even if the oil is punctured by high-voltage surges, the insulation seals itself again as soon as the excess voltage is removed. The construction follows the practice for transformers that would normally be wax impregnated, e.g. paper-interleaving supplemented by solid discs or tubes of insulation between points having high potential differences.

The high-voltage leads must be well separated or suitably sheathed in porcelain tubes where they leave the surface of the oil, the normal method being to make the output bushings sufficiently long inside the case to dip well below the oil level.

Vacuum Impregnation

Although it might be feasible to deal with very small transformers in limited quantities by long immersion periods in the impregnating fluid, it is necessary to follow out carefully a regular schedule of processes including vacuum treatment if good results are to be obtained in quantity production. This is more than ever true of the use of molten compounds which form a relatively viscous fluid. The assemblies prepared for treatment may be coils only, or in the case of smaller transformers be complete with core stampings. There has, in recent years, been an increasing tendency to impregnate complete assemblies so as to avoid the risk of cracking the sealing compound during the necessarily rough handling which occurs during the insertion of the last few stampings when the core space has been nearly filled. In any case the lead-out wires should be brought out to tags or terminals mounted on the bobbin itself so as to avoid any chance of their being broken off during the process. There is also the additional advantage that there is no chance of movement of the wires causing channels to form into the interior of the winding. The first process is a thorough drying out performed by baking first at atmospheric pressure and then *in vacuo.* When all traces of moisture have been removed, the impregnating fluid is allowed to enter the vacuum vessel containing the coils. When any frothing has ceased, atmospheric pressure is restored or even excess pressure applied, so as to cause the fluid to enter every space in the coils. After a suitable interval, the fluid is withdrawn from the vessel and the coils allowed to drain. If the impregnant is a molten compound, the process is now complete, but if a solvent has been used it must be driven off by vacuum baking. Much of the solvent can be recovered during this stage.

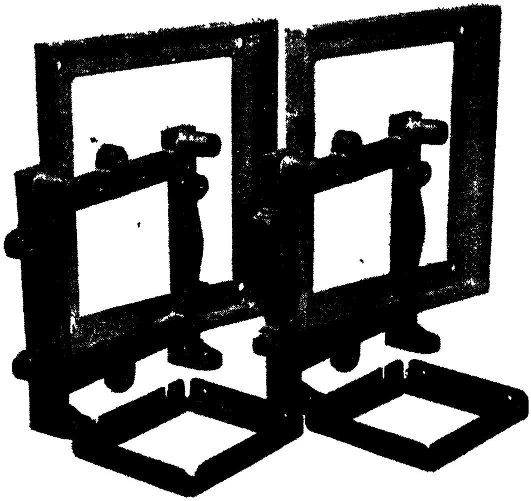
An almost essential accompaniment to vacuum impregnation is a "cold-dip." Owing to the high temperature attained by the body of the coil during the impregnation process, practically all the wax or other compound not actually soaked up by the paper or cotton, has time to drain away from the outer edges before solidifying. This is especially noticeable in those assemblies which have no side cheeks as in the paper-interleaved construction. After the impregnated assembly has cooled, it is given at least one (and often several) quick dips in molten bitumen. The dipping time should be quite short so that the body of the transformer is not heated up and consequently the bitumen solidifies almost at once without draining off. The dips leave a smooth unbroken surface all over the windings and so prevent the penetration of moisture. High m.p. waxes are sometimes mixed with the bitumen to reduce stickiness.

Oil-immersed transformers are in a sense vacuum-impregnated in so far that the windings are thoroughly dried out, the whole evacuated and oil admitted so as to fill all spaces without air bubbles being trapped. No dipping is required, of course, because the oil tank is itself effectively sealed.

If paper-interleaving is used in oil-immersed coils, special precautions are necessary to prevent the end turns of each layer from slipping down when the oil softens the paper. The addition of strips of paper or cloth placed across the winding so as to project from the end until the layer is complete, then folded over on top of the wire before commencing the next layer is one method of retaining the end turns.

4. CLAMPS

When a transformer is in operation, the individual laminations of the core tend to vibrate against each other, causing loud buzzing noises. Careful clamping is essential if these sounds are to be kept down to tolerable levels and the misplacement of a single stamping is often sufficient to cause trouble. The modern practice of impregnating small transformers complete with the core in position minimizes this trouble and enables simpler clamps to be used, which would not otherwise be satisfactory. The design of clamps for a particular core must take into account the positions and directions of the gaps between the sections from which it is assembled. The core plates must be firmly held at these points and arrangements must also be made to prevent the joints from opening during transit with consequent deterioration in performance. Although clamping can be more effectively carried out if the bolts joining the two sides pass through holes in the stampings, it is necessary to use insulating tubes around the bolts, at any rate in



Murphy Radio, Ltd.

FIG. 38

Examples of Pressed, Cast and Fabricated Core Clamps.

transformers of any size, because local heating may be serious if circuits are formed through the stampings and bolts.

Transformers of the smallest ratings are usually clamped by steel pressings applied on the sides of the laminations (Fig. 38).

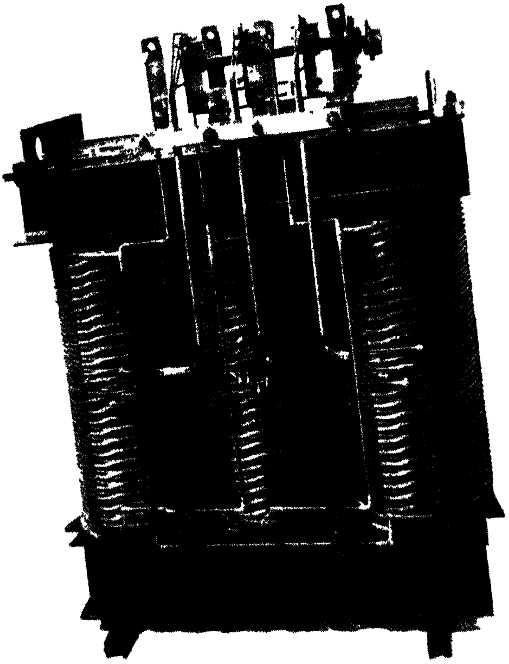


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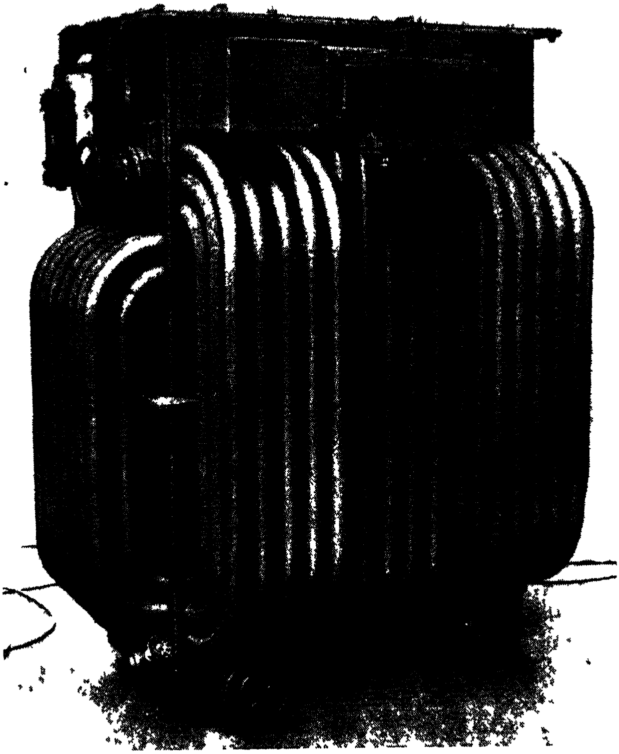
FIG. 39A

500-kVA 3-phase 50-cycle Transformer Fitted with Off-load Tap-changing Switch. Note that the Tappings are Taken at the Centre of the Winding. The Steps in the Yoke are at the Top of the Coil.

These pressings are stiffened by flanging the edges or drawing the central portion into a suitable shape to shroud the coil. The clamps are also arranged to carry terminal panels, tapping adjustments, etc. In this way very neat well-protected assemblies can be made and at the same time, production costs are low, owing to the ease with which pressed-steel parts can be produced. When the dimensions exceed a few inches in each direction, however, it becomes difficult, even with suitable flanging, to produce a sufficiently stiff framework to withstand the very large bending stresses produced when the bolts are tightened up fully, as they must be, to prevent noise from vibrating stampings. Cast clamping plates are often used for transformers up to a few kVA in rating, or alternatively very serviceable assemblies can be fabricated from angle- and tee-section rolled steel by arc-welding methods.



Johnson & Phillips, Ltd.

FIG 39B

The Complete Transformer Seen from the High-voltage Side. Tap-changing Control and Dehydrating Breather at Top Left.

Even castings become impracticable when dimensions are measured in feet instead of inches and the construction then adopted is one which seems to be very closely standardized amongst manufacturers. In these sizes, which are usually oil-cooled, arrangements are made to clamp the central limb or limbs of the core *within the coils*, by bolting on thick flat plates. The bolts which must, of course, pass through the stampings, are carefully insulated from the laminations and the clamping plates to prevent circulating currents. The top and bottom yokes are firmly clamped between channel-section girders whose depth is practically equal to that of the stampings. Only the central bolts need pass through the core, the end ones being arranged to pass across the end faces of the yokes. The channel irons are extended beyond the yokes to carry the end bolts in this way. Finally, the two sets of channel girders are kept in place by vertical tie rods passing through the side faces of the channels and secured by nuts (Fig. 39A).

5. SHIELDS

When power-distribution transformers were first introduced, it was the usual practice to insert an earthed metal shield or screen between the two windings. This shield was intended to prevent any possibility of dangerous voltages from the high-voltage primary side being transferred to the output circuits through a breakdown of the insulation; any insulation faults on the high-voltage side would then be confined to the one winding. A shield of this type must not of course be an electrically closed loop, as this would constitute a short-circuited winding and large currents would circulate round it. There is no difficulty, however, in leaving a small gap radially between the overlapping ends of the metal; the screening is then still complete electrostatically and no ill effects are produced magnetically (Fig. 40).

Screens are no longer fitted for reasons of safety, but in electronic and communications applications they are often required to prevent the transfer of unwanted frequencies from one winding to the other by electrostatic means. Examples are the fitting of a screen between the primary and other windings of the power transformer of a radio receiver to prevent high-frequency "noise" frequencies due to sparking commutators of motors, etc., from finding their way into the set. Again, in input transformers of audio-frequency amplifiers, screens are fitted to stop the entry of "hum" frequencies due to a potential difference between the valve grid circuits and ground. In these cases the shield is actually acting as its name implies as a screen or shield against unwanted frequencies. Recently, however, a device of similar shape, also described as a shield, has been introduced into very high voltage power transformers and potential measuring transformers for the purpose of modifying the distribution of capacity over the high-voltage winding in such a way as to minimize breakdowns due to lightning and switching surges.(6) The shield in this case is simply one plate of a condenser and it is connected to the line terminal. Its purpose is to increase the capacitance of the lower potential parts of the winding to the line and so ensure an initial voltage distribution on steep-fronted transients which does not differ greatly from the steady-state distribution of potential. Used in this way the metal plate is not a shield in the accepted sense, but the term has now become a recognized one. The subject of Transient Protection is considered in Chapter XI.

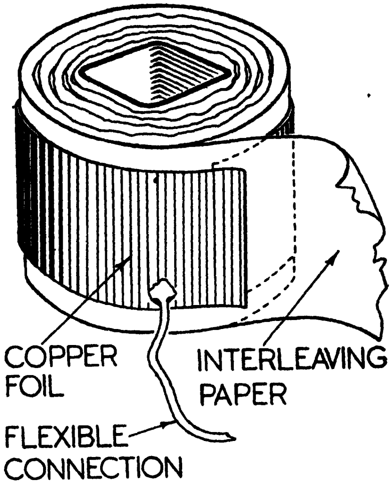


FIG. 40

An Electrostatic Screen of Copper Foil is often Inserted between Windings. A Turn of Paper Prevents the Ends from Touching and Causing a Short-circuited Turn.

6. INSULATING OIL

The purpose of the oil in a transformer may be twofold—

(*a*) To provide a higher degree of insulation between windings, and from windings to frame, than would be provided by an air space.

(*b*) To carry away the heat from the windings and other vulnerable points to the tank walls or to special cooling devices.

The oils in use at the present time are either mineral oil, refined from petroleum crude oils, or are specially prepared non-combustible compounds known under trade names such as "Pyranol."(7) The mineral oils used are saturated hydrocarbons, the unsaturated constituents having been removed as far as is practicable from the crude oil on account of their tendency to form "sludge" owing to oxidation at the high working temperatures.

The chief requirements of a transformer oil are—

(1) High dielectric strength.
(2) Freedom from "sludging."
(3) Low viscosity.
(4) Low volatility and high flash-point.
(5) Freedom from corrosive substances.

Dielectric Strength

There is a British Standards Institution test (B.S.S. 148) for the dielectric strength of transformer oils and this specifies the minimum breakdown voltage between 13 mm (½ in.) diameter polished spheres separated by a distance of 4 mm (0·157 in.). A figure of 30 kV obtained in the B.S.I. test is considered satisfactory for normal power transformers, but for specially high-voltage types a higher figure is advisable. The breakdown voltage is determined almost entirely by the proportion of moisture which has been absorbed by the oil and hardly at all by the quality of the oil. Almost any really dry oil will give the figures required, but only 0·01 per cent of moisture is sufficient to reduce the breakdown voltage appreciably and 0·1 per cent quite seriously. Another impurity which has a bad effect on the oil when present in minute proportions is fibrous matter. Under the influence of the electrostatic field, the minute fibres line themselves up between the high-potential points, so forming a semi-conducting bridge. The fibres may themselves be conducting or they may be made so by the condensation upon them of moisture films. These films form before the fibres find their way into the oil, remain in contact with them, and so contribute to a marked extent to the deterioration of the dielectric strength.

Moisture is most satisfactorily removed by mechanical filtration, but if necessary it can be dried out by the careful application of heat. It is, of course, evident that oil containers should be most carefully dried before the oil is poured into them and that the access of air to the surface of the oil should be restricted as much as possible. When in operation, too, it is advisable to prevent the ingress of fresh volumes of moist air during each heating and cooling cycle. The tank can either be completely sealed and a metal bellows fitted to allow for expansion, or breathing can be allowed through a silica-gel dehydrating system (Fig. 39B).

Freedom from Sludging

Sludging is not likely to be a serious trouble in low-power transformers, because oil-immersion is usually only adopted for insulation and not for cooling purposes. The oil temperatures attained are thus quite low and oxidation does not assume serious proportions below 75°C. For high-temperature working a special grade of oil with very low sludge-forming properties is available.

Low Viscosity

The need for a low viscosity is again connected with the cooling function of the oil and it hardly concerns the users of small high-voltage oil-immersed transformers.

Low Volatility and High Flash-point

It is clearly undesirable for the oil to give off appreciable vapour at even the highest working temperature, partly because the loss of liquid would in time become serious and also because of the risk of explosion should the vapour become ignited by a spark. The chances of such troubles are, however, very small in low-power transformers.

Corrosive Substances

The presence of sulphur compounds in the oil is most undesirable in view of their effect on the copper conductors and insulating materials. Acids and alkalis must also be carefully removed from the oil by refining processes during manufacture.

TABLE IV

MELTING POINTS OF IMPREGNATING COMPOUNDS

BITUMEN (Softening points*)		
Kingsnorth Compound†	440B	49°–54°C
Kingsnorth Compound	374	71°–77°C
Kingsnorth Compound	1 046	118°–121°C
WAXES		
Claude Campbell	AD20	84°C
	LPRM3	86°C
Seekay	R93	90°–95°C
	R113	110°–115°C

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* B.S.S. 688 (Ball and ring method).

† Manufactured by Berry Wiggins, Ltd.

CHAPTER VI

Temperature Rise

IT is usually the temperature rise which sets the limit to the output which may be taken continuously from most electrical machines, and this is particularly true of transformers. Direct-current motors may be limited in output for a given-sized frame by other considerations, such as commutation difficulties, but there are no factors of this kind in the operation of a transformer, which may be loaded until the most vulnerable materials reach their highest safe temperature. If a fixed load is applied continuously, the temperature of the transformer will continue to rise until equilibrium is attained between the heat generated by the losses and the heat lost to the surroundings. The heat produced is more or less independent of the temperature, but the heat lost from the surface increases with the temperature difference between the surface and the surroundings, so that the surface temperature rises until it reaches the temperature at which the rate of heat loss is equal to the rate at which heat is generated internally.

1. WORKING TEMPERATURES

The interior of the transformer is hotter than the outside surface from which the heat is finally lost, the extent of this temperature difference depending on the methods of construction, size, etc. The materials most affected by high temperatures are the insulating ones, for although the resistance of the windings is increased appreciably by the rise in temperature, no permanent deterioration takes place. The core materials now used are also little affected at temperatures up to 100°C, although in the early days of transformers considerable ageing did take place with serious deterioration in performance. The limiting safe working temperature for continuous operation is thus set by the capabilities of the insulating materials. The American Institute of Electrical Engineers has divided insulators into four classes and laid down maximum working temperatures. These range from 90°C for unimpregnated cotton, silk, paper, etc. (Class O), and 105°C for similar materials if impregnated in oil (Class A), to 130°C for mica, asbestos, and glass bonded with synthetic resins (Class B). No particular limit is set to Class C materials such as mica, porcelain, or glass. Class A also includes many thermo-setting moulded materials with wood fillings, varnishes, etc. The experience of transformer manufacturers over many years indicates that a maximum working temperature of approximately 90°C allows a reasonably long life to the insulating materials normally employed. This does not mean, of course, that failure would be immediate if the temperature were raised appreciably above this figure, but deterioration of the mechanical properties followed by electrical failure is likely to be relatively rapid.(1)

If a final temperature of 90°C is assumed, the permissible rise is decided by the maximum ambient (or air temperature) likely to be encountered. If the transformer forms part of an equipment such as a radio receiver, the ambient temperature to be considered is that inside the case of the apparatus. In temperate countries a maximum shade temperature of 35°C is reasonable so that, if a further 10°C rise inside the case is allowed, a transformer rise of 45°C is permissible. Under tropical conditions the permissible rise is reduced, because air temperatures (inside the case of the apparatus) may be 60°C or even higher.

Three significant temperatures are involved in the question of heat dissipation: the air temperature near the transformer, the surface temperature, and the temperature of the hottest point inside the windings. In any particular conditions the air temperature is fixed and so is the permissible temperature of the hottest point. The rated output is determined by the rate at which heat can be dissipated from the surface; this figure increases with the temperature so that it is desirable to allow the surface temperature to approach as near as possible to that of the hottest point—in other words, to keep the temperature gradient between inside and outside of the winding as small as possible. Calculations of the temperature gradient are complicated and unlikely to give more than an approximation to the correct figures. Since good thermal and electrical insulating properties usually go together, and good electrical insulation is essential, it follows that difficulties are to be expected in getting the heat away from the inside of a deep winding if thermal conduction alone is relied upon. For that reason the larger windings are usually built up in skeleton form with ample cooling ducts between individual coils. Oil or air blasts are then used to convey the heat away from the windings, but this does not necessarily ensure that the heat is finally disposed of. In the case of oil-cooled types, the oil itself has to be cooled and, if as is often the case in air cooling, the air is used in a closed-circuit arrangement, a similar problem arises there also. The cooling fluids thus serve only to keep the temperature gradient in the windings to reasonable proportions; the heat has still to be removed finally from the system. The difficulty in doing so becomes progressively greater as the size of the transformer goes up; for if constant flux-densities and current-densities are used, the losses increase as the cube of the linear dimensions of the transformer, but unless special arrangements are made, the surface area from which the heat can be dissipated increases only as the square of the dimensions. In large transformers the oil is circulated through tubes welded into the case, thus effectively increasing its surface area. Even so, forced air circulation through the cooling tubes is necessary in the largest sizes.

Cooling considerations are in fact largely responsible for the progressive change in the forms of construction which has been noted in Chapter V.

(*a*) Very small sizes (up to 1 kVA) can be wound with primary and secondary in one bobbin usually of the paper-interleaved type, the whole of the windings being impregnated with wax or other solid insulating substance. Heat reaches the surface purely by conduction and is then given up to the air by natural cooling methods.

(*b*) Larger sizes (1 to 5 kVA) are usually provided with air ducts between coils or sections. Air is circulated through the windings either naturally by convection or by a forced draught arrangement.

(*c*) Distribution transformers (5 to 30 kVA) are nearly always oil-cooled. The circulation of the oil through and between the windings ensures that no excessively hot spots occur. The hot oil is itself cooled by contact with the walls of the tank.

(*d*) Sizes over 30 kVA need external cooling tubes to cool the oil as the area of the tank walls is not alone sufficient; alternatively the case may be heavily corrugated.

(*e*) In the largest sizes, water pipes may be immersed in the tank to reduce the size of the radiators needed.

The rate at which heat can be successfully removed from a transformer determines the output which can be taken from it continuously. For short periods, higher loadings can of course be applied. If a particular efficiency is assumed, the continuous output rating can be calculated from the size and arrangement of the cooling surfaces; any increase in efficiency enables a greater output to be obtained for the same losses and so increases the nominal rating of a given sized frame. If, for example, a higher silicon-content core material is used, the flux-density for a given iron loss can be increased. The number of turns per volt can then be reduced so that larger section conductors can be used for a given output voltage, and so larger currents can be drawn for the same copper loss. In practice it is not economically feasible to go very far in the direction of improving efficiency by such means and the iron losses and copper losses per unit volume are quite well standardized. That being so, the output of a transformer of a given type of construction i.e. having a particular ratio of insulation to active material, can be calculated from its surface area and temperature, if the rate of heat loss per unit area is known.

2. Rate of Heat Loss from Surfaces

Heat is dissipated from the surface of a body maintained at a temperature of some 40°C above its surroundings by *radiation* and *convection* in rather similar proportions. The heat loss is conveniently expressed in watts per square inch, so eliminating any considerations of time which would be necessary if the usual thermal units were used.⁽²⁾

(*a*) *Radiation Losses*

Stephan's Law states that the heat lost by radiation from the surface of a body is proportional to the fourth power of the absolute temperature. This relationship can be written in the form

$$W_r = \sigma e\theta^4 \text{ watts per in.}^2 \qquad (32)$$

where σ is Stephan's constant ($3 \cdot 7 \times 10^{-11}$)
and e is the emissivity of the particular surface.

The value of e for an ideal black body is unity; values for other surfaces are given in Table V below—

TABLE V

Black body	1·00
Black paint (glossy)	0·90
Insulation (average)	0·90
Oxydized steel	0·70
Oxydized copper	0·60
Cast iron	0·25
Polished copper	0·04

At the same time as the body is radiating heat from its surface, it is also receiving heat from the surroundings which are at a different temperature. The effective loss of heat is the difference between these two quantities. If θ_1 is the absolute temperature of the surface in question and θ_2 that of the surroundings, then

$$W_r = \sigma e(\theta_1^4 - \theta_2^4) \qquad (33)$$

If θ_1 and θ_2 differ from each other by $\Delta\theta$ this becomes

$$\sigma e\{(\theta + \Delta\theta)^4 - \theta^4\} = \sigma e(4\theta^3\Delta\theta + 6\theta^2\Delta\theta^2 + 4\theta\Delta\theta^3 + \Delta\theta^4)$$

If $\Delta\theta$ is small compared with θ, then this expression becomes approximately

$$4\sigma e\theta^3\Delta\theta \text{ watts per in.}^2$$

If ambient temperatures of 15°C and 35°C are selected as representative of those in temperate regions and hot countries respectively, the following table of radiation loss in watts per

TABLE VI

HEAT LOSS BY RADIATION

Temp. Rise °C	Heat Dissipation in Watts per In.^{2} e = 0·90		
	15°C	35°C	50°C
10	0·034	0·044	0·048
20	0·070	0·086	0·100
30	0·113	0·134	0·153
40	0·156	0·188	0·215
50	0·204	0·247	0·281
60	0·258	0·310	0·355

*square inch of surface can be drawn up for a range of tempera-*ture rises. In these tables the emissivity of the surface is assumed to be 0·90.

(*b*) *Convection Losses from Surfaces*

In addition to the loss of heat from a surface by radiation which occurs even *in vacuo*, a hot body surrounded by air (or other fluid) at a lower temperature loses heat to the fluid by *convection.* The air in immediate contact with the hot surface becomes heated, suffers a reduction in density as a result of the thermal expansion which takes place, and so rises away from the surface. Cool air takes the place of the heated air and a convection current is set up. The rate of heat loss varies very considerably according to the shape and size of the body even at a fixed temperature rise above the surroundings because any factors which increase the air circulation by giving "chimney effects" put up the heat loss by convection very considerably.

If the heat loss by convection W_C is expressed in the form

$$W_C = \mathrm{K}\Delta\theta^n \text{ watts/in.}^2 \qquad . \quad . \quad (34)$$

representative values of the constants have been given as $n = 1{\cdot}25$, and $\mathrm{K} = 1{\cdot}6 \times 10^{-3}$ for a horizontal surface facing upwards, $1{\cdot}2 \times 10^{-3}$ for a vertical surface, and $0{\cdot}8 \times 10^{-3}$ for a surface facing downwards.(3) These figures are for large metal plates several feet square; the rate of heat loss is considerably greater from the much smaller surface with which we are concerned in small transformers rated at a hundred VA or less. This is to be expected because cool air can reach all parts of a small surface, whereas the air passing over a large proportion of a surface of considerable area has already been heated by contact with other parts and is therefore less effective in removing heat.

In the case of vertical surfaces the height has much more effect on the loss per unit area than the width. Even the height ceases to have much effect, however, if it is greater than about 18 in. This rather surprising result is thought by Griffiths and Davis(3) to be due to the onset of turbulent flow. When the height is only a few inches, the flow of the rising air is streamline; consequently the upper elements of the surface are in contact with air already warmed lower down, so that their heat loss is less. When a critical velocity has been reached higher up the surface, eddies occur and cool air is then brought in contact with the upper parts of the plate. The rate of heat loss by convection per unit area of surface increases steadily from the "large height" value at 18 in. to three times that value at 1·8 in.

The loss from vertical cylindrical surfaces is similar to that from vertical plane surfaces, provided that the diameter is over 6 in. The dissipation from cylinders of smaller diameter is rather greater. As a guide to the designer of small transformers, Table VII shows approximate values of Griffiths and Davis' figures converted to watts per in.^{2} The cylinders were all of the same diameter but varied in height.

TABLE VII

HEAT LOSS BY CONVECTION FROM VERTICAL CYLINDRICAL SURFACES*

TEMP. RISE °C	WATTS PER IN.^{2} FROM CYLINDERS OF DIFFERENT HEIGHTS				
	103·6 in.	22·9 in.	6·0 in.	3·15 in.	1·83 in.
10	0·021	0·024	0·041	0·056	0·073
20	0·053	0·057	0·092	0·131	0·169
30	0·090	0·095	0·148	0·211	0·276
40	0·133	0·136	0·209	0·295	0·39
50	0·179	0·179	0·273	0·38	0·52
60	0·224	0·224	0·34	0·47	0·64

Many other factors affect the convection loss—for example, the nature of the surface, the shape of surrounding bodies, and so on—so that no *general* figures can be quoted. Designers can, however, obtain empirically figures for the particular type of construction they employ, and apply them to other coils of comparable size.

The use of fluids of higher specific heat than air enables the rate of heat removal to be increased very materially.

(c) *Total Heat Loss from Surfaces*

The very considerable variations which occur in the rate of cooling by convection, and to a lesser extent in the rate of radiation, make accurate calculation of the total loss from the surface of a transformer out of the question. As a guide, however, Table VIII gives figures of total heat loss for various temperature rises.

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* Diameter of cylinder 6·9 in. in each case.

TABLE VIII

TOTAL HEAT LOSS FROM VERTICAL CYLINDRICAL SURFACES*

TEMP. RISE °C	WATTS PER IN.^{2} AT 35°C AMBIENT		
	2 ft high and over	6 in. high	1·8 in. high
10	0·068	0·085	0·117
20	0·143	0·178	0·255
30	0·229	0·282	0·41
40	0·32	0·39	0·58
50	0·43	0·52	0·77
60	0·53	0·65	0·95

3. RATING OF SMALL TRANSFORMERS

(*a*) *Calculating the Maximum Temperature*

It has already been noted (p. 115) that if the iron and copper loss densities are kept unchanged as the size of a transformer is increased, it is necessary to adopt progressively more and more effective means of getting the heat away from the coils since the ratio of surface area to volume is decreasing all the time. The reader will, however, be chiefly concerned with small transformers up to 1 kVA (class *a* on p. 115), wound in a single solid coil, and he has no option as to the method of cooling; he must rely entirely on natural air-circulation over the exterior. It is thus impossible to work to the same copper and iron loss densities in the larger sizes as the small ones, and reductions are necessary as the rating increases. This fact is often forgotten, and lists of laminations are given showing the relative outputs which they are supposed to be able to deliver, based on a factor obtained by multiplying the window area by the cross-sectional area of the core. If the same flux-density in the iron and current-density in the copper could be used, this would form a good indication of the capabilities of the core, but in simple surface-cooled types that supposition is not true. Not only is the comparison invalid between larger and smaller types of the same general proportions, but also between similar overall sizes having different proportions of window to core section. If the radial depth of winding space is doubled, so that wires of twice the sectional area could be used, it might be argued that the current output could also be doubled, but this would not in fact be true because the greater thickness of the winding makes it more difficult for the heat to escape from the inside. The effect of an increase in size is thus twofold—

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* Diameter of cylinder 6·9 in. each each case.

(1) The surface area does not increase in proportion to the volume, causing the *surface* temperature to rise if the same loss density is maintained.

(2) The temperature inside the winding increases *even if the surface temperature is kept constant by reducing the current-density.*

Of the two, the second effect is more insistent, and it becomes necessary, even in relatively small sizes, to fit cooling ducts for air or oil between the windings and even through the windings themselves. We are here, however, concerned only with the surface-cooled type and we are forced to reduce the losses per unit volume in large thick coils. The cost accordingly tends to go up disproportionately, but fortunately the iron working density need not be reduced. It has been pointed out on several occasions that considerations of magnetizing current usually force the designer of small transformers to work at a lower flux-density than the iron loss would suggest. The efficiency is thus lower than might have been expected, because the iron loss at full-load is much lower than the copper loss. When the difficulty of cooling in the larger sizes forces a reduction in loss densities, it is therefore best to reduce the current-density in the copper, leaving the flux-density in the core as high as possible.

The ultimate objective of this section is to indicate how a designer can decide upon a suitable-sized core upon which to build a transformer for a given output, for, as can readily be understood, the core is the foundation of any design. If the number of lamination sizes available were small, so that only one or two would be suitable for a given VA rating, there would be no problem, for a list of stampings and ratings could be published and the designer would select the right one. It would still be necessary for him to take account of such factors as operation at tropical temperatures, inefficient use of winding space in high-voltage types due to need for extra insulation, etc., but a relative figure would be available. Actually, however, there are some five hundred different designs listed in the catalogues of British manufacturers alone and more are still appearing. It is, therefore, proposed to show how the temperature rise of a transformer can be estimated after the windings have been calculated, and thus allow the designer to select for himself from amongst those laminations which are desirable on other grounds.

The crucial figure is the temperature of the hottest part of the windings. If the melting point of the impregnating compound is exceeded it will tend to run out (although local softening can be allowed if the outside is effectively sealed), and the paper, enamel, and other organic insulating materials deteriorate rapidly if the temperature exceeds about 100°C. On the other hand, the rate of heat loss is decided by the *surface* temperature, which is considerably lower than that of the interior of the coil. The problem of determining whether a particular output can be safely taken from a given size thus resolves itself into two parts—

(1) Find the surface temperature at which the proposed losses can be dissipated.

(2) Add the internal temperature difference from the hottest point to the surface.

The application of Section 2 of this chapter enables the first step to be taken. There are, however, complications due to the difficulty in assessing the division of heat between the core and windings. The core losses are usually smaller than the copper losses and being produced in a body having a surface similar in area to that of the coil, the temperature tends to be lower. Even if the core tended to become somewhat hotter than the windings, the thermal connexion between the two is fairly loose so that little transfer would take place. The temperature of the core is not important in itself and it will not be considered further. To simplify matters, therefore, let it be assumed that no heat transfer takes place between the coil and core; this assumption is unlikely to lead to serious errors because the barriers such as the central former are quite thick for mechanical reasons, and the interchange of heat through this route is likely to be slow, especially as the temperature difference will in any case be small. On this basis, the coil is assumed to dissipate all the copper losses from its own outer surface. Using the figures given in Sections 2*b* and 2*c* of this chapter, a curve can be plotted showing the heat lost from the coil exterior at different temperature rises of the surface above the ambient.

From the point of view of transformer rating, however, it is not the temperature of the surface but that of the hottest winding which determines the maximum loading, so that the temperature gradient within the coil must be determined. To do this accurately would require a good knowledge of the thermal conductivities of all the materials used and involve prolonged calculation.(4, 5) As the data are only approximately known in any particular case, long calculations are not justified. If, for instance, a layer-wound, paper-interleaved coil is being examined, a reasonable approximation is to divide the coil into tubular sections each representing one winding, e.g. primary, secondary 1, secondary 2, etc. The major insulation between the different windings is likely to be the most serious obstacle to the conduction of heat outwards, and a uniform temperature could be assumed throughout the thickness of each separate winding. (Although not a justification for the method, it may be noted that figures of temperature rise of a winding obtained from change of resistance measurements give these *average* temperatures.) It is not, however, advisable to neglect the effect of the minor insulation altogether, so that an estimate is made of the temperature difference across the winding by assuming that all the heat passes through half the total thicknesses of the paper, cotton, enamel, etc., between the individual layers of the winding. The factor of a half is introduced because not all the heat generated in a winding has to pass through all the minor insulation—some is developed on the outside of some of the layers. Since no heat is assumed lost to the core, all heat flow must be outwards. In a coil of normal dimensions most of this occurs radially, because the small surface exposed axially is covered with a considerable depth of insulation. Although the loss here will reduce the temperature of the end turns somewhat, this has little effect *on those in the centre of the limb and it is the temperature of* the *hottest* point which is required.

If the primary winding is inside, the heat lost in it is all forced to pass out through the tube of insulating material surrounding the winding. If, as is usually the case, the tube is thin relative to its radius, the temperature difference between its inner and outer surfaces can be calculated from

$$\theta_2 - \theta_1 = \frac{W_c'd}{ak} \quad . \quad . \quad . \quad (35)$$

where $\theta_1\theta_2$ are the outside and inside temperature,

k is the thermal conductivity,

a is the surface area through which the heat passes,

d is the thickness of the material,

W_c' is the primary copper loss in watts.

Representative figures for k for materials commonly used in transformers are given in Table IX. These values must, however, be used with caution as the particular grade used may differ somewhat. Great accuracy is, however, neither expected nor possible, and an average figure of 0·005 can often be assumed for the wax-impregnated materials of the paper, silk, or cotton type. In calculating the value of a, an area under the outer limbs may be deducted if this part of the surface cannot lose heat to the air.

TABLE IX

THERMAL CONDUCTIVITIES OF VARIOUS INSULATING MATERIALS

MATERIAL	CONDUCTIVITY (Watts per °C in.^{2})
Asbestos . .	0·0018
Cotton . .	0·0034
Ebonite . .	0·0025
Fullerboard (oiled)	0·0035
Mica . .	0·011
Paper . .	0·003
Paper (impregnated)	0·005
Paraffin Wax .	0·0035
Porcelain . .	0·001
Varnished Cambric	0·005

The next tubular section of winding is now examined in a similar way. The heat leaving this section must be equal to that entering it from the primary together with that generated in it by its own losses. The temperature difference between inside and outside of the next layer of major insulation is thus

$$\theta_3 - \theta_2 = \frac{(W_c' + W_c'')d_2}{a_2 k_2} \quad . \quad . \quad (36)$$

Proceeding in this way, an estimate of the total temperature difference between innermost winding and coil surface can be obtained by adding together the various separate temperature drops. It might be objected that this method assumes that the winding next to the core is necessarily the hottest, whereas a particularly heavily rated intermediate winding might get hotter. The idea that the innermost winding reaches the highest temperature is however implicit in the assumption that heat is only lost radially outwards. The effect of a hot outside winding must then be to slow down the conduction from the inside until that is even hotter! If a highly-rated outer winding of this kind were actually used, and it became hotter than the one below it, some heat would be getting out either radially inwards (i.e. to the core) or axially, and the initial assumption would not be justified. In most cases, however, it is satisfactory.

Having obtained the temperature difference between inside and surface, the curve of surface temperature against heat loss is consulted and the surface temperature is read off against the total copper loss. The temperature rise within the coil is now added to the surface temperature and the actual inside temperature obtained. If this exceeds the permissible figure for the insulation or the impregnating material, the design must be reconsidered. After a few trials, the designer will soon become familiar with the rating of a group of cores for the particular purpose and with the particular winding technique, impregnation and insulation which he intends to use. Once a range of cores has been selected in this way, he has only to select one of them for a new application according to the output desired. The examples given in the next chapter will form a guide to the ratings which may be expected from typical sizes.

EXAMPLE

It is desired to calculate the temperature rise of the innermost winding of a small transformer supplying a radio receiver. The core is a square stack of 1¼ in. No-waste stampings (Table XVII, p. 471) and the relevant details of the windings (in order, starting from inside) are as follows—

Primary

10 layers of SWG 29 Enam. interleaved with 0·001-in. paper between layers.
4 turns 0·002-in. paper.
Mean turn length 6½ in. Primary copper loss, 2·6 watts.

Screen

1 turn copper foil.
5 turns 0·002-in. paper.
Mean turn length 7 in. No copper loss.

H.T. Secondary

10 layers of SWG 38 Enam. interleaved with 0·001-in. paper between layers.
10 turns 0·001-in. paper.
Mean turn length 7½ in. H.t. secondary copper loss, 3·1 watts.

Heater Winding 1

1 layer of SWG 17 Enam.
5 turns 0·002-in. paper.
Mean turn length 8 in. Heater 1 copper loss, 1·1 watts.

Heater Winding 2

1 layer of SWG 20 Enam.
5 turns 0·002-in. paper.
Mean turn length 8¼ in. Heater 2 copper loss, 0·4 watt.

Tag Panel, etc.

1 turn 0·010-in. presspahn.
Turn length 9 in.

In this winding there is some space between the outside of the coil and the outer limbs. Although there will be some cooling of this portion by convection, radiation will be small, but this area will be included in full in the external coil area to compensate for the fact that the ends of the coil have been assumed not to lose any heat.

The primary loss of 2·6 watts is assumed to pass through half the thickness of the interlayer insulation and the whole of the insulation above it. The effective value of insulation thickness is 0·008 in. + 0·010 in./2, i.e. 0·013 in. The insulation area is 6½ × 1⅞ in.^{2} and k is taken to be 0·005. Hence the temperature difference is

$$\frac{W_c'd}{ak} = \frac{2{\cdot}6 \times 0{\cdot}013}{6{\cdot}5 \times 1{\cdot}875 \times 0{\cdot}005} = 0{\cdot}55\,^\circ\text{C}$$

Similarly the temperature difference across the insulation surrounding the screen is

$$\frac{2{\cdot}6 \times 0{\cdot}010}{7{\cdot}0 \times 1{\cdot}875 \times 0{\cdot}005} = 0{\cdot}40\,^\circ\text{C}$$

In the h.t. secondary, further losses take place so that the total rate of energy transfer through the interlayer and external insulation is 5·7 watts. Consequently the temperature difference is

$$\frac{5{\cdot}7 \times 0{\cdot}015}{7{\cdot}5 \times 1{\cdot}875 \times 0{\cdot}005} = 1{\cdot}22^\circ\mathrm{C}$$

In the same way the corresponding figures for the other portions of the coil are

Heater 1, $$\frac{6{\cdot}8 \times 0{\cdot}010}{8{\cdot}0 \times 1{\cdot}875 \times 0{\cdot}005} = 0{\cdot}91^\circ\mathrm{C}$$

Heater 2, $$\frac{7{\cdot}2 \times 0{\cdot}010}{8{\cdot}25 \times 1{\cdot}875 \times 0{\cdot}005} = 0{\cdot}93^\circ\mathrm{C}$$

Tag Jacket, $$\frac{7{\cdot}2 \times 0{\cdot}010}{9{\cdot}0 \times 1{\cdot}875 \times 0{\cdot}005} = 0{\cdot}85^\circ\mathrm{C}$$

The total temperature difference between the innermost turns and the surface is thus 4·9°C.

The total circumferential surface area of the coil is 16·9 in.^{2} The total copper loss is 7·2 watts, so using the figures of Table VIII for small areas, a temperature rise of 30·9°C will take place at the surface of the coil. The innermost layers will therefore have a rise of 30·9 + 4·9 = 35·8°C.

Tests made on an actual transformer gave the following results—

Primary (average, from change of resistance), 33·5°C.

Surface (from special winding embedded in wax dip), 29·8°C.

The calculated temperature difference between inside and outside is in good agreement with the measured figures, but the calculated surface temperature is slightly high. Apparently the heat dissipation per in.^{2} is somewhat greater than that assumed, or else some heat is being lost to the core. The agreement is, however, as good as can be expected, in view of the many uncertainties.

(*b*) *The Relation of Window Size to Core-section*

It is appropriate at this point to examine the relative advantages and disadvantages of two transformers of the same rated output, but having quite different proportions. The order of variation under consideration is that represented at one extreme by the no-waste stampings having window spaces and outer limbs of equal width, and at the other by the stampings having windows as wide as the central limb of a shell-type core (i.e. twice the width of the outer limbs). If the same flux-densities and current-densities could be used in each, it is evident that the product of window area and core-section would be a constant for a given rating, but, as already explained, this is not the case. Starting from intermediate proportions, if the core cross-section is reduced by x per cent, the window area would need to be increased by more than x per cent to keep the same maximum internal temperature, because the heat could get away less easily through the thicker winding. For a given rating the stamping with the smallest overall size will accordingly be somewhat nearer the large iron-section than the large copper-section type. Actually, over the useful range the product of window and cross-sectional areas for a given external sized lamination is practically constant, as will be seen from Fig. 41, which shows the value of this product against width of central limb for an external dimension of $3\frac{3}{4}$ in. × $3\frac{1}{8}$ in. The easier getaway of heat therefore biases the selection towards the smaller window. The overall size of the *transformer* becomes smaller since the coil bulges less over the sides.

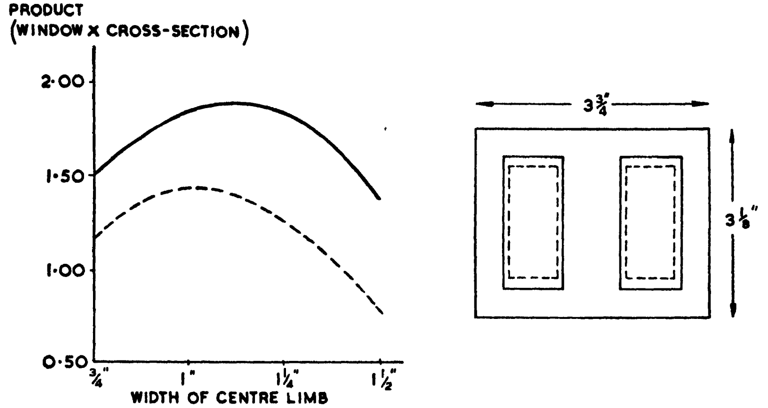

FIG. 41

The Effect on Rated Capacity of Varying the Width of the Central Limb of a Core Without Altering the Overall Dimensions.

Full Line Curve: Whole Window Utilized.
Dotted Line Curve: Space Allowed for Insulation.

Against this reasoning may be set two arguments. First, the effective area of the window for winding purposes is reduced by the need for insulation all round; the proportionate effect of this is worse on the small-window types. This is shown in Fig. 41 by the dotted curve which gives the product for the effective area obtained by taking off $\frac{1}{16}$ in. inside and outside and $\frac{1}{8}$ in. at each end of the window. Secondly, the assumption that the same flux-density would be used in the two cases is incorrect. Since the reactive volt-amperes are proportional to the weight of iron for a given value of B, and there is less iron in the large-window type, the flux-density can be increased for a given magnetizing current. It is then no longer necessary to increase winding space in proportion to the reduction in cross-section. These two effects undoubtedly offset to some extent the easier cooling of the thin coil.

On the matter of cost, the large section small-window type is definitely cheaper, because iron stampings are less expensive than a similar weight of copper, and the labour cost in winding is less with this type since fewer turns are put on. The regulation characteristics of the large-section type are also better owing to the lower winding resistances.

The cost and size aspect is dealt with by N. Partridge,(6) who deduces optimum proportions for transformer and choke laminations. He starts from the assumption that—

(1) The permissible output is, for a given form of construction, proportional to the outer circumferential surface area of coil outside the limbs (i.e. the flat ends of the coil and the surface under the outer limbs are neglected). This assumption has already been used in this chapter but the effect of an alteration in the radial depth of the winding was also taken into account.

(2) In assessing the cost of a lamination, the gross dimensions are used, i.e. the metal punched out for the windows is wasted. Although generally true, this is not applicable to the "no-waste" types.

Working on these lines, Partridge deduces that the height (axial length) of a stamping should be approximately equal to its breadth and that the depth of the lamination stack can with advantages in size and cost be up to 1·5 times the width of the central limb, although winding difficulties are of course greater than with the square stack. When examining the effect of increasing the stack depth, the coil surface available for heat dissipation is considered to remain constant. Since the resistance increases in proportion to the length of the mean turn, the current-density must be reduced, if the temperature rise is to remain unchanged. The power output capabilities of the transformer thus increase in proportion to

$$\{\text{stack depth}/\sqrt{\text{mean turn length}}\}$$

If a central limb 1 in. wide is assumed together with a typical width of window of $\frac{3}{4}$ in., and the depth of stack is denoted by x, the length of the mean turn is $\left(2 + 2x + \frac{3\pi}{4}\right)$. The volume of winding is $\frac{3}{4} \times (h - 1) \times \left(2 + 2x + \frac{3\pi}{4}\right)$ in.^{3} If Partridge's proportions of a square stamping are adopted, $h = 3\frac{1}{2}$ in. The volume of iron is for cost purposes $x \times 3\frac{1}{2}^2$ $(= 12{\cdot}25x)$ in.^{3} and for weight purposes $[12{\cdot}25 - 2(2\frac{1}{2} \times \frac{3}{4})]x$ $(= 8{\cdot}50x)$ in.^{3} Taking the costs per in.^{3} of winding and core to be equal, as is reasonably true, figures of relative outputs, weights and costs are given in Table X. It will be seen that using a rather narrower window than Partridge, as is the more usual practice, the optimum both as regards weight and cost, lies at a stack depth of approximately 1·5 times its width. Satisfactory transformers can, however, be made for any ratio between 0·75 and 2·0.

TABLE X

x	Mean Turn Length = m	$\sqrt{m}$	Relative Output	Relative Weight	Relative Cost	Relative Weight per Watt	Relative Cost per Watt
0·5	5·4	2·32	0·55	0·60	0·67	1·09	1·22
0·75	5·9	2·43	0·77	0·81	0·83	1·05	1·04
1·00	6·4	2·53	1·00	1·00	1·00	1·00	1·00
1·25	6·9	2·63	1·21	1·20	1·16	0·99	0·96
1·50	7·4	2·72	1·40	1·39	1·33	0·99	0·95
1·75	7·9	2·81	1·58	1·60	1·49	1·01	0·94
2·00	8·4	2·90	1·75	1·80	1·66	1·03	0·95

Core weight per in.^{3} = 0·27 lb.
Coil weight per in.^{3} = 0·08 lb.
Core cost per in.^{3} = Coil cost per in.^{3}

The next variation considered in the article is the effect of increasing the limb widths at the expense of the window width, leaving the overall width of the stamping unchanged. The surface area falls somewhat as the limbs are made wider, so that the current-density has to be reduced, but if a core-section of fixed proportions (e.g. square) is maintained the output rating, relative to cost, increases until a central limb width equal to 0·32 of the total width is reached. If a rectangular core-section 1·5 times as deep as wide is used, the optimum is reached at a rather lower figure of 0·30. If weight rather than cost is the criterion, the figures become 0·24 and 0·20, respectively. Now it will be remembered that these results are based on the assumption that the temperature difference between the hottest point and the surface does not increase as the coil is made thicker radially, an assumption which is certainly not strictly correct so that the optimum is actually rather farther in the direction of the large section with a small window. The "no-waste" stamping with a ratio of 0·33 is thus nearly optimum for cost with a square section, and when it is remembered that the metal stamped from the windows is *not* wasted in this case, it is seen to be a very economical type of lamination. On weight considerations it is rather past the optimum.

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CHAPTER VII

Designing Small Power Transformers

THE information given in the preceding chapters is sufficient to permit the calculation of all the quantities it is necessary to know in a low-power transformer working at normal distribution frequencies. Three assumptions have been made, viz.—

(*a*) that the magnetizing current wave is sinusoidal;

(*b*) that there is no flux leakage between primary and secondary;

(*c*) that winding and interwinding capacitances can be neglected.

In the particular application now under consideration, i.e. transformers delivering up to about 1 kVA at frequencies up to 100 cycles per second, these assumptions are generally justified although there may be special cases where (*b*) has to be modified.

The purpose of this chapter is to reverse the procedure so far followed, that is to say, we shall now attempt to build transformers which will, when completed, have certain specified characteristics; hitherto the object has been to determine the characteristics of an existing apparatus. When this reversed process is embarked upon, it immediately becomes evident that considerable latitude is available in respect of the individual characteristics which in combination determine the performance. For example, whereas the flux-density in a certain transformer is readily calculated from primary turns and core dimensions, the converse problem of deciding the flux-density at which to work a new transformer cannot be solved by the formula because the dimensions of the core are not yet known.

The art of transformer design depends to a considerable extent on experience. So many factors are involved that it is impossible to work along perfectly rigid lines, arriving by a definite sequence of calculations at a perfect transformer for the particular job. No two designers, given the same requirements, will produce identical transformers, because they will give different degrees of prominence to the many aspects of the problem. It is only by experience that a correct assessment of a suitable value for the various factors such as current-density, etc., can be made *quickly.* That does not mean that these quantities are finally decided by guesswork—far from it, for the correctness of each assessment has to be checked by going back after each step has been completed to see whether the design up to that point does in fact produce a figure compatible with the previous assumption. If not, then it is necessary to go back again, choose a more likely value and again see whether it gives the result expected. For instance, having decided the output required, an early step is to estimate the efficiency which can be expected from the finished transformer; this enables the primary winding to be calculated and hence the secondary. Right at the end, the efficiency must be calculated, and if it does not in fact agree reasonably with the estimated figure, another percentage must be assumed and the windings re-calculated throughout until the assumed efficiency matches up with the value calculated at the end. Experience thus helps to attain the right figure *quickly*, but eventually the correct figure would be reached in any case. This aspect of the process will be emphasized in the description which follows. It is not intended to suggest that the work of designing a transformer is any more tentative in nature than that of designing any other machine of similar complexity, but simply to indicate that designing generally calls for experience and perhaps even more for an ability to weigh correctly the opposed demands of different requirements such as cost and reliability, efficiency and size, etc.

1. THE PRINCIPAL STEPS

The principal steps in the sequence of designing a transformer are as follows—

(1) Determine the output requirements in volts and r.m.s. amperes from the secondary or secondaries. If the winding is not fully utilized, as for example when supplying a rectifier

which only takes current over a portion of the cycle, it is necessary to multiply the actual output by a factor to obtain the equivalent rating of the winding as regards space occupied. The appropriate factor must also be applied to the primary so that the equivalent rating for the whole transformer is obtained.

(2) A core size is selected which appears to be suitable for the volt-ampere rating arrived at in (1).

(3) The flux-density in the core is decided, on an iron loss basis.

(4) The primary turns are calculated so as to obtain the value of B which was selected in (3). Primary resistance is neglected at this stage.

(5) An estimate is made of the efficiency and primary power factor.

(6) Making use of the figures from (1) and (5), the primary current full-load is calculated.

(7) The no-load current is calculated to see that it is not excessive.

(8) A wire gauge is selected for the primary to suit the current calculated in (6).

(9) The turns and conductor sizes are calculated for the secondaries.

(10) Corrections are made to the primary turns in (4) and secondary turns in (8) to allow for their resistances.

(11) As it may not be possible to get sufficiently close to the desired secondary voltage with an integral number of turns (or half turns in a shell-type core) if the voltage is low, it may be necessary to revise (3) and (4) to suit.

(12) The efficiency is calculated from iron and copper losses —the result is compared with that assumed in (5) and adjustments made if necessary.

The more important steps in this procedure are now dealt with in rather greater detail.

Output Requirements

Although the output may be clearly stated in volts and amperes, this is not always the case and the designer may have to find out for himself the actual current and voltage required from the transformer in order to perform the duty which has been specified. The requirement may for instance be for a transformer to supply a full-wave rectifier with a capacitor filter delivering a stated d.c. output. It is then necessary to work back from the rectifier output and find the a.c. voltage to be applied to the rectifier and the *r.m.s. value* of the secondary current. Other cases where this is likely to arise are in supplying gaseous discharge tubes and in feeding motors driving slow-speed reciprocating pumps and planing or shaping machines with alternate power and idle strokes where the current fluctuates over a cycle of a few seconds in duration.

Since the heating of the windings is determined by the r.m.s. value of the current flowing through them, it is necessary in all cases to obtain the r.m.s. value before the design can proceed. The fact that the wire size of the winding may have to be larger than might have been expected because the r.m.s. value is greater than the mean value of the current does not, however, account for all the extra space taken up by a winding for a rectifier as compared with a straightforward resistance load. In certain methods of connexion, such as the centre-tapped winding supplying a full-wave rectifier, twice the normal number of turns has to be accommodated because each half is idle for one half-cycle. Although the r.m.s. value of the current over a cycle is reduced by this, the reduction is more than offset by the doubling of the turns required. The question of typical rectifier winding ratings is covered in Chapter XIII.

Selection of Core Size

The *smallest*-sized core which can be used for the output requirements decided upon is determined by heating considerations. Any larger size can usually be adopted with improved efficiency, but it is assumed that for reasons of cost the smallest size will be used. This means that the surface area of the transformer must be sufficient to dissipate the amount of heat generated by the losses without exceeding a definite internal temperature. Any factor which increases the efficiency, such as an increase in operating frequency, allows the output rating to be increased until the losses again reach the limit set by temperature rise.

Although a high operating temperature in a transformer may cause trouble in other equipment near, it is usually the effect on the internal insulation of the transformer itself which limits the permissible temperature. Material such as paper-boards, interleaving paper, bakelite, etc., deteriorate rather rapidly at temperatures above about 90°C. The effect is usually to make them brittle and so subject to mechanical failure rather than to any *direct* electrical troubles, although, of course, electrical breakdown is likely to follow cracking or splitting of insulation material.

If the paper, etc., is impregnated in wax or similar compounds, it may be the melting point of this wax which sets the upper limit to the temperature. It must also be remembered that the internal temperature is higher than the surface temperature of the transformer owing to the poor conductivity of the outer layers of windings and insulation. Heat from the inside of a winding can escape only by creating a temperature gradient from inside to outside; this temperature difference is naturally greater for windings of considerable radial depth than for thin ones. The matter of heat dissipation from a transformer was considered in detail in Chapter VI.

It is in this step of selecting a suitable core size that experience is particularly valuable, for it will usually be possible by reference to a past design of similar output to decide with reasonable certainty whether a particular frame size will meet requirements. In this assessment, account will be taken of the effect of such factors as supply frequency, extra insulation for high voltages, operation at high ambient temperatures, etc. There are, of course, an unlimited number of core shapes which will accommodate a transformer of given output, but each designer will have probably a relatively small range of sizes from which to select, because clamps, formers, terminal panels, etc., will be available already for certain laminations. The range of a particular manufacturer will thus contain probably only one size suitable for the output in question, although there will be many other patterns available of similar output capabilities; standardization would, however, rule out their use unless there were some special advantages to justify their introduction.

Flux-density

The flux-density which can be used depends on the supply frequency, the grade and thickness of the laminations, and the size of the transformer. It is assumed that it will always be working at or near full-load as is the case in most small transformers. If, however, there are special requirements specifying a low ratio of iron to full-load copper losses, the flux-density figure may require modification. In small transformers, as already noted, the ratio of no-load to full-load current is apt to be high so that the value of B is likely to be decided by considerations of magnetizing current rather than of iron loss. For outputs of less than several hundred watts, it is, therefore, best to bring step (7) in the above summary forward. The magnetizing current, which is at power frequencies much the larger component of the no-load current, is but little affected by the grade or thickness of the laminations so that where the limit of flux-density is set in this way, it is quite satisfactory to use quite low silicon content steel ($2\frac{1}{2}$ per cent or even lower) and relatively thick laminations (say, 0·020 in.). The increased iron loss resulting will still be sufficiently small at the value of B which has to be used. At 50 cycles a flux-density of 12 000 to 13 000 lines per cm^2 is a reasonable figure from which to start calculations. At higher frequencies a reduction will be necessary as soon as the iron loss rather than the magnetizing current begins to be the deciding factor. Even then, however, the first move is to use better grade iron and thinner stampings so as to reduce the iron loss; the flux-density must always be kept as high as the various considerations permit, because this allows the turns per volt on all windings to be kept down, so leading to lower copper losses and smaller frame sizes.

Estimating Efficiency and Power-factor

This step is really closely tied up with the selection of core size because it is known that the particular core selected will produce a transformer having a surface area capable of dissipating a given number of watts loss. Taken in conjunction with the output figure, the efficiency can be calculated. This method is applicable, however, only if it is assumed that the transformer will be run as hot as the frame size allows. If for any reason the losses are less than the maximum which the size can accommodate with the permitted temperature rise, estimates have to be made on other grounds, but in any case the method is quite rough and should be used only as a guide. Fortunately great accuracy is not necessary in this estimate, which in conjunction with the power-factor is used to calculate the primary full-load current. As a general guide the efficiency ranges from 85 per cent for 50 watts output to 95 per cent for 1 kVA output at 50 cycles.

The power-factor of the primary will not be unity even if that of the secondary is, owing to the presence of the magnetizing component which would lag 90° behind the load component for a resistance load. In those cases of small sizes where the magnetizing current is expected to be relatively large, its actual value will already have been calculated in order to arrive at a suitable value of B, so that the power-factor of the full-load current can be worked out from the relationship.

$$\text{P.F.} = \cos \psi = \frac{I_L'}{\sqrt{(I_L')^2 + (I_m)^2}}$$

where I_L' is the load component of the primary current.

In using this expression, the figure for I_L' will not yet be known accurately, but it can be obtained from the efficiency figure just estimated. It will be seen that even if the magnetizing current reaches 30 per cent of the load component its effect is only to contribute 5 per cent to the resultant full-load current. The value of the power-factor used to estimate the primary current thus differs little from that of the load being supplied.

Choice of Conductor Sizes

Certain conventions have grown up concerning the current-density which may be used in wires and strips used in coils. It is usually considered that in small transformers wires finer than about SWG 34 can be worked at about 2 200 amp per in.^{2}, SWG 34 to SWG 18 at 2 000 amp per in.^{2}, and so on down to only 1 000 amp per in.^{2} for large conductors of the order of an inch square. These figures are a reasonably good guide for fairly thin coils (up to ½ in. radial depth) for use in temperate conditions, but it must not be forgotten that the important consideration is the amount of heat developed in the winding as a whole. The heat is conducted relatively freely through the interior of a coil particularly if the air spaces have all been filled up by vacuum impregnation so that, provided a winding is not grossly overrun, the use of a high current-density in one winding and a low density in an adjacent one will not result in any great temperature difference. Both windings will be found to settle down to a figure intermediate between the temperatures which might have been expected. This effect is noticeable in 230-volt primary windings which have been tapped half way down for 110-volt supplies. The first half of the winding is generally arranged to be of a larger wire than the rest to deal with the doubled current at the lower voltage. When working on 230 volts the same current flows through both parts in series, but the temperatures of the two sections are found from resistance measurements to be more or less equal. This levelling-up of temperatures is helped by the fact that the thicker wire is usually nearer the centre and, therefore, cannot lose its heat so readily.

These remarks are not intended to suggest that wire sizes should not be chosen on a current-density basis; it is merely pointed out that discretion is necessary. An inner winding in a thick coil may run much too hot at 2 000 amp per in.^{2} whilst a single layer winding on the outside of a bobbin can be run considerably higher and still remain comparatively cool.

Voltage-drop Corrections

A general idea of the number of turns required in the primary and secondary windings can be obtained from the expression relating primary turns and flux-density (equation 22), together with the knowledge that the induced e.m.f.s in primary and secondary windings are in the same ratio as the turns of the windings. A transformer calculated in this way would, however, give rather lower output voltages than desired because no account has been taken of the resistances of the windings. On the basis that the primary should occupy half the winding space, estimates of the length of the mean turn can be made for each winding. In conjunction with the number of turns and the resistance per yard figure obtainable from wire tables, the resistance of the whole winding can be calculated. By multiplying by the current, the voltage drop in each winding is arrived at, and this can then be translated into the number of extra turns required to compensate for the drop. In the case of the secondaries, these turns are *added* to the number originally calculated, but in the normal way they are *subtracted* from the primary turns, because the effect of the resistance drop is to reduce the effective voltage applied. If, however, the transformer is expected to be used frequently at light loads or without secondary load at all, the deduction of the primary full-load voltage drop would make the flux-density excessive when operating lightly loaded. In that case the primary turns are left uncorrected, but an additional correction is made to each secondary by adding a percentage to the turns equal to the percentage resistance drop in the primary.

Insulation

Taking first of all the insulation between neighbouring conductors (minor insulation, as it is called), this is provided in all small transformers by the wire covering which may be enamel or other synthetic resinous material, textile, or paper. Owing to the possibility of cracks or blemishes, coupled with the considerable fire risk if shorted turns develop, it is not good practice to use plain enamelled wire without additional insulation between layers. Turns from the outer end of the winding may slip down the edges of the coil and come in contact with turns near the inner end, so putting the full winding voltage across the film of enamel. Although this film is usually extremely tenacious, it is being unduly stressed under these conditions. If a bobbin winding (without interlayer insulation) must be used, then one of the new tough plastic coverings such as "Lewmex" or "Pigenam" should be adopted. These polyvinyl formal coverings are available in various thicknesses which should be selected to suit the conditions.

The normal method of winding is likely to be in layers with a turn or turns of paper between. The greatest voltage between layers occurs at one end where the second layer returns over the first; the potential difference is then equal to twice that generated in one layer. A reasonable figure upon which to decide the thickness of the paper is 50 volts (peak) per mil (0·001 in.), although this can be somewhat exceeded if it is essential to do so. In practice, however, the thickness is often determined by mechanical rather than electrical considerations. The paper has to be sufficiently strong to withstand the quite considerable forces exerted upon it by winding on the upper layer under tension and this often entails a thicker paper than electrical conditions would demand. Two-mil kraft paper is generally used for gauges heavier than SWG 34, but a heavily calendered paper is essential when one mil thickness is used for the finer gauges. Two layers of thin paper are more satisfactory than one turn of thicker paper, because there is little chance of two flaws coinciding in the two layers.

Turning to the major insulation between windings, this again may be based on 50 volts (peak) per mil, but this should not be exceeded because the breakdown voltage per mil tends to decrease in greater thicknesses of insulation. In assessing the interwinding voltage stresses, it should not be forgotten that either the neutral or the phase (live) side of the input may be connected to a particular end of the primary, causing a different p.d. to other windings.

2. Practical Examples

The method described above can best be demonstrated by working through a number of examples. The earlier ones are given in greater detail than the later ones. The figures in brackets correspond to the steps outlined earlier.

Example 1

A transformer is required to supply a 12-*volt* 48-*watt projection lamp. The primary is to be tapped so as to be suitable for voltages between* 200 *and* 250 *and the supply frequency is* 50 *cycles per second.*

(1) This is a case in which the transformer designer has to find out the exact rating for himself, because after discussion with the customer he finds that the lamp is to be connected through 15 ft of flexible cable which calculation shows would cause a voltage drop of 0·6 volt (lead and return). The transformer output is thus 12·6 volts, 4·0 amp, i.e. 50·4 watts.

Being a normal resistive load, no further allowances are required.

(2) A suitable core would appear to be a square-section stack of Sankey No. 4A laminations. (Dimensions under 404A in Table XVIII and Fig. 173 on p. 472.) A thickness of 0·020 in. would be satisfactory in 42-quality material.

(3) As the transformer will always be working at full-load, a fairly high iron loss is no disadvantage. A figure of 12 000 lines per cm^2 is a reasonable figure from which to start.

(4) The gross area of a square stack of these stampings would be

0·88 in.^{2} With a stacking factor of 0·94 the effective cross-section would be 0·83 in.^{2} The number of primary voltage taps must next be decided. In order that the life of the lamp shall not be unduly shortened by overrunning, the output voltage should not differ by more than a few per cent from the nominal value when a suitable tapping is used. On the other hand it would be academic to work to better than, say, half the probable variations of the mains voltage from its declared value. On this basis three taps would be used at average voltages of 205, 225, 245, to cover the ranges 200–210, 220–230, 240–250. This deals with all the voltages in normal use. The uncorrected primary turns for these voltages are 1 438, 1 578, 1 717 from $B_{max} = \frac{3\,490\ V_p}{T_p af}$.

(5) Assume a figure of 85 per cent for the efficiency and 0·95 for the primary power-factor.

(6) The primary current at the lowest of the three voltage taps is $\frac{50{\cdot}4}{205 \times 0{\cdot}85 \times 0{\cdot}95} = 0{\cdot}305$ amp.

(7) The gross area of the stamping is 11·38 in.^{2}; deducting the window area of 4·05 in.^{2} leaves a net area of 7·33 in.^{2} In a square stack with a stacking factor of 0·94 the volume of iron is 6·46 in.^{3} and the weight (at 0·276 lb per in.^{3}) is 1·78 lb. The volt-amperes per lb are given as 6·3 at a flux-density of 12 000 lines per cm^2. The magnetizing current at 205 volts thus works out at $\frac{6{\cdot}3 \times 1{\cdot}78}{205} = 0{\cdot}055$ amp. This is quite satisfactory and we can, therefore, proceed further on these lines.

(8) As the winding space is rather wide in a radial direction, it is advisable to keep the current-density fairly low in the inner winding. It is desirable to put the primary on first as the surface would be apt to be uneven if the thick wire secondary had been made the inner winding. Taking 1 500 amp per in.^{2} SWG 26 would be suitable.

(9) The secondary turns required to give an *open-circuit* voltage of 12·6 are 88·3 and a suitable wire size would be SWG 17.

(10) Assuming that the primary and secondary each occupies half the winding space, the lengths of the mean turns are 5·6 in. and 8·9 in. respectively. The resistances are $\frac{1\,438 \times 5{\cdot}6 \times 94{\cdot}3}{12 \times 3 \times 1\,000} = 21{\cdot}1$ ohms (to first tap) and $\frac{88{\cdot}3 \times 8{\cdot}9 \times 9{\cdot}75}{12 \times 3 \times 1\,000} = 0{\cdot}214$ ohm. The corresponding voltage drops are 6·45 and 0·86 volts and these are equivalent to 45·3 turns and 6·03 turns.

(11) The total secondary turns $88{\cdot}3 + 6{\cdot}03 = 94{\cdot}3$ are too far from an integral number to be satisfactory and the primary turns should be adjusted to give a whole number of turns on the secondary. Increase the primary by 0·6 per cent, i.e. to 1 447 and the secondary to 88·9. The voltage drops are now revised, if necessary, and the turns allowances become 45·6 and 6·06 turns. These allowances are *subtracted* from the primary and added to the secondary, making 1 401 (for 205 volts)

and 95, respectively, to the nearest turn. By proportion the 225- and 245-volt points on the primary are 1 536 and 1 672 turns respectively.

(12) The iron loss is 1·56 watts per lb on 1·78 lb, i.e. 2·77 watts total. The primary copper loss is 2·00 watts and the secondary 3·44 watts. The total losses are thus 8·21 watts and the efficiency is

$$\frac{50{\cdot}4}{50{\cdot}4 + 8{\cdot}2} \times 100 = 86{\cdot}0 \text{ per cent.}$$

This checks sufficiently well with the figure assumed in (5) and no corrections are required.

(13) A check is now necessary to make sure that the windings will go in. Assume that the square resin-impregnated centre tube is $\frac{1}{32}$ in. larger inside than the lamination stack and is $\frac{1}{16}$ in. thick. The 26-gauge enamelled wire winds 50 turns per inch, so that if the winding can be carried to within $\frac{3}{16}$ in. of each end, about ninety-seven turns will be obtained per layer. With part layers due to tappings being brought out, the primary will take nineteen layers. With one turn of 0·002 in. paper between layers a depth of 0·42 in. will be occupied. Between windings five turns of 0·002 in. paper are put on, making a total depth of 0·43 in. The secondary wire (SWG 17 enamel) winds 16·9 turns per inch. With this wire a rather larger distance from the ends should be allowed particularly as it is at the outside, so that four layers will be

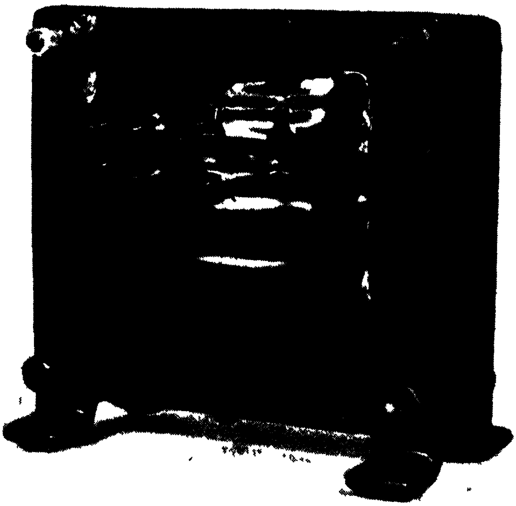

FIG. 42

The 48-watt Transformer Designed in Example 1.

required which, with interleaving paper, will take up 0·24 in. The total radial depth of the coil and former is thus—

0·015 in.	space former to core
0·064 in.	former wall thickness
0·420 in.	primary
0·010 in.	interwinding insulation
0·240 in.	secondary
0·749 in.	

It will be seen that ample space is left for the outer wrapping of insulation, as the window is 0·875 in. wide.

The summary of design particulars is thus—

Core. Square stack of No. 4A laminations in 0·020 in. 42 quality (Sankey).

Primary. 1 672 turns SWG 26 (0·018 in.) enamel-covered wire tapped at 1 401 and 1 536 turns for 205 and 225 volts. One turn of 0·002 in. paper between layers.

Major Insulation. Five turns of 0·002 in. paper.

Secondary. Ninety-five turns SWG 17 (0·056 in.) enamel-covered wire, interleaved with 0·002 in. paper.

Test Results

In order to show the reader the order of accuracy which can be expected from the design methods described, a transformer was wound in accordance with the data given. The results obtained were as follows—

	MEASURED	DESIGNED
Open-circuit voltage	13·85 volts	13·9 volts
Full-load voltage	12·6 volts	12·6 volts
No-load current	0·073 amp	0·055 amp
No-load loss	2·54 watts	2·77 watts
Primary resistance (200–210 tap)	23·3 ohms	21·1 ohms
Secondary resistance	0·20 ohm	0·21 ohm

Tested on full-load, the transformer showed a temperature rise after nine hours of 34°C. This figure was obtained from the change of resistance of the primary winding.

EXAMPLE 2

A transformer is required to supply six low-voltage heating elements from a 240-*volts* 50-*cycle supply. The elements each take* 175 *watts at* 50 *volts and are switched off and on by thermostats on the low-voltage side. In addition a* 3-*watt warning lamp is to be supplied at* 6 *volts.*

The method of construction proposed is to build up a bobbin from ⅛ in. thick square section resin-impregnated tube and ⅛ in. cheeks of the same material. The primary will be wound on first, then several layers of pressboard, followed by a flat strip secondary and the small indicator-lamp winding. Double-cotton covered wire can be used throughout and the whole will be impregnated in wax compound.

(1) An allowance of 1 volt is made for drop in the connexions. The output is thus $\frac{6 \times 175}{50} = 21$ amp at 51 volts, i.e. 1 071 watts from the main winding and 3 watts for the indicator lamp.

(2) A square stack of Magnetic and Electric Alloys No. 41A stampings in 0·014-in. Silcor II is suggested. (Dimensions under No. 441A in Table XIX on p. 472.) The nominal cross-section is 6·25 in.^{2} and the stacking factor 0·91.

(3) A fairly low flux-density of 11 000 lines per cm^2 is selected as it is desired to leave the primary connected for long periods when on light load.

(4) The primary turns are calculated to be 268. Take 270 for convenience.

(5) An efficiency of 95 per cent and a primary power-factor of 0·95 appear reasonable.

(6) The full-load primary current is found to be 4·95 amp from (1) and (5).

(7) The weight of the core is 27·8 lb, assuming a stacking factor of 0·91. The reactive volt-amperes are 4·8 VA per lb, so that the magnetizing current is 0·55 amp.

(8) A suitable wire for the primary at 1 500 amp per in.^{2} is SWG 16.

(9) The uncorrected secondary turns are 57·5 for the main winding and 6·75 for the indicator lamp. Suitable conductors are 0·24 in. × 0·06 in. strip (at 1 500 amp per in.^{2}) and SWG 20. (A heavier gauge than is necessary electrically is used for mechanical rigidity.)

(10) The primary mean turn length is approximately 13 in. The resistance is thus 0·72 ohm and the voltage drop 3·6 volts. The secondary mean turn is 16 in., giving a resistance of 0·042 ohm and a drop of 0·9 volt. The drop in the lamp winding can be neglected. As the transformer will often be used on light load, the corrections for primary as well as secondary are made to the secondary. 1·0 turn is added for the secondary drop and a further 0·86 for the primary drop; 0·1 turn is added to the lamp winding to allow for the primary drop.

(11) The nearest whole numbers are 60 turns and 7 turns. Adjust the primary to 273 turns.

(12) The iron loss at 0·74 watt per lb . . = 20·5 watts
The primary copper loss is 4·95 × 3·6 = 17·8 watts
The secondary copper loss is 21·0 × 0·9 = 18·9 watts

Total 57·2 watts

$$\text{The efficiency is } \frac{1\,071}{1\,071 + 57} \times 100 = 94{\cdot}9 \text{ per cent}$$

This is in good agreement with the value assumed in (5).

(13) The primary wire (SWG 16 double-cotton covered) winds 13 turns to the inch. Allowing for rigid end cheeks, the useful length of

each layer can be about 4 in., i.e. 52 turns. Hence 6 layers are required, occupying a radial depth of 0·46 in. The secondary strip will wind 3½ turns per inch, i.e. *14 turns per layer and* will, therefore, occupy 4 layers of radial depth 0·39 in. The lamp winding can be filled in at the end of the incomplete final layer and does not accordingly take up any depth. The space allowance is therefore—

0·031 in. core to former clearance
0·125 in. former wall thickness
0·46 in. primary winding
0·10 in. insulation
0·39 in. main secondary winding

1·11 in. Total

It will be seen that there is ample space outside for a lapping of tape for insulation and mechanical protection. It might in fact appear possible to increase the conductor size somewhat, but in view of the difficulty in pulling the strip really tight, it is probable that the space allowances above will be somewhat exceeded.

Summary of Data

Core. Square stack of M. & E. A. No. 41A stampings in 0·014-in. Silcor II—27·8 lb.
Primary. 273 turns of SWG 16 D.C.C.
Main Secondary. 60 turns 0·24 in. × 0·06 in. D.C.C. strip.
Indicator-lamp Winding. 7 turns SWG 20 D.C.C.

Test Results	MEASURED	DESIGNED
No-load voltage	52·7 volts	52·7 volts
Full-load voltage	51·1 volts	51·0 volts
No-load current	0·50 amp	0·55 amp
Full-load current	4·70 amp	4·95 amp
Primary resistance	0·68 ohm	0·72 ohm
Secondary resistance	0·04 ohm	0·042 ohm

It will be noted that although the results are quite close enough for practical purposes, the output voltage on load is slightly high. This is due to an overestimate of the primary current caused by taking the primary power-factor to be 0·95. Results show it to be practically unity in this case.

EXAMPLE 3

A transformer is required for a radio receiver. The heaters of the receiving valves take 2·25 *amp at* 6·3 *volts and the rectifying valve heater takes* 1·4 *amp at* 4·0 *volts. The anode currents of the valves total* 60 *mA at* 360 *volts (D.C.) to be supplied from a capacitor-filter full-wave rectifier circuit of the centre-tapped type. The transformer is to operate from* 110 *or* 200–250 *volts,* 50–100 *cycles, and reliability is to be as high as is reasonably practicable. The primary is required to be screened from other windings.*

NOTE. The design of this transformer is quite straightforward as regards the heater supplies and, as far as applicable, it follows the lines laid down in previous examples. The design for the winding supplying the rectifier anodes has not, however, been dealt with before. This subject is considered in detail in Chapter XIII, but for the present it is sufficiently accurate for most purposes to assume that the voltage and current ratings of the winding are the average values found in practice for this type of circuit. A full-wave centre-tapped rectifier winding passes current first in one half-winding and then in the other during alternate half-cycles. But the current does not flow through the valve and half-winding even for the whole half-cycle, but only for that portion of the cycle during which the induced e.m.f. exceeds the voltage on the reservoir capacitor. The current wave-form is thus a series of sharp peaks (Fig. 44) and the r.m.s. value is much higher than the mean value, i.e. than the d.c. output current; an average value for the r.m.s. value of the current *in each half-secondary* is 1·2 times the d.c. output.

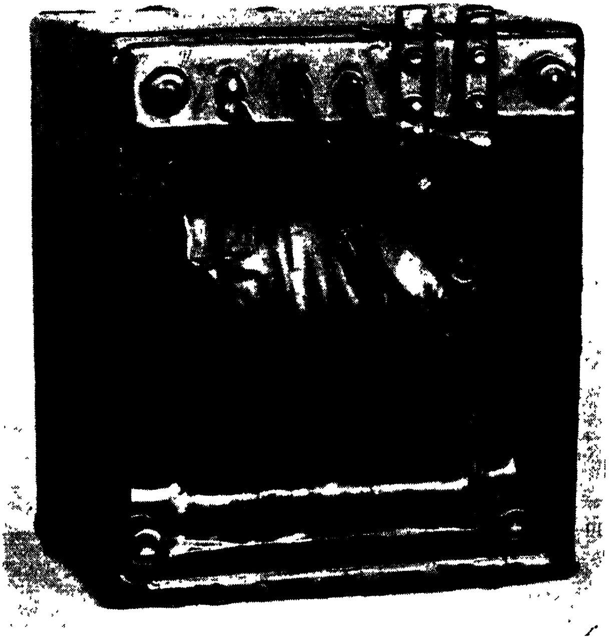

FIG. 43

1-kVA Transformer Supplying 21 Amp at 50 Volts, as Designed in Example 2.

The assessment of the voltage required from the transformer is also not quite straightforward. If no current were drawn from the rectifier, the voltage across the reservoir capacitor would build up to the *peak* value of the applied voltage. When supplying current, the voltage falls to an extent depending on the impedance of the rectifier and the transformer supplying it. For components in general use, the d.c. output voltage at 60 mA will be about 80 per cent of the peak a.c. voltage from the transformer *measured on open circuit.* If the winding is calculated on this basis, no extra turns are added to allow for the voltage drop when passing current; the effect of the winding resistance has already been taken into account in the 80 per cent figure.

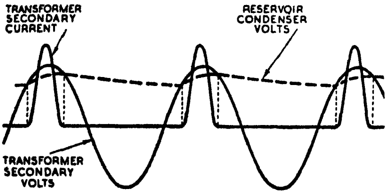
TRANSFORMER SECONDARY CURRENT
RESERVOIR CONDENSER VOLTS
TRANSFORMER SECONDARY VOLTS

FIG. 44

A Rectifier with a Capacitor Filter Passes Current only at those Points in the Cycle where the Secondary Voltage Exceeds the Output Voltage. The Secondary Current is thus a Series of Sharp Pulses and the r.m.s.-to-Mean Ratio is Large.

This method of deciding the probable voltage and current ratings of the rectifier winding is necessarily only approximate; when tentative winding data have been obtained and the resistance of the coil has been calculated, a check of the r.m.s. current and voltage can be made either from the rectifier valve maker's published curves or from charts given by Schade to which reference is made in Chapter XIII. It is not possible to start from these curves, because a knowledge of the winding resistance is required—an approximate value must first be obtained as described and corrections applied later.

The calculation of the primary current is complicated somewhat by the peaky wave-form of the rectifier current. If the rectifier were the only load, it would be best to decide the primary current by dividing the r.m.s. value of the secondary current by the turns ratio (Chapter III), but as the primary in this case also passes load component currents due to other secondaries, the r.m.s. value of the resultant current must be found. A quicker method, giving sufficiently good results, is to assume a figure of 1·3 for the ratio of rectifier input to output watts, so that the rectifier is assumed to draw 1·3 times its d.c. output.

General Construction

To reduce the risk of breakdown between turns, enamel and single-silk covered wire will be used. The layers will be interleaved with acid-free paper, the whole vacuum impregnated in bitumen-base varnish and multiple wax dipped to seal the winding against the ingress of moisture. Taps will be taken at 110, 205, 225, and 245 volts for ranges 105–115, 200–210, 220–230, and 240–250. In view of the screening requirement, the primary is best made the inner winding.

(1) To allow for voltage drops in the set wiring, the l.t. outputs will

be 6·5 and 4·1 volts. The rating of the h.t. winding is considered in detail in Chapter XIII; for the present it will be assumed that owing to its peaky wave-form the r.m.s. value of the current in each half-winding is 1·2 times the d.c. current, i.e. 1·2 × 60 mA = 72 mA, and that the a.c. peak voltage required is 1/0·80 of the d.c. output voltage. Thus the a.c. voltage is 315—0—315 r.m.s. The core size will be decided by the space taken up by the windings although the actual power output from the h.t. secondary is lower than would be the case if it supplied a resistance load. The apparent outputs for space purposes are thus—

6·5 volts 2·25 amp = 14·6 VA
4·1 volts 1·4 amp = 5·7 VA
630 volts 72 mA = 45·3 VA

65·6 VA

In addition the wave-form of the primary current is somewhat affected by the peaky nature of the rectifier current and its r.m.s. value will be higher than it would be if supplying an equal number of watts to a resistive load.

(2) It is desired to use "No-Waste" stampings, and in view of the extra space required for a 110-volt tapping which must be of larger wire and the use of silk and enamel-covered wire, the M. & E. A. No. 152 is suggested, using a square stack. (Dimensions under 1⅜ in. No-waste in Table XVII and Fig. 172 on p. 473.) Silcor III in 0·020-in. thickness will be satisfactory.

(3) A flux-density of 10 600 lines per cm² is proposed.

(4) The effective cross-sectional area of the core is 1·78 in.² on a stacking factor of 0·94. The primary turns work out to be 406 for 110 volts, 758 for 205 volts, 832 for 225 volts, and 905 for 245 volts.

(5) Efficiency 85 per cent; power-factor, 0·95 full-load.

(6) The figure for consumption purposes must not be confused with that obtained in (1) which is the figure obtained as a guide to the space taken up by the windings. For the present purpose, assume that the rectifier input is 1·3 times the d.c. output watts. The valve-heater loads being resistive remain as in (1) so that the loads are—

6·5 volts 2·25 amp = 14·6 VA
4·1 volts 1·4 amp = 5·7 VA
360 volts 60 mA × 1·3 = 28·0 VA

48·3 VA

The primary current at full-load is thus 0·54 amp at 110 volts, 0·29 amp at 205 volts, 0·265 amp at 225 volts, and 0·24 amp at 245 volts.

(7) The core weighs 4·05 lb, so that at the value of 4·0 VA per lb it is 16·2 for the whole core. The magnetizing current is thus 0·15 amp at 110 volts or 0·08 amp at 205 volts. This is satisfactory in relation to (6).

(8) The 110-volt portion of the primary being inside should be run at rather lower current-density than the outside, say 1 700 amp per in.^{2}, i.e. SWG 25. The 200–250 volt portion can be SWG 29 (at 2 000 amp per in.^{2}).

(9) The h.t. secondary turns are $\frac{315}{205} \times 758 = 1\,165$ (each half) and a suitable gauge for 72 mA is SWG 37.

The 6·5 volt winding requires $\frac{6{\cdot}5}{205} \times 758 = 24{\cdot}1$ turns of SWG 19 and the 4·1 volt winding requires $\frac{4{\cdot}1}{205} \times 758 = 15{\cdot}2$ turns of SWG 21.

(10) The estimated mean-turn lengths of primary, h.t. secondary, and l.t. windings are 6·7 in., 8·2 in., and 9·0 in. respectively. The resistances of the sections of the primary are 5·7 ohms (110-volt portion), 10·8 ohms (between 110- and 205-volt taps), and 2·3 ohms each for the two sections 205–225 and 225–245 volts, making total resistances of 16·5 ohms up to the 205-volt point, 18·8 to 225 volts, and 21·1 for the full primary. The voltage drops are 3·1, 4·8, 5·0, and 5·1 respectively, at 0·54, 0·29, 0·265 and 0·24 amp requiring turns corrections of 11·1, 17·8, 18·5, and 18·8. The l.t. winding resistances are 0·11 and 0·11 ohms, producing voltage drops of 0·25 and 0·15. The corresponding turns corrections are 0·92 and 0·55, making the corrected turns figures 25·02 and 15·75 respectively. The 6·5-volt winding is evidently as close to an integral number of turns as could be hoped for, but the rectifier heater winding would give rather a high output if 16 turns were used. Half-turns do not help in this case either; the best way is to reduce the wire diameter somewhat so as to increase the resistance. If SWG 22 is used, the current-density although high will not be excessive for an outer winding. The resistance would then be 0·14 ohm and the turns correction 0·72, making a total of 15·92, which is sufficiently close to 16 turns. The h.t. secondary resistance is 175 ohms per half-winding.

(11) No revision of primary turns is required to secure an integral number of turns on the secondaries. The winding, after voltage drop corrections are subtracted, is as follows—

110 volts—395 turns SWG 25 E. & S.S.C.
205 volts—740 turns (110-volt coil + 345 turns 29 E. & S.S.C.).
225 volts—813 turns (205-volt coil + 73 turns 29 E. & S.S.C.).
245 volts—886 turns (225-volt coil + 73 turns 29 E. & S.S.C.).

(12) The iron loss is 1·1 watts per lb, i.e. 1·1 × 4·05 = 4·45 watts
Primary copper loss at 225 volts is 0·265 × 5·0 = 1·33 watts
6·5-volt secondary copper loss is 2·25 × 0·25 = 0·56 watt
4·1-volt secondary copper loss is 1·4 × 0·15 = 0·21 watt
315-volt secondary copper loss is 175 × 0·072^2 × 2 = 1·82 watts

Total 8·37 watts

The efficiency is thus $\frac{45{\cdot}3}{53{\cdot}7} \times 100$ per cent = 84·5 per cent.

The h.t. secondary resistance is 175 ohms per half-winding. As already noted no correction is made in this winding for voltage drop, but it is advisable at this stage to check the correctness of the d.c. to a.c. ratios assumed for voltage and current.

Take the conditions when operating at 205 volts on the 200–210-volt tap. The primary resistance to this point is 16·5 ohms. We next require the total effective internal impedance of the source as seen from one rectifier, that is the sum of the half-secondary resistance, the primary referred to the half-secondary, and the rectifier impedance.

Turns ratio Prim./Half Secondary $= \dfrac{740}{1\,165}$

Primary resistance $= 16{\cdot}5$ ohms

Primary resistance referred to half-secondary $= 16{\cdot}5 \times \left(\dfrac{1\,165}{740}\right)^2$

$=$ 41 ohms

Half-secondary resistance $=$ 175 ohms

Rectifier impedance (from valve data) $=$ 200 ohms

Total impedance $=$ 416 ohms

Capacitance of reservoir capacitor $= 16\ \mu$F

R_L (360 volts 60 mA) $=$ 6 000 ohms

$\omega C R_L = 30$

Ratio of internal to load resistance $= \dfrac{416}{6\,000} = 7$ per cent

From Schade's curves (pp. 478–9) we find that the ratio of d.c. to peak a.c. volts for these conditions is 80 per cent, hence the d.c. output volts are $205 \times 1{\cdot}41 \times 0{\cdot}80 \times \dfrac{1\,165}{740} = 363$.

As this calculation takes no account of the heater loads which cause a small drop in the primary winding, the figure of 363 volts must be considered very satisfactory.

The r.m.s. current is now determined.

The value of $n\omega C R_L$ is of course 60, since $n = 2$ for full-wave rectifiers. Schade's curves give an r.m.s. to d.c. ratio of 2·3; since the D.C. per half-winding is 30 mA, the r.m.s. value will be 69 mA. This is in satisfactory agreement with the figure of 72 mA assumed.

(13) The insulation between the various windings is calculated on a basis of 50 volts per mil thickness of paper. The peak voltage differences are used in all cases. The data for the construction are summarized as follows—

Core. Square stack of M. & E. A. No. 152 stampings in 0·020-in. thick Silcor III.

Primary. 395 turns SWG 25 E. & S.S.C. + 491 turns SWG 29 E. & S.S.C. tapped at 345 and 418 turns.

Insulation. 4 layers 0·002-in. paper.

Screen. 1 turn copper foil.

Insulation. 5 layers 0·002-in. paper.
Secondary 1. 2 330 turns 37 E. & S.S.C. tapped at centre.
Insulation. 5 layers 0·002-in. paper.
Secondary 2. 25 turns SWG 19 E. & S.S.C.
Insulation. 5 layers 0·002-in. paper.
Secondary 3. 16 turns SWG 22 E. & S.S.C.
Outer lapping of tape as insulation and protection.

Test Results

Input—245 volts to full primary.

Output voltages at full load—	MEASURED	DESIGNED
H.T. (D.C. on reservoir capacitor)	357 volts	360 volts
6-volt secondary	6·47 volts	6·5 volts
4-volt secondary	4·07 volts	4·1 volts
No-load current	0·058 amp	0·066 amp
Full-load current	0·23 amp	0·24 amp
Primary resistance (to 110-volt tap)	5·2 ohms	5·7 ohms
Primary resistance (full-winding)	20·8 ohms	21·1 ohms
H.T. secondary resistance (half-winding)	193 ohms	175 ohms
6-volt secondary resistance	0·10 ohm	0·11 ohm
4-volt secondary resistance	0·12 ohm	0·15 ohm

3. General Comments

To reassure the reader, in view of the opening remarks about "trial and check-back methods,' it should perhaps be stated that no preliminary trials were made in the three examples quoted. Any corrections which had to be made to obtain agreement at the checking-back stage have been included so that a true idea can be obtained of the time taken over a particular design. In one important respect, however, the writer has had an advantage, namely, in having a good idea of the output which can be expected from a given-sized core. This is the most difficult estimate to have to make in the design, but once more let it be emphasized that there is no final uncertainty about the suitability of a core. The checking up in the later stages will clearly reveal whether an excessively large size or an inadequate one has been selected. Some suggestions are now made as to how to set about becoming familiar with the output capabilities of different core sizes.

First of all, the examples given in this chapter will form a useful starting point. They have been chosen to cover the range up to 1 kVA: first of all in Examples 1 and 2 on simple resistive loads, and in Example 3 with more complex multiple loading. It will, of course, be necessary to make due allowance for special factors such as tropical operation, or in the opposite direction for short-period duty. When a suitable rating has been established for one core size, the next ones to be used should be simply related in shape to the first, so that some quantitative relationship can be established. They may, for example, be the series of laminations represented by Magnetic and Electric Alloys Nos. 97, 93, 24, 189, which differ from No. 4 (used in Example 1) only in height. When only one dimension is varied it is much easier to assess its effect on the permissible output than when all three are changed together. Since all these stampings have $\frac{15}{16}$-in. central limbs, the mean-turn length of the windings will remain the same, so easing calculations. In addition some measure of standardization of bobbins may be possible between the various sizes. Another series of related transformers might be built up from the no-waste type (used in Example 3). In this case each dimension changes, but by the same percentage, because each member of the series is of the same general shape but drawn to different scales. A relationship can accordingly be worked out between the different members so that even before a particular one has been used, its output can be estimated by interpolation. In drawing up a chart of such output figures, it is essential to take as the normal figure for a given core the permissible output for a particular type of winding used for a specified class of duty.

For many designers the most convenient conditions to adopt would be: resistance loading, continuous operation, temperate climate. When other conditions obtain, the first step is to derive an equivalent loading to that specified, but in terms of a resistive load. For example, if the load has a power-factor less than unity, the transformer windings will have to be arranged to carry the same currents as though it had been resistive, but the *power* output will, of course, be reduced. The first step then is to convert the load wattage to volt-amperes in order to fix upon a suitable core. Similarly, if the transformer is feeding a rectifier, the r.m.s. value of the current through the windings is, in some cases, several times greater than the d.c. output. Since it is the r.m.s. value of the current which determines the heating of the windings, a transformer size has to be chosen which is considerably larger than might have been expected from the d.c. watts supplied by the rectifier. (This is illustrated in Example 3, in which the actual output required is 48·3 VA, but a frame size must be used which is capable of dealing with 65 watts in a simple load. This subject is considered in detail in Chapter XIII.)

CHAPTER VIII

Magnetic Leakage in Transformers

When its basic principles were described, the transformer was introduced to the reader as a device in which the passage of a fluctuating current through the primary induces an e.m.f. in the secondary that is magnetically coupled to it. When an impedance is connected across the secondary the effective primary reactance is reduced, so allowing more primary current to flow. At quite an early stage, however, it was pointed out that the primary current could be regarded as being made up from two components, an exciting (or magnetizing) component and a load component. Since the exciting component was considered to remain unchanged whether or not a load was applied, the "shunt" equivalent circuit was brought into being to represent the behaviour of the transformer. The equivalent circuit method of dealing with problems has been shown to be an extremely valuable one; by its use load and winding reactances can be "referred" from one side to the other merely by the use of a factor equal to the square of the turns ratio, and furthermore the variable permeability of the core over the cycle does not affect the output characteristics since the excitation elements of the circuit are shunt ones.

Nevertheless, it is well at this stage to remind the reader that the equivalent circuit is to some extent an artificial device; in an actual transformer there is only one path through the primary which has a reactance value of its own. The apparent change in primary impedance when a load is applied to the secondary is caused by the introduction of an additional e.m.f. into the primary circuit by the flow of the secondary current acting through the mutual inductance. The "coupled circuit" viewpoint, as this presentation is called, is therefore a more literal interpretation of the phenomena of transformer working. Using this theory, no difficulties arise in recognizing the fact that in practice some of the primary flux does not link the secondary, and vice versa. It is only necessary to modify the value of the mutual inductance M relative to the self-inductance of the winding, to represent any desired degree of coupling. The equivalent circuit technique in the form in which it has so far been presented, starts from the assumption that there is no flux leakage between the primary and secondary, i.e. that all the flux produced by the primary current is assumed to link the secondary and vice versa. This assumption has been made in all previous chapters, and is one which is valid in quite small transformers supplying resistance loads. Now it has already been indicated that although in many respects a magnetic circuit resembles an electrical one, there are two important differences—

(*a*) The reluctance of the magnetic circuit, unlike its resistance counterpart, is not constant with variations in the m.m.f.

(*b*) There is no such thing as a magnetic insulator.

We are not concerned in this chapter with (*a*) except in so far as its effects influence to some extent the results of (*b*). It is with this second statement that this chapter is concerned. Since the ratio of permeabilities of silicon steel and air is only of the order of a thousand to one (as compared with the ratio of the electrical conductivities of conductors and insulators of some 10^{20} times), it follows that leakages of flux from the core into the air space around it will be quite appreciable, particularly when the permeability tends towards the lower values it assumes at high values of flux-density.

It is not immediately obvious how the equivalent circuit should be modified so as to allow for the fact that the flux linkage is in practice incomplete. A return is, therefore, made to the coupled-circuit theory, and an equivalent circuit is then developed from it.

1. COUPLED CIRCUITS

In the coupled-circuit method of examining the behaviour of a transformer, currents having instantaneous values i_p and i_s are assumed to be flowing in the primary and secondary windings. These windings have resistances and self-inductances R_p, L_p, and R_s, L_s respectively, and are coupled by a mutual inductance M. The instantaneous values of the terminal potential differences are then

$$v_p = R_p i_p + L_p \frac{di_p}{dt} + M \frac{di_s}{dt} \quad . \quad . \quad (37a)$$

$$(-v_s) = R_s i_s + L_s \frac{di_s}{dt} + M \frac{di_p}{dt} \quad . \quad . \quad (37b)$$

Since the permeability of the core is assumed to be constant, the inductances L_p, L_s, and M are constant, so that if the voltages and currents are sinusoidal, these equations can be written vectorially as—

$$\mathrm{V}_p = R_p \mathrm{I}_p + j\omega L_p \mathrm{I}_p + j\omega M \mathrm{I}_s \quad . \quad . \quad (38a)$$

$$-\mathrm{V}_s = R_s \mathrm{I}_s + j\omega L_s \mathrm{I}_s + j\omega M \mathrm{I}_p \quad . \quad . \quad (38b)$$

In order to see the effect of the secondary constants on the behaviour of the primary circuit, the arrangement is best examined under the conditions of a "short-circuit test," i.e. the secondary is short-circuited and a reduced voltage applied to the primary so that the normal full-load currents flow. This test has already been described in connexion with efficiency testing (Chapter IV), but it also enables an effective value to be measured for the impedance of the transformer as a whole.

Equivalent Impedance

If in equation 38b V_s is equal to zero (as is the case when the secondary is short-circuited) then

$$\mathrm{I}_s = \frac{-j\omega M \mathrm{I}_p}{R_s + j\omega L_s}$$

Substituting in (38a)

$$(R_p + j\omega L_p)\mathrm{I}_p = \mathrm{V}_p + \frac{(j\omega M)^2 \mathrm{I}_p}{R_s + j\omega L_s}$$

$$\therefore \qquad \mathrm{V}_p = \mathrm{I}_p \left[R_p + jX_p + \frac{\omega^2 M^2}{R_s + jX_s} \right]$$

Hence the short-circuit impedance of the transformer seen from the primary is

$$\mathrm{Z}_{scp} = \frac{\mathrm{V}_p}{\mathrm{I}_p} = R_p + jX_p + \frac{\omega^2 M^2}{R_s + jX_s}$$

$$= R_p + jX_p + \frac{\omega^2 M^2 (R_s - jX_s)}{R_s^2 + X_s^2}$$

$$= R_p + \frac{\omega^2 M^2}{R_s^2 + X_s^2} R_s + j\left(X_p - \frac{\omega^2 M^2}{R_s^2 + X_s^2} X_s\right) \quad (39)$$

The equivalent resistance of the whole transformer is thus seen to be

$$R_p + \frac{\omega^2 M^2}{R_s^2 + X_s^2} R_s \quad . \quad . \quad . \quad (40)$$

and the equivalent reactance is

$$X_p - \frac{\omega^2 M^2}{R_s^2 + X_s^2} X_s \quad . \quad . \quad . \quad (41)$$

In other words, the apparent resistance of the primary is increased by a quantity equal to that of the secondary multiplied by a factor $\frac{\omega^2 M^2}{R_s^2 + X_s^2}$, whilst the reactance of the primary is *reduced* by that of the secondary multiplied by the same factor. If the measurement is made at a sufficiently high frequency for R_s to be neglected in comparison with X_s, this factor becomes $\frac{\omega^2 M^2}{X_s^2} = \left(\frac{M}{L_s}\right)^2$. In large transformers this condition is satisfied even at the rated frequency, but in small transformers, it is usually necessary to make the measurement at a higher frequency.

The Coupling Factor

It is convenient at this point to introduce the quantity known as the "coupling factor," which indicates the "tightness" or closeness of coupling. It is denoted by k and is defined as

$$k = \frac{M}{\sqrt{L_p L_s}} \quad . \quad . \quad . \quad . \quad (42)$$

The maximum possible value of k is unity, i.e. the condition in which all the flux links both windings completely. In air-cored transformers the values of k are generally less than 0·5, and values above 0·6 are obtained only with considerable difficulty. In an iron-cored transformer, however, k is usually above 0·99. If k is taken to be 1, then the approximate value of the equivalent resistance $R_p + \left(\frac{M}{L_s}\right)^2 R_s$ becomes $R_p + \frac{L_p}{L_s} R_s$. Since in the absence of magnetic leakage, the inductances of the two windings are proportional to the squares of the number of turns, the equivalent resistance is equal to $R_p + n^2 R_s$. This is, of course, the expression deduced in Chapter IV using equivalent circuit technique. The equivalent reactance using the approximate expression becomes $X_p - \frac{L_p}{L_s} X_s = 0$, again in accordance with our equivalent-circuit theory, for on the assumption of complete magnetic linkage, no resultant inductance appears.

Leakage Inductances

Having checked that the two methods of viewing the working of a transformer do in fact give the same results in the case in which complete coupling is assumed, we must now extend the equivalent circuit theory to cover the state in which k, although high, is not quite unity (e.g. 0·99). If this can be done, we shall retain the advantages which the method possesses in avoiding the use of quantities which do not remain constant over the cycle, without being restricted to an ideal coupling figure.

The fluxes due to primary and secondary circuits are divided into two components. The flux which links both coils of the transformer is known as the *mutual* or main flux, whilst that which links one coil or the other, but not both, is called the leakage flux. That which links the primary but not the secondary is the primary leakage flux, and similarly that due to the secondary current which does not link the primary, is the secondary leakage flux.

In equation 38*a* the e.m.f. induced in the primary by the flux linkages due to the primary current is represented by $j\omega L_p \mathrm{I}_p$. But not all this flux due to the primary current links the secondary as well. That part of the primary flux which does link the secondary, induces in it the e.m.f. represented by $j\omega M \mathrm{I}_p$ (i.e. the third term of equation 38*b*). But the voltages induced by a given flux are proportional to the turns of the winding involved, so that if this same portion of the primary flux which links the secondary acts on the primary it induces an e.m.f. $j\omega M I_p \frac{T_p}{T_s}$. But the whole primary flux, as we have seen, induces an e.m.f. of $j\omega L_p I_p$ in the primary, so that the difference $j\omega\left(L_p - M\frac{T_p}{T_s}\right)$ represents the voltage induced in the *primary* by that portion of the primary flux which *does not* link the secondary, i.e. the primary leakage flux.

Equation 38*a* can now be re-written in the form

$$V_p = R_p I_p + j\omega\left(L_p - M\frac{T_p}{T_s}\right)I_p + j\omega\frac{M}{T_s}(I_p T_p + I_s T_s) \quad (43)$$

The second term represents the e.m.f. induced in the primary by the flux which links the primary but not the secondary. This voltage is equal to that which would be dropped across an inductance of value $\left(L_p - M\frac{T_p}{T_s}\right)$ henrys connected in series with the primary winding. The third term is the e.m.f. induced in the primary by the resultant mutual fluxes due to the primary and secondary combined. This is the primary induced voltage or back e.m.f.

Equation 38*b* can be treated in the same way, so showing that the effect of the secondary leakage flux can be represented by connecting an inductance in series with the secondary winding. The inductances are known as the *leakage inductances* of the two windings and they give rise to *leakage reactances.*

According to this treatment, the transformer can be regarded as one having unity coupling factor between the primary and secondary (i.e. no magnetic leakage) but with inductances $\left(L_p - M\frac{T_p}{T_s}\right)$ and $\left(L_s - M\frac{T_s}{T_p}\right)$ connected in series with the two windings. It is then only a short step to the equivalent circuit in which the mutual inductance is replaced by a shunt reactance and the secondary elements are replaced by their equivalent values referred to the primary. The T-network shown later in Fig. 47 is thus exactly equivalent to the coupled circuits from which we started, except that an additional element R_w has been introduced to represent the core losses.

It should be noted that the quantities $\left(L_p - M\frac{T_p}{T_s}\right)$ and $\left(L_s - M\frac{T_s}{T_p}\right)$ are the leakage inductances of each winding *with respect to the other*; they must not be confused with the *total leakage inductance* (or *transformer leakage inductance* as it is sometimes called). The total leakage inductance is the sum of the leakage inductance of one winding and the leakage inductance of the other winding *referred to the first winding*; consequently it is also known as the *equivalent leakage inductance* of the transformer. The corresponding reactance is of course known as the *equivalent* or *total leakage reactance of the transformer*. Hence the equivalent reactance referred to the primary is

$$X_{eqp} = X_{lp} + n^2X_{ls}$$

and referred to the secondary is

$$X_{eqs} = X_{ls} + X_{lp}/n^2$$

Although the values of the leakage inductances of the two windings with respect to each other have been quoted in terms of the self and mutual inductances of the windings, it should not be assumed that they can in fact be calculated from measurements of these quantities, partly because in an iron-cored transformer the inductance values have little meaning since the permeability varies considerably over the cycle, but also because these expressions $\left(L_p - M\frac{T_p}{T_s}\right)$ and $\left(L_s - M\frac{T_s}{T_p}\right)$ are the difference between two very nearly equal quantities. The degree of accuracy required in the measurement of the inductances would in fact be out of question. The individual leakage reactances of the windings cannot in any case be separated effectively from each other in practice(1) so that the quantity which is usually measured is the equivalent or total reactance. This is usually measured with one winding short-circuited, and it is then known as the *short-circuit reactance*. The value obtained under these conditions differs slightly from the true equivalent reactance because the exciting current is lower under short-circuit conditions than under normal voltage conditions, but the difference can often be neglected (see Section 6 of this chapter).

Leakage Coefficient

The short-circuit reactance can be expressed very conveniently in terms of the open-circuit primary reactance and the coupling factor. The approximate form for the short-circuit reactance given in equation 41 is—

$$X_{scp} = X_p - \frac{\omega^2 M^2}{X_s}$$

But
$$\frac{M^2}{L_p L_s} = \frac{\omega^2 M^2}{X_p X_s} = k^2$$

Hence
$$X_{scp} = X_p(1 - k^2)$$

The factor $(1 - k^2)$ is known as the *leakage coefficient* denoted by σ, so that

$$X_{scp} = X_p \sigma$$

If for instance the coupling factor is 0·99, the leakage coefficient will be 0·02, so that the short-circuit or equivalent reactance will be 2 per cent of the primary or open-circuit reactance.

Other "Equivalent Circuits"

The circuit shown in Fig. 47 is known as the "exact" equivalent of a pair of mutually coupled circuits. If the inductances of the separate circuits are L_p and L_s and the mutual inductance M, then as we have seen, L_{lp} and L_{ls}' have the values $L_p - \frac{T_p}{T_s} M$ and $n^2 \left(L_s' - \frac{T_s}{T_p} M\right)$ and L_m has the value $\frac{T_p}{T_s} M$. The fact that the shunt arm is connected to the junction of the two series impedances complicates calculations, and for many purposes it is sufficiently accurate to transfer L_m to bridge either the input or output terminals of the network. The two series impedances can then be treated as one element, viz. the short-circuit impedance. This simplification is more applicable to larger transformers than very small ones because the exciting current is then a smaller proportion of the total current, and the excitation voltage drops, which are either neglected or exaggerated by the approximation, are less important. Nevertheless the simplified equivalent circuit is frequently sufficiently accurate for use in many small transformer problems and its validity is in fact implicit in the adoption of terms such as the equivalent impedance of a transformer.

Although the transference of the shunt arm to either end of the network results in an "approximate equivalent" if the values of the elements are left unchanged, another "exact equivalent" can be found if the values of the elements and the ratio of the imaginary ideal transformer are changed. The series elements are then, of course, no longer equal to the leakage impedances, but this may not be of any significance. If, for example, the shunt arm is placed directly across the input terminals, it assumes the same value as that of the primary self-inductance L_p and the series inductance (total) is equal to $L_p \frac{1-k^2}{k^2}$. The secondary winding and load impedances are then referred to the primary by the factor $\left(\frac{L_p}{M}\right)^2$ instead of n^2 as formerly. $\left(\frac{L_p}{M}\right)^2$ is very close to $\frac{n^2}{k^2}$ and, as k is seldom less than 0·99 in an iron-cored transformer, the "exact equivalent" values differ little from the "approximate equivalent" ones.

2. LEAKAGE FLUX

It will be remembered that this discussion was based on the fact that the core had to be of constant permeability in order that the quantities L_p, L_s, and M should be constant over the cycle. This limitation need not, however, be applied to the modified picture which has been introduced by the conception of leakage inductance.

Since the leakage fluxes link their own winding but not the opposite winding, their return paths must be through the air and not completely in iron as is the path of the mutual flux. For this reason the leakage inductances, unlike the winding inductances, are practically constant because the large air-gap makes the reluctances of the magnetic circuits largely independent of the permeability of the core. The conception of leakage fluxes thus enables the variable magnetic properties of the core to be eliminated from any calculations of output voltage and regulation. This is a most important advantage.

The leakage flux in a normal iron-cored transformer is quite low, being only a few per cent even at full-load and much less at no-load. The secondary voltage on open circuit is of course lower than it would have been had all the primary flux linked it, but as the proportion of leakage is very small indeed at no-load, the voltage drop is also very small. The reason why leakage produces a larger drop on load can be seen in a general way from the fact that it spoils the self-adjusting behaviour of a transformer in taking just the right amount of primary current to keep the main flux unchanged by the demagnetizing effect of the secondary current. If all the secondary flux does not link the primary, the rise of primary current is not sufficient to maintain the original state of affairs and the secondary voltage falls.

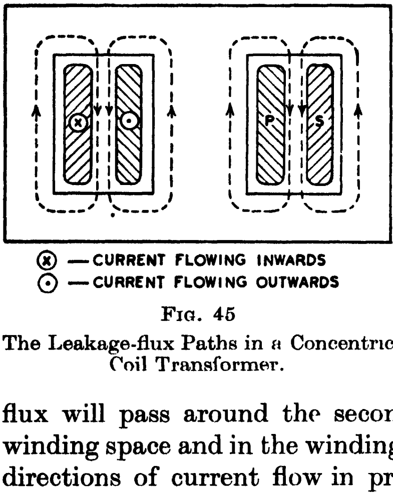

FIG. 45
The Leakage-flux Paths in a Concentric Coil Transformer.

If the primary and secondary windings are in the form of concentric cylinders round a limb of the core as in Fig. 45, the primary leakage flux will complete its path through the air in the space between the windings and in the primary space itself. Similarly the secondary leakage flux will pass around the secondary and return in the interwinding space and in the winding itself. Now the instantaneous directions of current flow in primary and secondary are such that the fluxes they produce are in opposition in the core which remains in a more or less constant state of magnetization whatever the load, but this is not true of the two leakage fluxes which are in the same direction. This arises from the fact that the interwinding space is, of course, *outside* the primary, but *inside* the secondary. Currents in opposite directions in the two coils thus produce fluxes in the same direction in the space between them. The leakage fluxes, unlike the mutual flux, thus increase in proportion to the load. The leakage fluxes are moreover in phase with the load current and the voltages they induce in the windings will accordingly be in quadrature with the load currents; this is, of course, in accordance with the representation of inductances in series with the windings. The appearance of e.m.f.s in quadrature with the load currents produces much less effect when the load is resistive than when inductive, because the load current is then in phase with the main secondary voltage induced by the mutual flux. The addition of a small voltage in quadrature with the main voltage vector has little effect on the resultant output voltage. If, however, the load is inductive so that the current and, therefore, the leakage flux has a component in quadrature with the secondary voltage, the leakage flux will induce an e.m.f. a further 90° out of phase and therefore in direct opposition to the main induced e.m.f. An inductive load thus causes much worse regulation than one of unity power-factor.

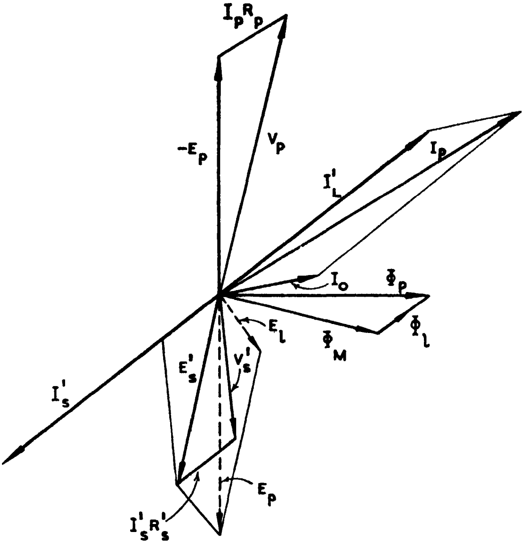

FIG. 46
Vector Diagram of a Transformer Supplying an Inductive Load, showing the Effects of a Leakage Flux Φ_l on the Output Voltage.

The vector diagrams for transformers given in Section 3 of Chapter IV can now be extended to show the effect of the e.m.f.s due to leakage.

3. VECTOR DIAGRAMS

A vector diagram is drawn first on the basis of the physical explanation given above, in which the leakage flux is regarded as being diverted from the primary flux as the secondary flux appears when on load (Fig. 46). The starting point is as before the flux Φ and, as this must now be specified more precisely, it becomes Φ_p the total primary flux. This flux links the primary and induces an e.m.f. E_p in lagging quadrature. Next $-E_p$ is drawn 180° out of phase, and in combination with the resistance drop due to the primary current gives V_p the applied voltage. This primary current is as usual the vector sum of the exciting current I_0 and the load component I_L'. Now on load some of the flux Φ_p is diverted from the iron path into the air space and this diverted or leakage flux is in phase with I_p. Φ_l is accordingly drawn parallel to I_p to represent the leakage flux and, if this is subtracted vectorially from Φ_p, the remainder is Φ_M the mutual flux. The secondary e.m.f. is induced by this mutual flux so that E_s' is now drawn lagging 90° behind Φ_M. Since Φ_M is smaller than Φ_p, it will be observed that E_s' is smaller than E_p in spite of the assumption of a one-to-one turns ratio. A vector $I_s'R_s'$ representing the resistance drop in the secondary is now drawn parallel to I_s' and subtracted from E_s', giving V_s' the secondary terminal voltage.

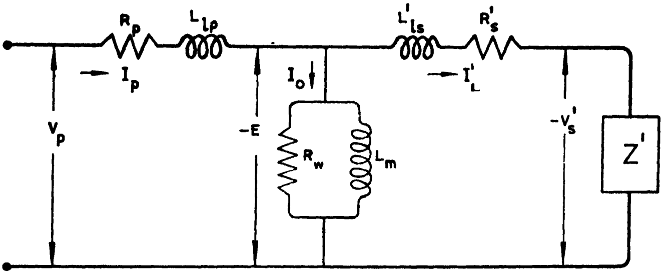

FIG. 47
Equivalent Circuit of a Transformer Referred to the Primary. The Leakage Reactances are now included.

The primary back e.m.f., E_p is proportional to Φ_p, since the whole flux links the primary, but it can also be regarded as being the vector sum of the e.m.f.s generated by the mutual flux Φ_M and the leakage flux Φ_l, i.e. numerically equal to the resultant of E_s' and E_l. The latter is the e.m.f. generated by Φ_l linking the primary, and it lags 90° behind Φ_l.

Although this method of picturing the effects of magnetic leakage on the magnitudes of the induced e.m.f.s is useful for explanatory purposes, it is more convenient in practical computations to work on the leakage reactance principle. Fig. 47 shows the equivalent circuit of a transformer in which the leakage inductances have not been neglected as has hitherto been the case. (The separation of the total leakage inductance into two separate sections is somewhat arbitrary since they cannot be measured independently, but only as a sum. This follows from the fact that there is no sharp line of demarcation between the primary and secondary leakage fluxes. It has accordingly become the convention to allocate the leakage reactance equally between the two windings.)

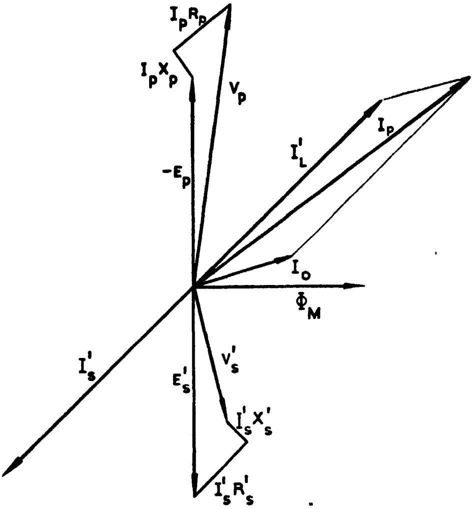

FIG. 48

Vector Diagram for the Same Load Conditions as in Fig. 46 but Using the Leakage Reactance Method of Calculation.

In drawing the vector diagram on these lines (Fig. 48) the mutual flux Φ_M which links both windings is now taken as the starting point and E_s' the secondary induced voltage drawn 90° lagging behind it. The reversed primary induced voltage $-E_p$ is drawn 90° ahead of the mutual flux and two voltage drop vectors added to it to obtain V_p the applied voltage. The drops are I_pR_p parallel to I_p as before, and $E_l = I_pX_p$ the reactance drop perpendicular to I_p. The corresponding resistance and reactance drops $I_s'R_s'$ and $I_s'X_s'$ are *subtracted* from E_s' to give V_s' the secondary terminal voltage (referred to the primary). Comparison of Figs. 46 and 48 shows that the final results obtained are the same, apart from a small discrepancy caused by neglect of the voltage drops due to the exciting current.

These vector diagrams have been drawn for inductive loads of moderate power-factor (i.e. the load current lags on the secondary voltage by about 45° so that $\cos \psi = 0{\cdot}71$), and the reactive drops have been seen to cause a considerable fall in output voltage. The lower the power-factor of the inductive load the more nearly the leakage reactance drop becomes in direct opposition to the induced e.m.f., and hence the lower the output voltage. If, on the other hand, the load is a resistive one so that the vectors I_s' and hence I_p become nearly parallel with the induced voltages, then the reactive drops are drawn perpendicular to the induced voltage vectors whilst the resistive drops approach direct opposition. The reactances thus have little effect when the load power-factor is near unity, and the regulation is chiefly determined by the winding resistances. Since many applications of small transformers are for resistive loads, it was decided in the interests of simplicity to neglect the effects of flux leakage in the chapters preceding this. In further justification of this decision it should be pointed out that the ratio of resistance to leakage reactance tends in any case to be high in small transformers.

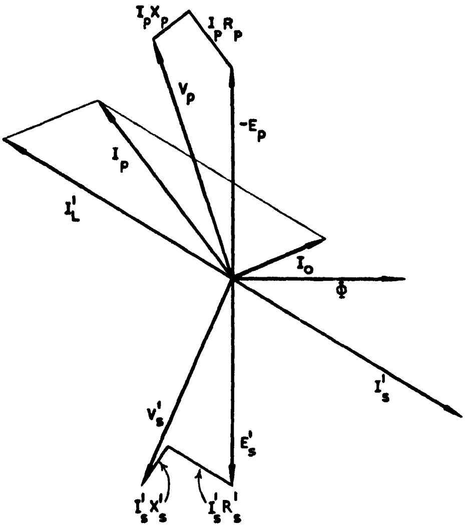

FIG. 49

Vector Diagram of a Transformer Working into a Capacitative Load. Note that the Secondary Output Voltage on Load Exceeds the e.m.f.

Going to the other extreme of a low power-factor capacitative load, i.e. one in which the current leads the voltage, the effect of the leakage reactance drop is to offset the resistance and at a certain power-factor the regulation is nil. At greater leading phase angles still, the voltage on load rises above the open-circuit voltage. This case is illustrated in Fig. 49. The flux, induced voltage, and exciting current vectors are drawn as before. The load current vector, however, is in advance of the secondary induced voltage and consequently the primary load component current and resultant primary current are in advance of the applied voltage. The reactive voltage drops, drawn perpendicularly to their respective currents (and in advance of them), are thus in such directions as to add to the induced secondary voltage and subtract from the primary induced voltage. The resistive drops tend as usual to reduce the secondary output voltage.

4. VOLTAGE REGULATION

This is a convenient place to take up again the question of regulation which was considered in Chapter IV, only from the point of view of resistive voltage drops. When drawing the vector diagrams of the previous section of this chapter, the drops have been allocated partly to primary and partly to secondary. Whilst there may be some justification for separating the resistive drops in this way, that is certainly not true of the reactive drop, which can be divided only in an arbitrary manner. It is accordingly the usual practice to refer the primary resistance and reactance to the secondary (or vice versa), and, adding them to the corresponding quantities on the secondary side, an effective transformer resistance and reactance are obtained. When this is done it can be imagined that the input voltage is transferred to the secondary without any losses, and after being divided by the turns ratio is applied to the load through the total resistance and reactance. Alternatively all the drop can be imagined to be deducted from the applied voltage, which is then transformed without loss and applied to the load.

Using that artifice together with the assumption that the exciting current can be neglected in comparison with the load component current, a very convenient diagram can be drawn to show the voltage regulation at any power-factor load (Fig. 50). This is due to Kapp and is evolved from the vector diagrams of the previous section.

The starting point is the current vector which is drawn vertically down from the origin *O*. Along the continuation of I in an upward direction is drawn *OA* representing I*R*, the

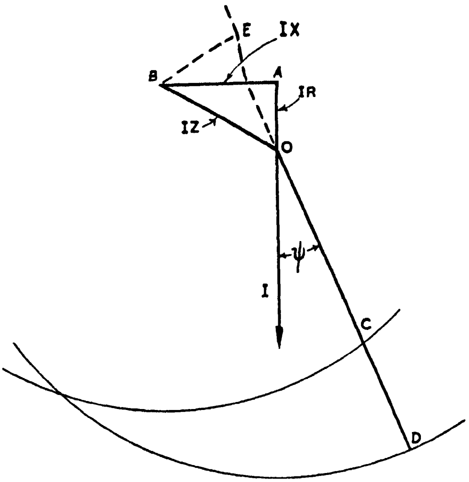

FIG. 50

The Kapp Regulation Diagram for the Rapid Estimation of Output Voltage at Loads of Various Power-factors.

total resistive drop, *and IX, the total reactive drop, is added* vectorially to it, to give *OB* representing IZ, the total voltage drop. From both *O* and *B* arcs are drawn with radius equal to the open-circuit voltage. If a line *OCD* is now drawn to cut the two arcs at *C* and *D* so that it makes an angle with the current vector equal to the phase angle of the load, then *OC* will be the voltage on load and *CD* the regulation voltage drop. With a lagging power-factor an increasing phase angle gives a larger drop whilst with a leading power-factor it gives a smaller drop. If the leading phase-angle is large enough it causes a rise in voltage with load.

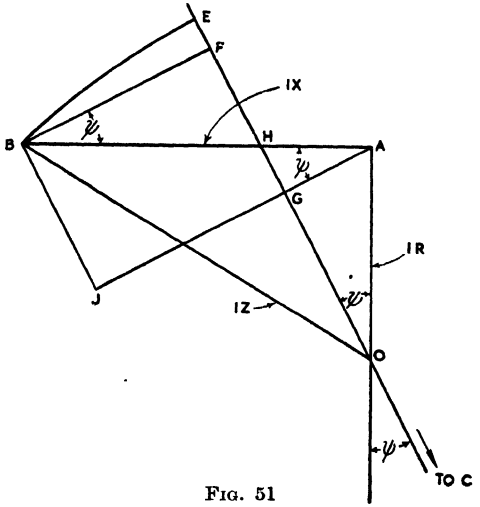

FIG. 51

The Upper Portion of Fig. 50 to a Larger Scale.

A slight addition to the Kapp diagram enables a formula to be derived for the calculation of the regulation in those cases in which a graphical method is not acceptable. With centre *C* and radius *CB* describe an arc to cut *CO* produced at *E*. Then *OE* also represents the voltage drop. The upper part of the diagram is reproduced on a larger scale in Fig. 51. Perpendiculars *BF* and *AG* have been dropped on to *OE*, and *AG* has been produced to meet a perpendicular from *B* at *J*. The intersection of *OE* and *AB* is denoted by *H*.

Angles *HOA* and *FBH* are both equal to the phase angle ψ.

$$\begin{aligned} \text{Now}\quad OE &= OG + GH + HF + FE \\ &= OA \cos\psi + AH \sin\psi + HB \sin\psi + FE \\ &= IR \cos\psi + IX \sin\psi + FE \end{aligned}$$

Since *CB* and *CE* are radii of a circle and *BF* is perpendicular to CE, then

$$EF(2\,EC - EF) = BF^2$$

$$\therefore \qquad 2\,EC \,.\, EF - EF^2 = BF^2$$

Neglecting EF^2 compared with the other quantities,

$$\begin{aligned} EF &= \frac{BF^2}{2\,EC} \\ &= \frac{(JA - GA)^2}{2\,EC} = \frac{(\mathrm{I}X \cos\psi - \mathrm{I}R \sin\psi)^2}{2\,(\text{open circuit voltage})} \end{aligned}$$

Thus the voltage drop *OE*

$$= (\text{resist. drop}) \cos\psi + (\text{react. drop}) \sin\psi$$
$$+ \frac{[(\text{react. drop}) \cos\psi - (\text{resist. drop}) \sin\psi]^2}{\text{twice open-circuit volts}} \quad . \quad (44)$$

The last term can often be neglected without serious error.

Using the percentage system, as is usual in most regulation applications—

$$\text{Percentage regulation} = \frac{\text{voltage drop}}{\text{open-circuit voltage}} \times 100$$

$$\therefore \frac{OE}{EC} \times 100$$

$$= \frac{100\,(\text{resist. drop}) \cos\psi + 100\,(\text{react. drop}) \sin\psi}{\text{open-circuit voltage}}$$

$$+ \frac{1}{200} \frac{[100\,(\text{react. drop}) \cos\psi - 100\,(\text{resist. drop}) \sin\psi]^2}{\text{open-circuit voltage}}$$

$$\therefore \text{ Percentage regulation}$$
$$= (\text{percent. resist. drop}) \cos \psi + (\text{percent. react. drop}) \sin \psi$$
$$+ \frac{[(\text{percent. react. drop}) \cos \psi - (\text{percent. resist. drop}) \sin \psi]^2}{200} \quad . \quad . \quad . \quad . \quad (45)$$

5. CALCULATION OF LEAKAGE REACTANCE

It is often necessary to calculate the leakage inductance to be expected for a particular design of transformer, and although the results are not of a high order of accuracy useful working figures can be obtained.

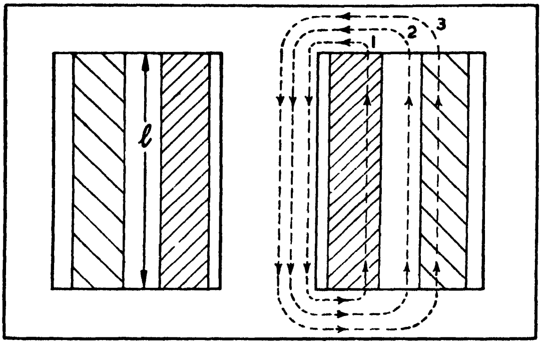

FIG. 52

Representative Paths of the Leakage Flux in a Concentric Coil Transformer.

The arrangement of a typical concentric coil transformer is shown in Fig. 52, the primary winding being the inner one. The mutual or main flux which links the whole of both coils is omitted for clarity, but three representative leakage flux paths, 1, 2, and 3 are drawn. These all pass through the central iron core, but instead of completing their paths entirely in the iron, they pass from one limb to the other across the window space. It is satisfactory to assume that the air paths of these flux lines are parallel and uniform across the window space, and that the reluctance of the iron section of the path is negligible compared with that in the air. The problem is to calculate the total flux which links one winding (say the primary) but not the secondary. In practice some of the lines do not link even the whole of the primary (path 1), whilst others, such as

3, link the whole primary and some of the secondary. It is accordingly necessary to derive equivalent fluxes which link the whole of the primary and none of the secondary. Flux which follows paths such as 2 between the two windings already meets the stated condition and no modification is required.

Consider a transformer having concentric windings of equal length l cm as shown in Fig. 52, and let the radial thickness of the two windings be b_1 and b_2 and that of the space between them b_0 cm. Let the lengths of the mean turns be m_1 and m_2 and the mean circumference of the interwinding space m cm. Let the ampere-turns of the two windings be assumed equal and let them be IT. The leakage flux is for convenience split up into three portions, that passing through the primary, that through the secondary, and that through the space between. Only the latter will have the full effect, since in the other portions all the lines either do not link all the primary turns or link also some of the secondary. The component leakage fluxes are considered to have return paths through the primary, secondary and interwinding space respectively, and the reluctance of the iron path is assumed to be negligible compared to that in the air.

The interwinding leakage flux Φ_0 (path 2)

$$= \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{4\pi IT}{10} \times \frac{m_0 b_0}{l} \quad . \quad . \quad (46)$$

The primary leakage flux (path 1) spreads over the whole width b_1, so that an equivalent flux Φ_1 must be derived which links the whole winding. Consider an element of radial width δx situated at a radius distance x from the inner edge of the primary, then the m.m.f. $= \frac{4\pi I}{10} \times \frac{x}{b_1} T$, since only $\frac{x}{b_1}$ turns are operative. The flux is then $\frac{\text{m.m.f.}}{\text{reluctance}} = \frac{4\pi IT}{10} \times \frac{x}{b_1} \times \frac{m_1 \delta x}{l}$. The equivalent flux linking all the primary turns would be $\frac{x}{b_1}$ times as great, so that

$$\Phi_1 = \frac{4\pi IT}{10} \frac{m_1}{l b_1^2} \int_0^{b_1} x^2 dx.$$

$$= \frac{4\pi IT}{10} \times \frac{m_1 b_1}{3l} \quad . \quad . \quad . \quad (47)$$

As regards paths similar to 3, the m.m.f. is that due to the whole of the primary turns opposed by that due to $\frac{b_2 - x'}{b_2}$ of the secondary turns, where x' is the distance from the outer edge of the secondary.

$$\text{Hence m.m.f.} = \frac{4\pi I}{10} \times \left(T - T\frac{b_2 - x'}{b_2}\right) = \frac{4\pi IT}{10}\frac{x'}{b_2}$$

This expression corresponds to that derived for the primary leakage m.m.f., so that, by analogy,

$$\Phi_2 = \frac{4\pi IT}{10} \times \frac{m_2 b_2}{3l} \quad . \quad . \quad . \quad (48)$$

Hence the total leakage flux $\Phi_l = \Phi_0 + \Phi_1 + \Phi_2$

$$= \frac{4\pi IT}{10l}\left(m_0 b_0 + \frac{m_1 b_1 + m_2 b_2}{3}\right) \quad . \quad . \quad (49)$$

Now the inductance of any circuit in henrys is equal to 10^{-8} times the number of linkages of the flux with the circuit when it is passing a current of 1 ampere so that the leakage inductance

$$= \frac{T\Phi_l}{I} \times 10^{-8} \text{ henrys}$$

$$= \frac{4\pi T^2}{10^9 l}\left(m_0 b_0 + \frac{m_1 b_1 + m_2 b_2}{3}\right) \text{henrys} \quad . \quad (50)$$

(All dimensions being in cm.)

When using this expression the appropriate winding turns must be used, that is to say the number of primary turns must be inserted if it is desired to calculate the leakage inductance referred to the primary side, and similarly for the secondary side.

The leakage reactance is, of course, readily obtained from the leakage inductance by multiplying by ω $(= 2\pi f)$.

6. MEASUREMENT OF LEAKAGE REACTANCE

Reference to Fig. 47 (which shows the equivalent circuit of a transformer) indicates that if the output terminals are short-circuited, the impedance measured between the input terminals would be the equivalent impedance of the transformer referred to the primary, modified slightly by the shunting effect of the exciting circuit. The latter has, however, a fairly small effect even at the working voltage, and if the primary voltage is reduced so that the current flowing through the short-circuiting lead is equal to the normal full-load current, then the shunting effect of R_w and L_m becomes even less important. For practical purposes, therefore, the short-circuit impedance is equal to the total series impedance of the transformer.

At first sight it might appear that impedance values measured under reduced voltage conditions would differ considerably from those at normal voltage, but it must be remembered that as the path of the leakage flux is largely through air, the value of the leakage inductance is only very slightly dependent on the applied voltage. The measurement of impedance is usually combined with that of efficiency (see p. 80), since the necessary readings are obtained with the same set-up. The secondary winding is short-circuited and a reduced voltage applied to the primary of such a value as to pass the full-load current. The current is actually measured in the primary, but the secondary current will also reach the full-load value at the same input voltage. Readings are then made of primary voltage, current, and power. The voltage required to produce full-load current under short-circuit conditions is known as the *impedance voltage* or *impedance drop*, and the value of the equivalent impedance is obtained from the current and voltage readings. The equivalent resistance is obtained from the voltage and power readings and hence the equivalent reactance can be calculated. As already explained, the reactance cannot really be allocated between the two windings, since it is only apparent when both are carrying current.

The values of impedance, resistance and reactance obtained in the manner described are, of course, those referred to the primary, but if the test had been conducted by short-circuiting the primary and applying a reduced voltage to the secondary, equivalent values relative to the secondary would have been obtained.

7. Typical and Special Values

The effect of the leakage reactances on the performance of a transformer has already been examined in some detail in

Section 3, in which it was shown that it is partly responsible for a drop in output voltage as the load is increased. This is particularly noticeable at low power-factor inductance loads. In view of this undesirable result, it might seem advisable to reduce the leakage reactance to the lowest possible value, and many of the older power transformers were made with quite low reactances. The reactance does, however, fulfil an important function in limiting the short-circuit current under fault conditions, and consequently it is unwise to make the value particularly low. In large transformers working at 50 cycles the reactance voltage drop (at full-load) is usually made between 4 per cent and 10 per cent of the voltage, the higher value being used for the largest sizes. In small transformers rated at a few hundred watts, the figure is usually between 0·5 per cent and 2 per cent. In these applications the resistance is often much more important than the reactance.

Although the reactance of ordinary transformers is often kept somewhat above that which could be obtained without undue difficulty in order to safeguard them from short-circuit stresses, the increase is not large. There are, however, instances of the deliberate introduction of leakage on a considerable scale in order to assist the operation of certain devices, notably gaseous discharge tubes.(2) A certain minimum potential is necessary to make a discharge tube, such as a neon sign or mercury-vapour tube, pass any current at all. Once a discharge has started, however, the current which flows is limited practically only by the impedance of the source. If a low-impedance transformer were used as a source, the current would be so large that the tube would be blown to pieces or the transformer damaged, so that special transformers are used which limit the current to reasonable values. The wave-form of the tube current is far from sinusoidal under these conditions. The tube potential rises at first in a sinusoidal manner until the tube "strikes." Up to this point the current has, of course, been zero. The potential across the tube now remains nearly constant, whilst the current which has suddenly jumped to a fair value rises to a peak and then falls until the tube is extinguished when the transformer e.m.f. becomes too low to maintain the discharge. This voltage is somewhat lower than the "striking voltage." Depending upon whether the design of the electrode system is symmetrical or not, this operation may repeat itself during the negative half-cycle or possibly not until the next positive half-cycle. In either case the current is limited by the reactance of the transformer, and this is deliberately increased by inserting a "magnetic shunt" between the primary and secondary windings. The design is usually made by trial, packets of laminations being inserted till the short-circuit current in the secondary (with full input volts) is the desired value.

EXAMPLE

A transformer for a small cold-cathode mercury-vapour tube is required to provide an open-circuit voltage of 1 000 (r.m.s.) *and when short-circuited the current is to be* 100 *mA. The r.m.s. current through the tube is* 90 *mA. The supply is* 230 *volts* 50 *cycles.*

The order of the steps in the design follows that given in Chapter VII.

(1) The equivalent impedance referred to the secondary is clearly 1 000/0·1 = 10 000 ohms. Heating and efficiency considerations demand that this shall be almost entirely reactive, so that the leakage inductance must be 10 000/$2\pi \times 50$, i.e. approximately 32 henrys. To obtain so large a figure will probably involve some loss of volts as compared with the turns ratio even on open-circuit, so that when selecting a suitable core, the apparent rating should be increased by say 10 per cent. The VA rating is thus 1 000 × 0·09 × 1·1 = 99 VA.

(2) To enable a sufficiently high leakage to be obtained easily, a core with long narrow windows is chosen, and the side-by-side method of winding is used. A suitable core of this type appears to be one having a 1¼ in. × 1¼ in. centre limb with windows 3¾ in. × ¾ in., making an overall size of 5 in. × 4 in.

(3) A flux-density of 12 500 lines per cm^2 is selected.

(4) The uncorrected primary turns are 915.

(5) and (6) The data given enable the load component of the primary current to be estimated from the assumed turns ratio as

$$0{\cdot}09 \times \frac{1\,100}{230} = 0{\cdot}43 \text{ amp.}$$

(7) The estimation of the magnetizing current is more important in this case than usual, because it will flow through part of the leakage reactance, so causing a drop in *open-circuit* output volts. For Sankey's 42 Quality 0·018-in. laminations, the VA per lb are given as 7·3 at $B = 12\,500$, and the weight of the core is 4·37 lb. The magnetizing current is thus $\dfrac{7{\cdot}3 \times 4{\cdot}37}{230} = 0{\cdot}14$ amp. If it is assumed that the leakage reactance is equally divided between primary and secondary, and that for

the purposes of transferring from secondary to primary the ratio is taken to be $\frac{230}{1\,100}$, then the primary leakage reactance is

$$\frac{10\,000}{2} \times \left(\frac{230}{1\,100}\right)^2 = 219 \text{ ohms.}$$

The primary reactance voltage drop on open-circuit is thus 0·14 × 219 = 30 volts. This is reasonably close to the figure of 10 per cent assumed in steps (1) and (6), but if great accuracy is wanted, a recalculation can be made on the $\frac{30}{230} \times 100$, i.e. 13 per cent basis.

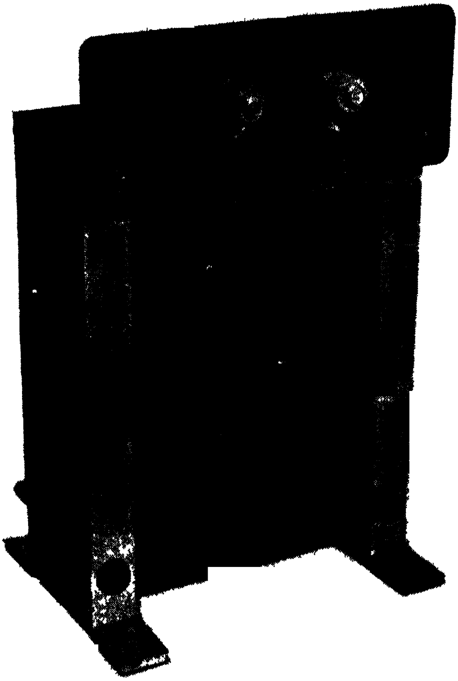

FIG. 53

The High-leakage Transformer (Designed on Page 177) for a Cold-cathode Discharge Tube.

(8) The load-component of the primary current has been found in (6) to be 0·43 amp and the magnetizing current in (7) is 0·14. Now in this application the two are nearly in phase so that the maximum primary

current will be nearly 0·57 amp. A suitable size at 2 200 amp per in.^{2} would be SWG 26.

(9) The secondary wire size should be SWG 36 on the same basis, and the turns required to produce an open-circuit pressure of 1 000 volts are determined from an effective input voltage of 230 — 30 volts. The secondary turns are thus $\frac{1\,000}{200} \times 915 = 4\,575$.

(10) It is not practicable to proceed further by calculation, so the bobbins should be wound and the laminations fitted. Packets of rectangular stampings are now inserted between the primary and secondary to such a depth that the desired short-circuit current is obtained.

Tests

A transformer wound to these figures was found to require packets of laminations $\frac{7}{16}$ in. thick inserted on each side between the primary and secondary coils to reduce the short-circuit current to 100 mA. The open-circuit voltage was then 1 005 and the exciting current 0·12 amp.

8. MAGNETIC LEAKAGE AND CORE LOSSES

The general principle has frequently been laid down in the earlier chapters that the magnitude of the iron loss remains unchanged between no-load and full-load. This statement is dependent on the theory that the magnetic condition of the iron remains the same when the transformer is loaded up, a state of affairs realized in the absence of magnetic leakage. If the whole of the secondary flux does not link the primary circuit, however, some modification of the theory may be required, although in normal transformers the effect is slight and can usually be neglected. To enable exceptional cases to be considered, the matter is discussed briefly below.

Consider a transformer having a sufficiently large leakage reactance to cause an appreciable reactance drop on full-load current. If the same output voltage is to be obtained at full-load as no-load, the primary voltage will have to be raised. The question is—does this higher input voltage increase the iron loss, and if so how much? The answer is that it raises the iron loss in some parts of the core, but not in others. In a general way it can be taken that the iron loss is modified by the ampere-turns of the winding nearest to it. If in a small shell-type single-phase transformer, the primary is next to the central limb, the flux-density in that limb will be determined by the primary whilst that in the yokes and outer limbs will be affected by the secondary. If the primary voltage is raised at full-load to produce the same secondary terminal voltage on load the loss in the central limb will accordingly increase somewhat, and that in the rest of the core will be unchanged. Alternatively, if the input voltage is kept constant, and the output voltage allowed to fall, the central limb losses will remain constant whilst those of the yokes and outer limbs will fall on load. The effect of these changes on the iron losses as a whole is not great, for over the normal range of input voltages the affected part of the core is only likely to change its losses by some 20 per cent, so that the proportion over the whole core is less than 10 per cent.

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(2) F. G. SPREADBURY: *Electric Discharge Lighting.* Sir Isaac Pitman & Sons, Ltd., 1946.

CHAPTER IX

Tappings and Tap-changing

A "TAPPING" is the term applied to a connexion made to a winding at a point other than at its ends. Tappings are often used to obtain the effect of varying the number of turns on the coil, and in that case the winding beyond the tap, although still maintained at its normal potential, passes no current. Sometimes, however, the tap may be used to allow two loads to be supplied simultaneously at different voltages, or as in certain rectifier circuits at different instants. An auto-transformer is an example of a tapped winding which is carrying current in sections on both sides of the tap, and Chapter XV is devoted to this type.

Tappings are most frequently used for the purpose of varying the input or output voltages over a small range. For example, portable electrical equipment of various kinds has to be made so that it can be easily adjusted to give the same results when connected to any of the supply voltages in normal use. Tappings are accordingly taken at suitable points on the primary winding so that the number of primary turns appropriate to that voltage can easily be put into circuit by means of a switch or plug and socket. Again, tappings may be used to alter the output voltage over a small range, or to keep the output voltage constant in spite of variations in load current which alter the internal voltage drop in the transformer. Tappings of this kind may be taken either on the primary or the secondary, the number of turns on the primary being decreased to obtain a higher output, whilst if the transformer is tapped on the secondary they would be increased.

Regulating tappings can be dealt with quite simply as regards the electrical side, because the effect is just that of an alteration in the number of turns on the winding, whilst the portion that is not carrying current can in most cases be ignored. Information on the rating of tapped windings of the other kind is contained in the several chapters dealing with the applications, e.g. in Chapter XIII, dealing with rectifier transformers, and also in Chapter XV, dealing with auto-transformers, where the question of the power ratings of the windings is of the first importance.

1. Regulating Tappings

Choice of Winding

There is no hard-and-fast rule as to whether primary or secondary windings should be tapped, but when all relevant factors are taken into account there is usually no doubt which should be used in any particular case.

(*a*) In a general way, it may be taken that the high-voltage side is the more suitable; in large transformers where the volts per turn are high, because there would often be difficulty in selecting points on the winding at the particular voltages desired, since it would involve fractional turns, an expression which, at any rate in core types, has no meaning, for all the induced voltage appears in the portion of the turn within the plane of the core; in small transformers because of the voltage drops likely to occur in heavy-current circuits at the tap-changing device, e.g. in a spot-welding transformer where the low-voltage winding is carrying thousands of amperes at a pressure of several volts only.

(*b*) Where neither winding has a small number of turns, the considerations of (*a*) do not apply, and there is then some advantage in tapping the secondary rather than the primary because heavy-current surges are likely to occur if the magnetizing current circuit is disturbed as would be the case if the primary were switched. A further disadvantage of primary tappings used to obtain varying secondary voltages is that the flux-density in the core is altered according to the output required. Since there is a fairly definite figure for the maximum flux-density which can be used, any variations must be in the downward direction. It follows that the iron is not fully utilized under these conditions; consequently more turns per volt are required than is necessary and the copper losses are unduly high, or alternatively the rating has to be reduced considerably.

There is, however, one case in which this variation in iron loss can be put to good use. A distribution transformer in a sub-station is left with its primary connected continuously to the supply. When the load is small, the iron loss still continues unchanged if the secondary is tapped down to keep the voltage constant, but if the primary turns are increased to achieve this result, the iron loss falls and a high efficiency is obtained on light loads.

(*c*) Where tappings are used to make a transformer suitable for use on different input voltages, they should be taken on the primary. This is the opposite case to that given in (*b*); to maintain a constant flux-density the primary turns have to be varied. Tapping the *secondary* would in this case result in poor core utilization and consequent high copper losses. The argument against secondary tappings is, of course, unanswerable if there are several secondaries on the transformer supplying several voltages simultaneously, as each would have to be tapped.

2. Methods of Tap-changing

The simplest arrangement of tap selection is to bring out connecting leads from the windings to a grouping point or terminal panel, where terminals or connecting bolts are attached to the ends of the leads. In oil-immersed transformers, these connecting points are taken up to oil level and access can be obtained by removing the top cover. External cables are connected to the appropriate tappings at the time of installation and are not normally altered. Naturally this method of connexion is not suitable for such applications as voltage maintenance under varying load conditions where rapid changes have to be made at frequent intervals; it is chiefly employed for the selection of the correct number of primary turns on a transformer arranged for operation on one of a number of voltages. It does not take advantage of the possibility offered by a series of closely-spaced tappings of adjusting the primary turns to correct fluctuations of the supply voltage.

This simple system can be made rather more convenient without any great increase in cost by the provision of a link arrangement by which a permanent external connecting point can be coupled to any of the tappings at will. This arrangement merges imperceptibly into the complete *off-load* tap-changing switch. This may be a rotary-arm type moving over studs or through clips as in small transformers, or it may take the form of a number of separate switches operated in sequence by a series of cams on a common shaft rotated by a hand wheel. Whichever type of switch is used, the arrangement is essentially an "off-load" one, because each contact must break before the next one is closed, otherwise a number of turns between the two adjacent taps would be short-circuited through the switch and the heavy current resulting from this short-circuiting of the e.m.f. induced in the turns would burn the contacts badly and damage the windings. If, on the other hand, the switch interrupts the full line current at each change, it will again be damaged by sparking as it is not economic to make each of the breaks capable of dealing with the full-load. A single circuit-breaker is accordingly provided to interrupt the circuit whilst tap-changing and this is frequently interlocked with the tapping switch, so that incorrect operation cannot be made. These precautions are not necessary for the smaller transformers up to about 1 kW at normal domestic distribution voltages, and tapping switches with breaks between each point can usually be safely operated without disconnecting the circuit elsewhere.

It might appear that if a tapping switch is always to be operated with the transformer circuit broken, the contacts could be of the "shorting" type, but this is inadvisable on account of the risk of the switch being left inadvertently in an intermediate position owing to a failure of its locating mechanism.

The most comprehensive system of tap-changing is that employed when it is required to carry out the operation *on-load*. It is not then admissible to interrupt the supply at a circuit-breaker to avoid short-circuiting during the transition, and other arrangements have to be made. These consist in limiting the short-circuit between tappings by interposing either a resistance or a reactance. If the latter is used it may be left continuously in circuit and used to obtain an intermediate voltage between the two taps across which it is connected. A resistance is, however, usually short-period rated and is only in circuit for a fraction of a second during the changeover.

The switch, like its off-load equivalent, may be of the rotary-arm type or may consist of a number of separate switches, suitably coupled, but in either case the method of introducing the limiting impedance is in effect the same. For ease in explanation the rotary type is chosen, but the method used with other types of switch will be readily understood. The principle is to divide the moving switch arm into two sections insulated from each other and of such a width relative to the contact studs that a given half of the arm can touch only one stud at a time; the gap between studs is thus slightly greater than the width of each half of the arm. The spacing of the two portions of the arm is such that one or other, and sometimes both, is in contact with a stud or studs. The two are connected externally to the impedance which bridges them and the external connexion is taken from one end of the resistance, or if a reactance is employed, a centre connexion can be used. Dealing first with the use of a resistance (Fig. 54),

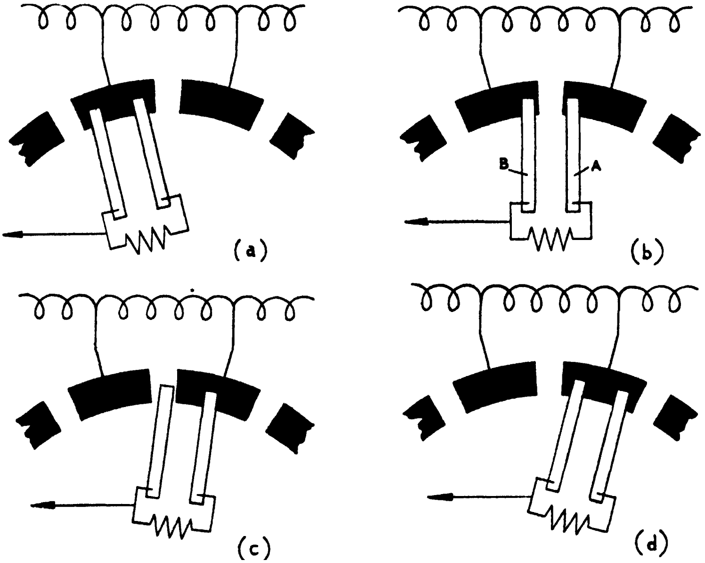


FIG. 54

Four Stages in the Operation of an On-load Tap-changing Switch. (*a*) Initial Tap. (*b*) Bridging Two Taps—Output Voltage Still that of Lower Tap. (*c*) Output at Higher Tap, but with Limiting Resistance in Series. (*d*) Resistance Short-circuited—transition completed.

the first movement of the switch is to connect one of the arms *A* to the next tapping, and this happens before the other arm *B* leaves the previous tap. The resistance is thus placed across the tappings and its value is so chosen that it limits the current to a sufficiently low value. The output voltage at this stage is still that of the first tap. Further movement of the switch (stage two) now breaks the connexion to the first tap and leaves the circuit completed to the second tap with the resistance in series. The third stage consists in bringing arm *A* on to the second stud at the same time as arm *B*, thus short-circuiting the resistance. The output voltage is now that of the second tap.

The centre-tapped reactance method is similar in action, except that the switch may be left in the stage two position continuously without introducing a high impedance into the circuit, because the load current flows from both tappings round the two halves of the reactance in opposite directions, so leaving the reactor core unmagnetized and the voltage drop very small. The output voltage at this stage is intermediate between the two taps and thus only half the number of transformer taps is required for a given number of voltage steps. A plain reactor (without a centre-tap) may be used in place of a resistance, but in that case, of course, no intermediate voltage can be obtained.

3. Bringing Out Tappings

Heavy strip-windings can be tapped without difficulty by attaching lead-out connexions at suitable points and bringing the bars vertically upwards to the connecting points. The windings in the locality of the tappings must be well supported to avoid making weak spots in the coils which would be likely to be damaged during fault conditions. The mechanical forces exerted on the conductors when a short-circuit occurs on the secondary are enormous and suitable supports must be provided.

In small transformers where round wires are used, the T-connexion method is not suitable, largely because of the difficulty of re-insulating the wires after attaching the lead-out. Windings in wires between about SWG 18 and SWG 30 are best dealt with by taking out the wire to the connecting point and bringing it back along the same path to start the next section. The actual joint is then made at the terminal board. The two wires forming the tapping lead can with advantage be twisted together to improve the mechanical strength and rigidity. The exact arrangement used for anchoring and insulating the lead-out wires depends on the type of former and general method of construction. For instance, the details of bringing out leads from a multiple-wound paper-interleaved coil have been described in Chapter V. Wires finer than about SWG 30 are too fragile for use as lead-outs and flexible connecting wires should be soldered to the winding proper, and a turn of flexible wire taken round the bobbin to prevent strains applied to the lead-outs being transmitted to the fine windings. Due allowance must, of course, be made for this turn when calculating the output.

4. Position of Tappings in a Winding

Although tappings are usually shown diagrammatically as being taken at the end of a winding, this is only done in practice in small transformers or those not subjected to surges produced by lightning or faults on distribution networks. These surges are dealt with in Chapter XI, but for the present purpose it is sufficient to note that the impact of a voltage surge on a transformer winding is felt most severely at the end turns, and the inter-turn insulation is specially strengthened at these points to withstand considerably higher voltages than those generated in the winding. If the transformer is likely to encounter surges of this kind (as any transformer connected to over head wires must), then it is essential for the safety of the winding that the specially strengthened end turns should always be kept in circuit. Tappings in a transformer of any size are therefore taken from the middle of the winding in a symmetrical fashion. Even if tappings are taken at the centre, it is usual to provide extra inter-turn insulation on the tapped turns because the abrupt changes in spacing and electrostatic capacity which occur at these points may cause high voltages to build up under transient conditions.

Another reason for taking tappings from the middle is to minimize the variations in reactance with tapping selection. A pair of coils with specified numbers of turns on each has a minimum reactance when the lengths of the two coils are equal, so that if the length of one of them is reduced by connecting to a low tapping at the end, the reactance will increase considerably. This will not be the case if the tappings are taken centrally.

CHAPTER X

The Core Excitation

IN the hypothetical conception of an *ideal transformer* two distinct assumptions were made regarding the performance of the core material, namely that the working value of flux was attained in the iron at an infinitesimal value of the magnetizing force and that there were no losses due to hysteresis or eddy currents in the material. In practice, of course, neither assumption is completely justified so that in certain respects an actual transformer behaves differently from an ideal one. These divergences from the ideal are often apparent only at light loads; consequently they can be neglected in many transformers operating at or near full-load all the time. Even if the no-load current cannot be neglected, it is often sufficient to assume that it is sinusoidal in form, and this assumption is, of course, implicit in any vector diagrams showing the exciting current as a normal vector. A treatment of the subject on this basis was made in Chapter III, when the method of calculating the r.m.s. value of the exciting current was given. If the transformer primary is regarded as a reactor, the assumption that the magnetizing current is sinusoidal really amounts to assuming the permeability of the core material to be constant.

There are, however, some applications in which it is necessary to take account not only of the existence of the exciting current but also of its divergence from the sinusoidal wave-form. These instances occur most frequently in communications work, where in the interests of good performance at high audio-frequencies, the flux-density at low frequencies is apt to be high. Also in communications work, the impedance of the source (usually a thermionic valve) is high and, as will be seen shortly, this has a serious effect on the amount of distortion. Even in power transformers, a departure of the output voltage from the sinusoidal form may be important; an example is in a star/star three-phase shell-type transformer. It is the purpose of this chapter to examine in detail the effects of the departures of the core material from the ideal. Since the appearance of a secondary current always brings into being a corresponding load component of primary current and these two largely neutralize one another's effect on the core, it is sufficient in this chapter to examine the no-load performance only. For this purpose it is thus only necessary to regard the transformer as having a primary winding; in other words, the examination of the behaviour of an iron-cored reactor is all that is required.

Under no-load conditions, an *ideal* transformer would pass no current; the current which flows in the primary of an *actual* transformer under no-load conditions is known as the *exciting current.* Since there are departures from the ideal in two distinct respects, the exciting current can be regarded as containing two separate components: one a watt-less one reflecting the fact that a finite value of magnetizing force is required to produce the desired flux in a material having a finite permeability, and the other an in-phase or loss component supplying the power to make up the losses in the core due to hysteresis and eddy currents. The core losses have been examined in some detail in Chapter IV. In transformers of normal design working at the usual power-supply frequencies, the loss component is usually much smaller than the magnetizing component, so that its effect on the magnitude and wave-form of the resultant exciting current is quite small. It will not be considered here in detail, but the extent of its effect on the exciting current wave-form will be noted later.

1. THE GENERAL PROBLEM

If a source of alternating e.m.f. is connected to a transformer primary having a resistance R_p, then the current i_p flowing at any instant is governed by the relationship

$$v_p = R_p i_p - e_p \text{ (equation 8)}$$

where v_p and e_p are the terminal potential difference and back e.m.f. respectively. Now the back e.m.f. is equal to the rate of change of linkages of the main flux with the primary winding so that

$$e_p = - T_p \frac{d\phi}{dt}$$

The value of ϕ at any instant depends on the value of i_p, and since ϕ is proportional to the flux-density B and i_p is proportional to the magnetizing force, the shape of the curve relating ϕ and i_p is similar to that of the B-H curve for the material of the core. (This statement takes no account of the effects of air gaps in the magnetic circuit and eddy currents in the core.) It will be remembered that the B-H curve takes the form of a loop, since the value of B is greater for a given value of H when the latter is decreasing than when it is increasing. This is, of course, the well-known magnetic hysteresis effect and the work done in taking the iron round the loop is the major cause of the loss component of the exciting current.

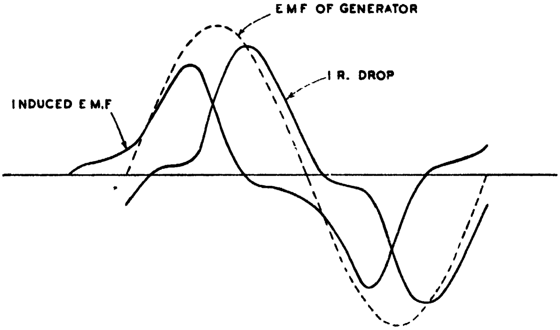


FIG. 55

Wave-forms of Induced Voltage and *IR* Drop in an Iron-cored Transformer Primary Having a High Winding Resistance.

Owing to the peculiar shape of the B-H curve and the fact that it varies with the maximum value of H, even graphical methods of plotting the exciting current wave-form are laborious, since they involve point-by-point or successive approximation methods. A typical result is shown in Fig. 55. This also shows the back e.m.f. wave e_p induced in the primary by the flux wave. Since the latter is non-sinusoidal, e_p is, of course, also non-sinusoidal. The i_pR_p drop is of the same wave-form as i_p and since equation 8 must be satisfied, the sum of the two voltages must be sinusoidal and equal to the applied voltage wave.

In those cases, fortunately rather rare in power transformers, in which both the *IR* drop and the induced back e.m.f. are of the same order, neither is even approximately sinusoidal, as

Fig. 55 shows. If, however, one or the other is sufficiently small to be neglected, then either the exciting current or the flux (and hence the induced voltage) are sinusoidal. These particular cases are now examined.

2. Primary-circuit Resistance Assumed Negligible

Taking first the case in which all the resistances (including that of the supply) are assumed to be negligible, it will be seen that the induced back e.m.f. must be sinusoidal in form, since it is at all instants equal and opposite to the applied e.m.f. The truth of this statement follows from equation 8 when $R_p = 0$. The induced e.m.f. is numerically equal to the rate of change of linkages of the flux with the circuit so that the flux wave is also sinusoidal since the integral of a sine wave is itself a sine wave displaced 90°. If the flux-density in the iron is high, the exciting current wave will, however, be far from sinusoidal. At normal power frequencies where the eddy-current effects in laminations of the usual thickness are small, the a.c. magnetization characteristic is very similar to the *B-H* curve obtained by static methods. The magnetizing current can then be determined graphically from the *B-H* loop for the core material taken under the particular peak-flux conditions in the transformer being examined.

Fig. 56 shows the method used. On the left is a curve of B against i_m. Since H is proportional to i_m, this loop is of the same form as a *B-H* loop and can be drawn from it by a suitable alteration to the horizontal scale. The particular loop chosen is one which reaches to the correct peak value of B, and on the right is drawn the time variation of B to the same vertical scale and to any suitable time scale. As noted above, the curve B against time is sinusoidal. Ordinates are erected at a number of points, a, b, c, etc., on the right-hand curve, and where these ordinates meet the flux curve, horizontal lines are drawn across to the magnetization curve. Since there are two possible values (apart from the initial central line of the loop) care must be taken to read off from the correct side. At a, for instance, the flux, and hence the current, are positive and increasing. The right-hand (ascending) side of the magnetization loop is, therefore, chosen and the corresponding value of current read off. This value is measured up the ordinate which was first raised at *a* on the right-hand curve and a point on the current time curve has thus been plotted. Similarly, values of the current at *b* and *c* are also obtained. On the descending side of the flux loop another ordinate has been raised (not lettered) and the horizontal construction line has, of course, been continued to the descending or left-hand side of the magnetization loop. The corresponding current is seen to be negative in this case, so when transferring back to the right-hand curve, the value of the current is plotted below the axis. Further points are shown on the increasing negative-flux curve and finally a point on the decreasing negative portion of the flux curve, shown actually at the commencement of the curve as only one cycle has been drawn.

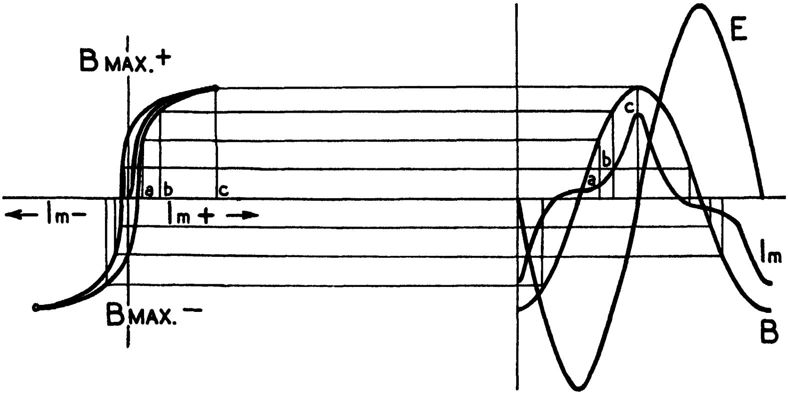


FIG. 56

The Construction for Determining the Shape of the Magnetizing Current Wave I_m for a Sinusoidal Flux Variation.

From the current wave-form thus derived, it will be seen that there is marked distortion from the sinusoidal shape; the wave-form is characterized as "peaky." It is symmetrical about the time axis, the negative half-cycles being inverted counterparts of the positive ones. In the language of harmonic analysis, the wave contains only odd harmonics (see p. 197). A point of some interest is the slight departure from symmetry about the peak-current point, i.e. the second quarter-cycle is not a left-to-right reversal of the first quarter-cycle. This follows from the fact that the current required to produce a given value of B is less when decreasing than when increasing, as can be seen in the hysteresis loop. The effect of this hysteresis on the current wave-form is that the points of zero current occur earlier in time than they would have done had the magnetization curve been a single-valued one. In other words, the current cycle is not a full quarter-cycle behind the applied voltage. There is accordingly an in-phase component of the current and this appears as a net loss of power in the circuit. Had the current been an exact quarter-cycle behind the voltage, the power stored in the magnetic system during one-half of the cycle would have been completely restored to the supply during the other half. As it is, there is more power absorbed than is subsequently returned and this is the mechanism by which the hysteresis loss is supplied from the power source.

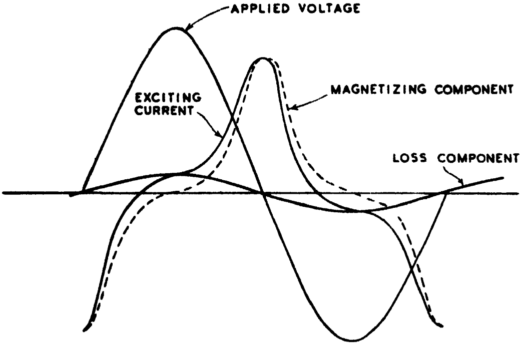


FIG. 57

Derivation of the Exciting Current Wave-form from Those of Magnetizing Current and Loss Component Current.

The loss component of the exciting current has already been mentioned earlier, where it was stated that for many purposes it could be neglected. To do so would, of course, mean that the current wave would have to become symmetrical about the peak value, and then it would assume the proper 90° phase-lag on the applied voltage.

The loss component is the component of the exciting current which is in phase with the applied voltage. To carry power, the current must be of the same frequency as the applied voltage· and as that is sinusoidal, i.e. of the fundamental frequency, the loss component current is also sinusoidal. (Harmonics of current generated in a circuit by a non-linear impedance cannot carry power—p. 201.) The loss component and the true magnetizing current components are shown in Fig. 57. It will be seen that together they make up the exciting current wave derived above. The loss component is sinusoidal and in phase, the magnetizing component is 90° out of phase and is peaky due to odd harmonics.

The losses referred to in the previous paragraph are hysteresis losses only, since eddy-current losses were specifically excluded from the discussion. Eddy-current losses can be allowed for, however, by modification of the shape of the flux-exciting current loop. The loop instead of being a *static* magnetization curve then becomes a dynamic or a.c. magnetization curve. As might be expected, the additional loss has the effect of further widening the loop, since the area within the loop represents the work done in one magnetization cycle. The way in which the widening of the static loop occurs is as follows—

Eddy currents are the circulating currents round the individual laminations of the core and they are assumed to be in phase with the e.m.f. which produces them. This e.m.f. is induced in the current path by the main flux and is accordingly proportional to the rate of change of flux with time and in a negative direction when the flux is increasing. This can be seen directly from Lenz's Law. The effect of the eddy-currents on the magnetization loop is thus to subtract from the flux when it is increasing and to add to it when decreasing by an amount which is proportional to the rate of change of flux. The loop thus becomes broader and the area within it is proportional to the total core loss per cycle. The assumption that the eddy currents are in phase with the e.m.f.s which produce them, is substantially valid at power frequencies, but becomes increasingly in error as the frequency increases. This in effect means that the inductive reactance of the eddy-current paths becomes appreciable compared with their resistance. When this occurs the phenomenon of the screening of the interior of the iron by the circulating currents near the surface becomes important; this is the well-known "skin-effect" of high-frequency work.

3. PRIMARY-CIRCUIT RESISTANCE ASSUMED VERY HIGH

The condition assumed in this section is that in which the resistance of the source is so high that the back e.m.f. is small compared with the resistance drop. The primary current is then determined by the resistance and is accordingly sinusoidal. A simple graphical method corresponding to that adopted in Section 2 enables the flux, and hence the induced voltage wave-forms, to be derived. The method is shown in Fig. 58. The flux-current loop is first drawn on the left and the sinusoidal current wave on the right. The vertical scale of the current wave is such that the peak value is equal to the *horizontal* distance representing the peak current on the flux-current loop. Ordinates are raised to the current curve at suitable intervals, e.g. *a*, *b*, *c*, etc., and their heights to the current curve stepped off horizontally along the current axis of the left-hand diagram. At these points corresponding to *a*, *b*, *c*, more ordinates are erected and the values of *B* read from the appropriate portion of the magnetization loop. Horizontal construction lines enable these values to be transferred to the right-hand side, where the intersections with the original ordinates indicate the shape of the flux wave. This is seen to be "flattened" in shape. The induced wave is obtained from the flux by differentiating, since $e_p = -T_p \frac{d\phi}{dt} = -T_p a \frac{dB}{dt}$. In this case the induced voltage wave differs radically from the flux wave and is "peaky." (The particular case in Section 2 of a sine wave

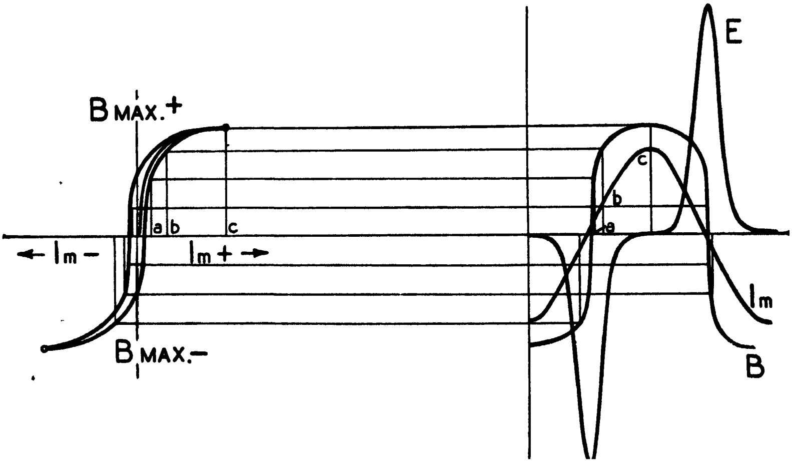


FIG. 58
Graphical Method of Deriving the Induced Voltage Wave-form for a Sinusoidal Exciting Current.

and its differential coefficient being of the same shape should not be allowed to give the mistaken idea that the flux and voltage wave-forms are always similar.)

The effects of hysteresis are as before evident as a phase displacement between the current and the flux. The flux lags slightly on the current so that the induced voltage (equal to the terminal voltage in the case of negligible primary resistance) is less than 90° in phase angle from the current.

4. FOURIER ANALYSIS

The magnetizing current, which has formed the subject of this chapter, is the only example of a non-sinusoidal wave-form introduced by the working of the transformer itself; other examples of non-sinusoidal currents do occur, however, as the result of the operation of non-linear elements external to the transformer, and it is accordingly necessary to examine the properties of non-sinusoidal wave-forms. For many purposes it is convenient to analyse these wave shapes into a series of sinusoidal components or *harmonics*. This process is known as a Fourier analysis.

Any periodically recurring wave-form, provided that it is single valued, can be analysed into a constant term together with an infinite series of sine and cosine waves, the frequencies of which are integral multiples of the basic or *fundamental* frequency. These components are known as "harmonics" of the fundamental, and are called second, third, fifth harmonics if their frequencies are twice, three times, five times that of the fundamental. In many examples of complex wave-forms, some of the component frequencies are absent and in addition the series is usually rapidly convergent so that only a small number of components is required to represent the function to a sufficiently high degree of accuracy for all practical purposes. For example, the magnetizing current wave, such as that shown in Fig. 56, has no constant term, and only odd harmonic frequencies; both sine and cosine terms, however, are present.

Using mathematical symbols, Fourier's analysis is expressed as—

$$f(t) = \frac{a_0}{2} + a_1 \cos \omega t + a_2 \cos 2\omega t \ \ldots$$
$$+ b_1 \sin \omega t + b_2 \sin 2\omega t \ \ldots \quad . \qquad (51)$$

where $f(t)$ is a periodic function of time and a_0, a_1, b_1, etc., are constant coefficients having values equal to *twice* the average value of the component with which they are associated. For example—

$$a_2 = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} f(t)\cos 2\omega t\,dt \quad . \quad . \quad (52)$$

$$a_0 = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} f(t)\,dt \quad . \quad . \quad . \quad . \quad (53)$$

Since $\sin \omega t = \cos\left(\omega t - \frac{\pi}{2}\right)$, it is possible to combine the sine and cosine terms so that

$$f(t) = \frac{a_0}{2} + C_1 \cos(\omega t + \varepsilon_1) + C_2 \cos(2\omega t + \varepsilon_2) \qquad (54)$$

This form of the equation is convenient in many ways, but it should be noted that the phase of the harmonics is no longer simply related to that of the fundamental as was the case when the sine and cosine components were kept separate.

Various methods have been developed for the determination of the coefficients of the various harmonic terms representing a given wave shape, and textbooks on practical mathematics should be consulted for details. In certain special cases, however, it is possible to state by inspection what terms are present in the series and sometimes even to estimate their relative magnitudes. Fig. 59 shows some of the recognizable types.

(*a*) If the area of the positive part of the cycle is equal to that of the negative part, there will be no constant term, since the mean value over a complete cycle is zero.

(*b*) If the wave is symmetrical about the zero line, i.e. the shape of the negative half-cycle is the same as that of the positive, or $f(t) = -f\left(\frac{\pi}{\omega} + t\right)$, then not only will the constant term be zero, but there will also be no *even*-order harmonics. This is the state of affairs in the wave-forms produced in a.c. magnetization of iron cores, and there are then only odd-order harmonics present, principally third, fifth, and seventh. Of these harmonics, the third predominates.

(*c*) If the wave is symmetrical about an ordinate raised at the commencement of a cycle or at a half-cycle point, so that

$f(t) = f(-t)$, then only cosine terms are present. In Fig. 59*c*, for instance, the value at any instant θ is equal to that at $-\theta$. In the case of the sine components, however, $\sin(-\theta) = -\sin\theta$, so they cannot appear in this particular function. The cosine components are possible because $\cos(-\theta) = \cos\theta$.

(*d*) If the shape is such that $f(t) = -f(-t)$, then only sine components are present. The definition of this wave-form necessarily includes case (*a*), so there is no constant term either.

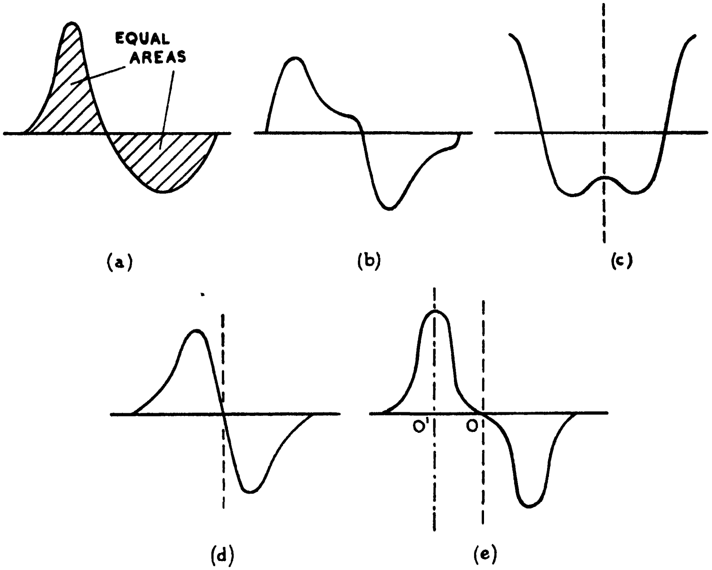


FIG. 59

Non-sinusoidal Wave-forms Having Easily Recognized Characteristics. (*a*) No Constant Term. (*b*) No Even Order Harmonics. (*c*) No Sine Terms. (*d*) No Cosine Terms. (*e*) Sine Terms Only of Odd Order Harmonics.

It is, of course, possible for a wave-form to satisfy more than one of the particular conditions described, so several groups of components are eliminated. For example, wave-form (*e*) in Fig. 59*e* meets the conditions $f(t) = -f\left(\frac{\pi}{\omega} + t\right)$ and $f(t) = -f(-t)$ given in (*b*) and (*d*) respectively, so that only odd-order sine components are present. The constant term, cosine terms, and even-order sine terms are all absent. Some wave-forms (of which Fig. 59*e* is an example) can be made to satisfy either condition (*c*) or condition (*d*) depending upon the position of the origin. If situated at 0′, then $f(t) = f(-t)$ and only cosine terms are present, whereas if the origin is at 0, then $f(t) = -f(-t)$ and only sines appear. In steady state conditions, the origin can be chosen at will, so that whichever is the more convenient can be used. It will be realized, of course, that changing a sine to a cosine term really amounts only to a change of phase of 90°.

In a general way it may be said that a wave having sharp points, e.g. a triangular or rectangular wave-form, has high harmonics of considerable amplitude, whereas a smooth curve can be represented sufficiently well by low-order harmonics only.

Power in Non-linear Circuits

The instantaneous power in any circuit is given in watts by the product of the voltage and current at the instant in question. If either or both are periodic functions, the expression for the power is also periodic. Usually, however, it is the average power rather than the instantaneous value which is of interest. The average is calculated over an integral number of cycles. If both voltage and current are sinusoidal, as is the case when a sinusoidal voltage is applied to a constant impedance, the instantaneous power p is given by

$$\begin{aligned} p &= V_{max} \sin \omega t I_{max} \sin (\omega t + \psi) \\ &= \frac{V_{max} I_{max}}{2} [\cos \psi - \cos (2\omega t + \psi)] \\ &= \frac{V_{max} I_{max}}{2} \cos \psi - \frac{V_{max} I_{max}}{2} \cos (2\omega t + \psi) \quad . \qquad (55) \end{aligned}$$

i.e. a constant term and a periodic term having a frequency twice that of the supply.

The average power of an integral number of cycles is equal to the constant term, i.e.

$$P_{mean} = \frac{V_{max} I_{max}}{2} \cos \psi = VI \cos \psi \quad . \qquad . \quad (56)$$

This is, of course, the well-known expression for the power in a linear a.c. circuit.

If the circuit is not linear, that is to say, if the impedance varies over the cycle, then although the applied voltage is sinusoidal, the current which flows is not. Let the current be represented by a Fourier series of harmonics thus—

$$\begin{aligned} &I_0 + I_{1a_{max}} \sin \omega t + I_{2a_{max}} \sin 2\omega t \; . \; . \; . \\ &\quad + I_{1b_{max}} \cos \omega t + I_{2b_{max}} \cos 2\omega t \; . \; . \; . \end{aligned}$$

where I_0 is the direct current component (if any). Hence

$$\begin{aligned} p &= V_{max} \sin \omega t \, [I_0 + I_{1a_{max}} \sin \omega t + I_{2a_{max}} \sin 2\omega t \; . \; . \; . \\ &\qquad + I_{1b_{max}} \cos \omega t + I_{2b_{max}} \cos 2\omega t \; . \; . \; .] \\ &= V_{max} \, [I_0 \sin \omega t + I_{1a_{max}} \sin^2 \omega t + I_{2a_{max}} \sin \omega t \sin 2\omega t \; . \; . \; . \\ &\qquad + I_{1b_{max}} \sin \omega t \cos \omega t + I_{2b_{max}} \sin \omega t \cos 2\omega t \; . \; . \; .] \\ &= V_{max} \, [I_0 \sin \omega t + \frac{I_{1a_{max}}}{2}(1 - \cos 2\omega t) + I_{2a_{max}}(\cos \omega t - \cos 3\omega t) \\ &\qquad + \frac{I_{1b_{max}}}{2} \sin 2\omega t + \frac{I_{2b_{max}}}{2} (\sin 3\omega t - \sin \omega t) \; . \; . \; .] \end{aligned}$$

All these terms are periodic ones, with the exception of $\frac{1}{2} V_{max} I_{1a_{max}}$, so the average power taken over an integral number of cycles will be given by this term. Put into words, this result means that the average power due to a voltage of one frequency and a current at a harmonic of this frequency is zero, if taken over a whole number of cycles. The only currents and voltages which act together to produce power are those of the *same* frequency. In the particular case in which they are in quadrature, even they do not carry power. (The fact that a voltage of one frequency and a current at a harmonic frequency do not produce power, can be extended to read: "A voltage and a current at *any* differing frequencies produce no average power over a period representing an integral number of cycles of the difference frequency.")

Applying this statement to the power in an unloaded transformer connected on the primary side to a supply, it will be seen that the power taken is that represented by the product of applied voltage and the in-phase (or loss) component I_{1a} of the exciting current. (To obtain the power in watts, the peak values used above must, of course, be reduced to virtual (i.e. r.m.s.) values.) None of the other current components reacts with the applied voltage to produce power because it is either of a different frequency or in the case of I_{1b}, in quadrature. There is no constant term I_0 in this particular example, since the circuit contains no source of d.c. voltage nor a rectifier.

CHAPTER XI

Transients

AN electrical transient or surge is the abnormal state of voltage or current which occurs in any piece of apparatus when a change is made in the applied voltage or any of the other circuit conditions. As the name indicates, a transient has only a temporary existence. Before the change is made in the conditions, the values of voltage and current are assumed to be stable; when a sufficiently long interval of time has elapsed after the change, stable conditions will again be reached. The state during the interval is known as a *transient.*

Transients of finite duration can occur only in circuits that include an energy-storing element, such as an inductance or capacitance as well as an energy-dissipating element, i.e. a resistance. A network consisting of pure resistances only must respond instantly to any change in the applied voltage because no energy can be stored anywhere, and consequently final conditions are attained without any delay. But when the circuit contains elements capable of storing energy it requires time to complete the change in energy distribution. This fact is true not only of electrical circuits but of mechanical systems too. When a motor car is accelerated from rest it does not reach full-speed immediately; a finite period of time elapses and eventually a steady speed is attained. The condition which links the steady state of rest with the steady speed finally reached is a transient one corresponding to that which marks the growth of an electric current during the period which follows the closing of a switch connecting a battery to a circuit consisting of a resistance and inductance in series. The duration of the transient condition is theoretically infinite, for it requires an infinite time to reach the final steady current value. In practice it is assumed that the transient is over when its existence can no longer be detected, for the transient state merges gradually into the final state without any sharp distinction.

It is convenient in solving transient problems to consider that the final steady-state current begins to flow immediately the change takes place in the circuit conditions, and that superimposed on the steady-state current is a transient component. The value of the transient component continuously decreases and reaches zero after a (theoretically) infinite time. The form of the current which actually flows in the period following the application of the changed conditions is the sum of the steady-state component and the transient component. If the circuit includes either inductance *or* capacitance (but not both) it can give rise to what are sometimes known as "single-energy transients." These are always unsymmetrical about the zero line, and in fact the transient term itself cannot change sign, i.e. cross the zero line, although the resultant voltage (or current) resulting from the addition of the transient term to the final steady·state term will, in an a.c. circuit, be of an oscillatory form. The frequency will, however, in all cases be the same as that of the supply.

If the circuit contains inductance, capacitance, and resistance, energy can be stored in either the inductance or the capacitance elements, and it will tend to oscillate backwards and forwards between the two at a frequency determined simply by their values. The transients which result from changes in the conditions of such circuits are called "double-energy transients." The transient terms are in general oscillatory and in transformer work their frequency is usually much higher than that of the supply, but this is not necessarily so; double-energy transients may be unidirectional if the value of the resistance exceeds a certain critical value determined by the inductance and capacitance values.

Whilst both voltage and current transients can appear simultaneously in double-energy circuits, only one of them can have a transient form in a single-energy circuit. As will be seen later, for example when a transformer primary is switched into circuit, there is no voltage rise but only a current surge.

In transformer work, transients have to be considered from two rather different points of view. First of all, the combination of inductance and resistance in the transformer windings gives rise to current or voltage transients whenever the primary voltage is switched on or off, and these transients are liable to cause disturbances in the circuits and protective gear associated with the transformer. Secondly, voltage transients due to switching operations external to the transformer or to lightning discharges are liable to be applied to the transformer windings and damage may be caused unless the windings are designed to withstand them. The designer has to be aware of the nature of transient disturbances in order to be sure that the precautions he takes are adequate.

1. TRANSIENTS CAUSED BY TRANSFORMER SWITCHING

(a) *Current Surges when Switching On*

When a transformer primary is connected to the supply, a rather unexpected rush of current sometimes takes place. This surge is all the more puzzling to those who are not aware of its cause, because it does not occur every time the transformer is connected. The reason for this variability is the fact that the extent of the surge depends on the exact instant at which the circuit is made, and also on the state of magnetization of the iron, which is largely determined by the exact moment at which the transformer had last been switched off.

It is the possibility of this surge occurring which makes the protection of transformers by fuses uncertain, because a fuse wire has to be chosen which will not blow at the moment of switching on when a large current flows for a cycle or so, but does no harm: yet the fuse must go when a relatively small overload occurs. Special fuses having an appreciable time-lag are often used for this purpose.

When explaining the cause of this surge, it is not necessary to bring the secondary into the picture—the primary alone is involved, and no abnormal voltages are produced on the output side. A transient of this nature occurs when any inductive circuit is connected to a source of alternating current, but the effect is accentuated if the inductance has an iron core since the iron tends to saturate. To simplify matters at first, however, the case of an air-cored inductance will be examined.

Consider a circuit consisting of an inductance and a resistance in series to which an a.c. voltage is suddenly applied. Once a steady state has been reached, the current flowing will be of a certain value determined by the magnitude of the applied voltage and the circuit impedance. This current will be of the same value whatever the instant at which the circuit was completed, provided that a certain minimum time has elapsed. Let us follow the principle of imagining that this steady-state current immediately assumed its final value at the moment the voltage was applied, and that a transient current also commenced at the same moment of such a value that the resultant current satisfies the conditions of the circuit. Since the resultant current at the moment when the switch is closed ($t = 0$) is evidently zero, the value of the transient component at that moment is obviously equal and opposite to the instantaneous value of the steady-state component. If the circuit had been closed at some other instant, the instantaneous value of the steady-state current would have been different, and the initial value of the transient component would, therefore, also have been different. The magnitude of the transient current thus clearly varies with the timing of the switching operation. There will be one particular instant at which the voltage could be applied without causing any transient disturbance; this is the moment when the current would have been passing through zero if the circuit had already been completed some time before and steady conditions had been reached. At that instant the transient component (equal and opposite to the steady-state component) must also be zero, and as the transient continually decreases with time, it must in this case, of course, remain at zero. This result is also reached from energy considerations. At the moment when the current is zero, no energy is stored in the inductance (or indeed anywhere in the circuit) and no finite period is required to reach the final state because the distribution of energy at the instant of switching on is exactly that which it would have assumed at that same instant if the steady-state current had already been established.

This case is analysed very clearly by H. H. Skilling in *Transient Electric Currents.*(1) The treatment given here is taken, in a somewhat abbreviated form, from that publication.

Let the voltage wave be expressed by the equation

$$v = V_{max} \sin (\omega t + \alpha) \quad . \quad . \quad . \quad (57)$$

If the switch is closed at the instant $t = 0$, then the instantaneous voltage will be $V_{max} \sin \alpha$. The dotted curve in Fig. 60

shows the steady-state current which would have been flowing had the circuit been completed a sufficiently long time before. The value of this current is given by

$$i = \frac{V_{max} \sin (\omega t + \alpha - \psi)}{\sqrt{R^2 + \omega^2 L^2}} \quad . \quad . \quad (58)$$

where ψ the angle of lag of the current on the applied voltage is given by $\psi = \tan^{-1} \frac{\omega L}{R}$.

At every instant the applied voltage must equal the sum of the resistance drop and back e.m.f. induced in the inductance, i.e.

$$V_{max} \sin (\omega t + \alpha) = Ri + L\frac{di}{dt} \quad . \quad . \quad (59)$$

Now in the circuit being considered, the actual current must be zero at $t = 0$, hence $i = 0$. Reference to equation 58, however, shows that at $t = 0$ the steady-state current is not zero but is

$$i_0 = \frac{V_{max} \sin (\alpha - \psi)}{\sqrt{R^2 + \omega^2 L^2}} \quad . \quad . \quad (60)$$

In order to satisfy the condition that $i = 0$ at $t = 0$ there must accordingly also be a transient component of magnitude—

$$i_0' = -\frac{V_{max} \sin (\alpha - \psi)}{\sqrt{R^2 + \omega^2 L^2}} \quad . \quad . \quad (61)$$

But the transient current which flows does so without the application of any externally applied e.m.f. This will be evident from the fact that the transient current ceases in time without any change in the overall voltage conditions. We can, therefore, write for the transient current

$$Ri' + L\frac{di'}{dt} = 0 \quad . \quad . \quad . \quad (62)$$

Integrating $$\int \frac{1}{i'} di' = \int -\frac{R}{L} dt$$

i.e. $$\log i' = -\frac{R}{L} t + K$$

where K is the constant of integration.

$$\therefore \qquad i' = A\varepsilon^{-\frac{R}{L}t} \quad . \quad . \quad . \quad . \quad (63)$$

(The constant A now incorporates K.)

The form of the transient component is thus seen to be exponential and the value of A can easily be derived since the value of i' at the time $t = 0$ is already known (equation 61). From equation 63, A is seen to be the particular value of i' when $t = 0$, hence equation 63 becomes

$$i' = -\frac{V_{max}\sin(\alpha - \psi)}{\sqrt{R^2 + \omega^2 L^2}}\,\varepsilon^{-\frac{R}{L}t} \quad . \quad . \quad (64)$$

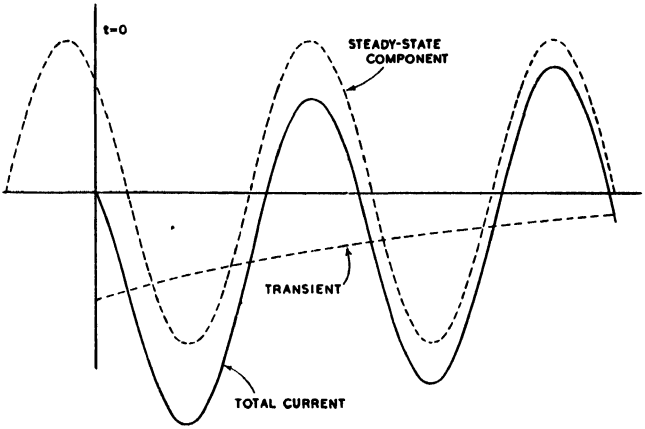


FIG. 60

A Switching-on Current Surge in an Air-cored Inductance.

The total primary current is given by adding the steady-state and transient components so that

$$i_p = \frac{V_{max}}{Z}\left[\sin(\omega t + \alpha - \psi) - \sin(\alpha - \psi)\,\varepsilon^{-\frac{R}{L}t}\right] \quad (65)$$

The transient and total current are indicated in Fig. 60 by dotted and continuous lines. It will be seen that when t becomes large, the second term tends to zero and the expression becomes the normal steady-state value.

As already shown on purely physical reasoning, the magnitude of the transient component varies according to the moment at which the switch is closed. If $\psi = \alpha$ the transient does not occur, whilst it reaches the highest value when $\alpha - \psi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$. The first peak of current following the closing of the switch at a moment when $\alpha - \psi = \frac{\pi}{2}$ approaches twice the height of the normal peaks, because if $\frac{R}{L}$ is reasonably small the transient component will not have fallen appreciably during the first quarter-cycle.

The expressions derived above assume that the inductance of the coil is constant, as would be the case if the magnetic circuit contains no iron. They have been derived in some detail in order to show clearly the nature and mechanism of transient production, but in practice the peak currents reached exceed those calculated in that way by a considerable factor because the effect of saturation of the iron is quite marked.(2) In addition there is the fact that the iron may already be magnetized before the voltage is applied, owing to the residual magnetization from previous operation.

When considering the effect of the iron core of a transformer on the value of the transient component it is convenient to recall the existence of the magnetic flux in the core. Although the e.m.f. induced in the coil is proportional to the rate of change of flux through it, the flux is no longer proportional to the current flowing. When this proportionality existed (as in the air-cored case), it was not necessary to recognize the intermediate step of flux production, as the induced e.m.f. was then directly proportional to the rate of change of current. Now, however, it is helpful to consider the part played by the flux. As the value of H (which is proportional to current) increases, B increases more and more slowly and tends to a limiting value. The shape of the hysteresis curve for a typical core material was shown in Chapter II, and is reproduced again in Fig. 61. A curve of this type is obtained experimentally for the particular material which is being used. The appropriate portion of the curve must, of course, be used according to whether the current is increasing or decreasing and whether the iron is initially demagnetized.

From the standpoint of flux considerations, the primary current can be regarded as adjusting itself to be of such a value that it produces just the right flux at any instant to induce an e.m.f. equal and opposite to the applied e.m.f. (neglecting the resistance drop in the primary). During normal operation the induced e.m.f., being proportional to the *rate of change* of flux, lags 90° behind the flux. In the half-cycle between an instant when the applied voltage is zero (going positive) and the instant at which it is again zero (going negative), the flux changes from its maximum negative value $-B_{max}$ to its maximum positive value $+B_{max}$, so inducing an e.m.f. equal and opposite to that applied. If, however, the circuit is closed at a moment when the voltage is zero and about to go positive, the flux, instead of being $-B_{max}$ as it would be in steady-state conditions, is zero, and during the next half-cycle instead of changing to $+B_{max}$ it has to change from 0 to $+2B_{max}$. From the flattening of the *B-H* curve it is evident that in order to produce this flux the current will have to be considerably greater than twice normal, as would have been the case if the curve had continued upwards in a straight line (Fig. 62).

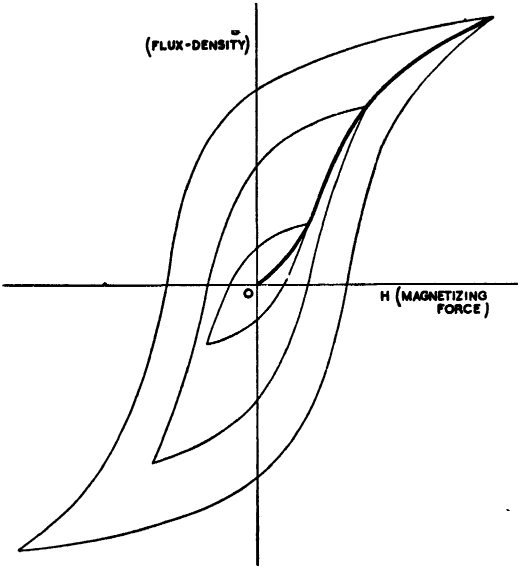


FIG. 61

A Disproportionately Large Magnetizing Force is Required to Reach High Values of B in a Ferro-magnetic Material.

When it is remembered that the iron may not be simply unmagnetized, but may be already magnetized to $+B_{max}$ and so required to increase its magnetization to $+3B_{max}$, it will be understood that the corresponding current value is very large indeed. Actually at the values of B normally employed in power frequency transformers it would not be possible to produce a flux of $2B_{max}$ and certainly not $3B_{max}$, because the current increases to such a value that the resistance drop in the winding (hitherto neglected) becomes sufficiently great to balance the applied volts.

It is quite possible for the residual magnetization in the iron to be of the same polarity as that which it would have had if the steady-state current had been flowing, although it is unlikely that the magnitude of the flux would be as large as that reached in operation. In that case, of course, the current rush which results when the circuit is closed at a moment of zero voltage is much lower than would have occurred if the iron had not been magnetized. On the other hand, the existence of a residual flux in the core will cause a surge of current if the circuit is closed at a moment when no current rush would have taken place with a neutral core.

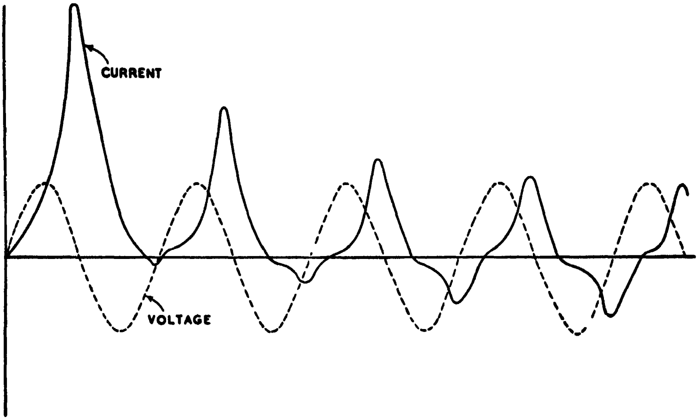


FIG. 62
The Exciting Current Wave of an Iron-cored Transformer Switched-in at an Instant of Zero Voltage.

Serious current surges of the type described do not usually occur in transformers working at the higher frequencies, such as 500 cycles, because the flux-density has had to be kept low to avoid excessive iron losses. If peak flux-densities of twice or even three times normal are called for during transient conditions, they can still be reached without drawing currents greatly exceeding two or three times normal, because the normal working point is low on the curve. It is only when the normal flux is already well on to the knee of the magnetization curve that heavy surges are experienced. For this reason, the automatic stabilizing transformers depending on magnetic saturation for their operation are particularly likely to give rise to heavy surges when switching on.

The most serious effect of these current surges in large transformers is the very large mechanical forces which are exerted on the conductors. Constant repetition tends to displace the insulation and in time causes a breakdown. In small sizes the surges are most troublesome in their effect on the protective gear, which is likely to operate as though a fault had occurred unless fitted with a time-delay device. It is then unable to deal quickly with real fault conditions. When current surges cause difficulties, they can best be controlled by the use of a special switch which first closes the circuit through a resistance and then short-circuits the resistance a moment later when normal conditions have been reached.(3)

It is important to remember that no untoward voltage effects accompany the current surge. The secondary voltage never exceeds its normal value; in fact, it tends not to reach the normal value immediately owing to the voltage drop which is likely to occur in the primary resistance and that of the supply.

(*b*) *Voltage Surges when Switching Off*

When a transformer winding is *disconnected* from the supply, a voltage surge is liable to be induced in the windings which may cause damage either to the transformer or to associated equipment. The windings of a transformer are, of course, highly inductive, and if the circuit happens to be broken at an instant when the current is not zero, the flux collapses rapidly and induces a correspondingly high voltage in the windings. The current cannot actually be interrupted instantaneously, but the small arcs which form at the switch contacts are usually quickly extinguished, so that the rate of change of current is much higher than that occurring during the normal periodic variation under a.c. conditions. The rate of flux change is proportionately high, and so voltage rises occur that are liable to cause flashing over at the terminals or between the turns of the windings.

This transient is again a "single-energy" one and consequently there is no current surge accompanying the voltage rise.

The voltage rise is best dealt with by arranging, as far as possible, that the normal load is left connected to the secondary when the primary is switched out. This amounts to switching the transformer and load together as a unit. When this cannot be arranged, it may be possible to connect an auxiliary load immediately before breaking the primary circuit. The effect of a load is to allow magnetizing current to continue to flow round one or other of the windings after the switch has broken, so reducing the rate of change of flux and limiting the voltage rise. The lower the resistance, the more effective is the limiting effect.

This particular transient effect is not usually considered by power engineers to be sufficiently frequent to cause much trouble, because most switching of currents of any size is carried out by oil-immersed circuit-breakers. These switches tend to break when the current is passing through its zero value, at which point the oil extinguishes the arc and the contacts meanwhile separate sufficiently far to prevent the arc from re-striking. Under no-load conditions (when the voltage rise is most serious) the primary current consists of the exciting current only, and this is practically in phase with the flux, so that interruption at zero current avoids any appreciable voltage rise. If the transformer is supplying a resistive load, the primary current contains a load component as well as the magnetizing component and so the resultant current is no longer in phase with the flux. Some voltage rise will then occur if the break takes place at a current zero, but it is unlikely to be serious because the surge is limited by the load, as already explained. Oil-break switches are, however, not often used for small transformers, and air-break types allow quite a considerable amount of arcing. The voltage rise takes place in the primary as well as the secondary to an extent proportionate to the normal voltage ratio, so that the arc tends to persist at the switch contacts, damaging them severely and also causing disturbances in the circuit when it does eventually break. These troubles are particularly evident in the small quick-break switches commonly used for low-power applications. They are usually of small physical dimensions and the quick-break action is necessary if they are to deal with even a non-inductive load of their full rated capacity on a d.c. circuit. The ability to control D.C. as well as A.C. is a result of standardizing the same switch for both purposes. Actually the quick-break is a disadvantage on a.c. circuits of an inductive nature, and a slow-break type, particularly if fitted with heavy contacts, is the best to employ for frequent operation in a transformer primary circuit.

If the flux-density in the transformer core is not high, so that the magnetizing current is reasonably sinusoidal, it may be possible to fit a condenser across one of the windings of such a value that the no-load power-factor of the transformer approaches unity. The combination then behaves like a resistance and no voltage rises take place. At the flux-densities usually employed in power-frequency transformers, however, it is not usually possible to find a condenser value which will bring about much improvement.

When the voltage rise cannot be prevented, its effect can be minimized by the use of a surge-limiting device either of the gaseous discharge type or of the "Thyrite" or "Metrosil" type. The former pass no current until a certain voltage (chosen to be above the normal operating figure) is exceeded and then they offer a very low resistance. "Thyrite" is a material which behaves in a similar manner, although the transition from the non-conducting to the conducting state is a less sudden one, so that some power loss has to be tolerated at normal working voltage.

2. SHORT-CIRCUIT CURRENT SURGES

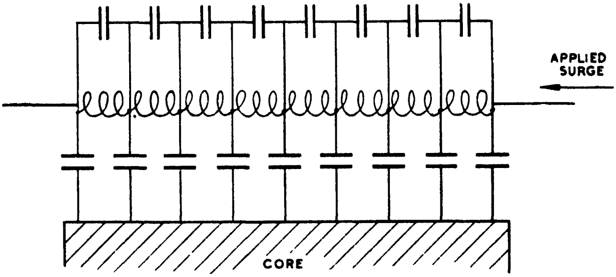
It is to be expected that if a transformer secondary is short-circuited whilst normal primary volts are maintained, a current greatly in excess of normal full-load current will flow in both windings. The magnitude of the current is determined by the resistance and reactance of the transformer and the latter is often made higher than necessary in order to limit the short-circuit current. It is not, however, the purpose of this section to consider the steady-state condition which would eventually be attained in these circumstances if the protective gear does not operate, but rather to examine the transient state which immediately follows the short-circuit.

The first current peak which occurs after a short-circuit has been applied at an instant of zero voltage is actually practically double those eventually occurring if a steady-state is attained. This result is sometimes known as the "doubling-effect," and, although the peak values fall relatively quickly and so do not contribute greatly to any further overheating, the mechanical forces, which are proportional to the square of the current and are already enormous in a large transformer, are increased a further four times.(4) The cause of the doubling is clear on general grounds, for the reasoning follows closely that given in detail for the switching-on current surge. Imagine the steady-state short-circuit current to flow at the value it would have reached if the short-circuit had been in existence for some time. If the value of this current wave is not zero at the moment when the short-circuit is actually applied, a transient component must appear of such a value that in combination with the steady-state current it produces a resultant current which is actually zero at the moment of short-circuiting. This means, of course, that the transient is equal and opposite to the steady-state current at that instant, and in the worst case of a short-circuit at zero voltage its value is equal to the peak short-circuit current. But once this transient has started it persists and, although its rate of decay is fairly fast, it will still be nearly at the peak value a quarter of a cycle later when the steady-state component is also at the peak value. The resultant is thus nearly twice as great as that to which it eventually settles down. On the negative half-cycles, the transient and steady-state components are in opposition and the resultant is small.

Short-circuit current rushes are of great importance in large transformers owing to their mechanical effects. These are not particularly serious in small sizes where a compact and solid winding is often used without cooling ducts which weaken the structure. Overheating on a short-circuit is in any case very rapid, and the doubling effect does not add greatly to the hazards which in any event require rapid clearance of the fault.

3. VOLTAGE SURGES APPLIED TO TRANSFORMERS

Transformers connected to transmission lines are likely to be subjected to transient over-voltages caused by lightning discharges and switching disturbances originating at distant points on the lines. The lightning surges are usually the more serious, and peak voltages of fifteen to twenty times normal are sometimes found. This is itself a serious enough trouble, but matters are made worse by the fact that this voltage is liable to be applied not simply across the whole winding, but is concentrated on the end turns only. This effect is due to the capacities between the various parts of the windings.(5)



H. L. Thomas, J.I.E.E.

FIG. 63

Diagrammatical Representation of a Transformer Winding for Purposes of Surge Application.

At normal power frequencies a transformer winding can be regarded as an inductance having a relatively low resistance, and the capacitances between turns or sections can be completely neglected. At very high frequencies, on the other hand, the impedance of the inductive elements becomes very high, whilst that of the capacitances becomes low. The actual distribution of capacitance is very complicated, but concentrated capacitances can be substituted between the coils and from the coils to ground without seriously upsetting the result. The diagrammatical representation of a transformer winding on these lines is shown in Fig. 63. At the highest frequencies, where the inductances assume very high impedances and can therefore be omitted, the winding appears to be simply a chain of series and shunt capacitances. If, therefore, a very high-frequency voltage is applied to the transformer it will distribute itself across the first series and shunt condensers roughly in inverse proportion to their capacitances. If the series capacitances are much smaller than the shunt ones, it can readily be understood that the greater proportion of the voltage will appear across the end turns whilst the turns towards the earthy end will be very little stressed. For simplicity let the transient consist of a rectangular pulse having a nearly vertical wave-front followed by a long horizontal portion. This is in effect a d.c. voltage suddenly applied, and it is mathematically equivalent to an infinite series of components ranging up to the highest frequencies. Imagine such a transient applied to the winding. When the steep wave-front reaches the terminal, it is the high-frequency components which determine the initial voltage distribution because it is only the high-frequencies that can attain appreciable values within the interval of a few microseconds that is being considered. The initial distribution is accordingly determined almost entirely by the capacitances. After an interval of time measured in hundreds of microseconds, the voltage will be distributed across the winding in a manner related to the distribution of inductance. This is usually approximately uniform, so that the final voltage distribution is also more or less uniform. Initially, however, the voltage was concentrated across the end turns so that during the interval following the impact of the transient on the winding, oscillations are likely to occur and these are liable to cause dangerously high stresses in the inter-turn and inter-coil insulation.

Until recently the normal method of avoiding breakdowns due to the concentration of voltage surges across the end turns was to increase the insulation strength at that point. Since this results in a reduction in the series capacitances, it tends to increase still farther the voltage gradient between the end turns, and the insulation must be increased to an extent which more than off-sets the increased voltage gradient in order to ensure a sufficiently large safety margin. This, however, does not deal with the oscillations which may be set up farther down the winding and so cause failures where there has been no reinforcement of insulation. A new school of thought has now assumed considerable importance and this asserts that the correct way to deal with the problem is so to modify the capacitances that the initial distribution of voltage is the same as that which is finally attained. There will then be no undue stress on the end turns, because the voltage distribution is uniform and there will also be no tendency to oscillations in the various sections of the winding. The method of securing the desired uniform voltage distribution is by what is called "shielding," although it does not constitute screening in the usually accepted sense. The "shield" consists of a metal cylinder or plate connected to the high potential terminal and shaped in such a way as to add materially to the capacitances between the line terminal and the lower potential parts of the winding. In this way a uniform voltage distribution is obtained initially as well as finally. A transformer built on these lines is known as a "non-resonating" transformer. Again the use of the term "resonating" is hardly the same as that generally accepted, although it is possible to regard the building up of excessive voltages in the lower sections of the winding of a normal type as being due to resonance of the sections with the high-frequency components of the steep-fronted impulse. "Non-oscillating" would appear to be a better description for this construction.

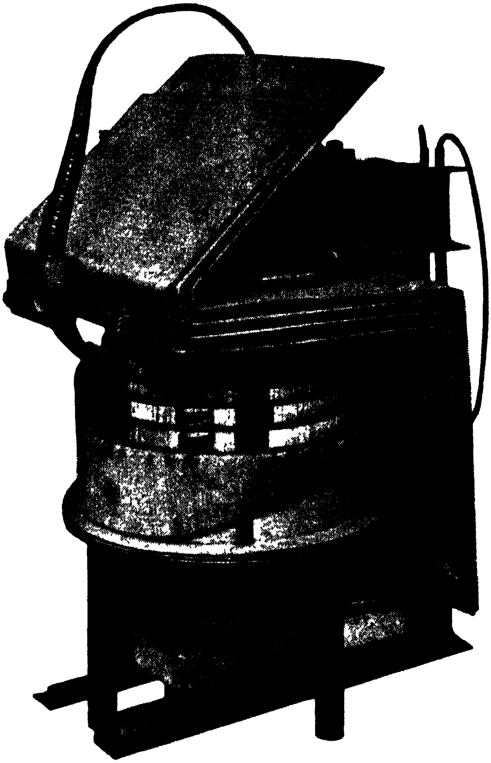


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FIG. 64

An Example of a Shielded (Non-resonating) Voltage Transformer Intended for 132-kV Operation.

As regards power transformers, the complete "non-resonating" system of shielding is usually applied only to those transformers that work at the highest voltages, i.e. 200 kV or higher. For lower-voltage transformers a simpler form can often be used which makes the voltage distribution much more nearly uniform than would be the case without it. At lower voltages the thickness of insulation is often determined more by mechanical considerations than electrical, and in that case some lack of uniformity in initial voltage distribution can be tolerated. "Non-resonating" shielding is, however, particularly helpful in voltage-transformers (for measuring purposes, see Chapter XIX), that can be very materially reduced in size by its use.

Although the application of transients to transformer windings has been discussed on the assumption that the surges are brought into the windings along transmission lines, this is by no means the only way in which they can appear. In light engineering work it is in fact unlikely that the transients will gain admission in that way, but there are often other sources of surges within the equipment. Unexpected insulation failures in high-voltage transformers apparently designed generously as regards insulation, are frequently due to surges generated either in the transformer itself by unsuitable switching arrangements or by the discharge of condensers elsewhere in the circuit. The designer and user should therefore be familiar with the behaviour of these surges and the ways in which their effects can be minimized.

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CHAPTER XII

Three-phase Transformers

ALTHOUGH the engineer engaged in the light electrical industries is chiefly concerned with single-phase circuits, there are occasions when he requires to use three-phase transformers. One of these is in the supply of D.C. for large R.F. oscillators, radio transmitters, etc. By using a three-phase supply (or six- or twelve-phase derived from it) for the rectifiers, much less ripple occurs in the output. Chapter XIII deals with the ratings of transformers used for this purpose.

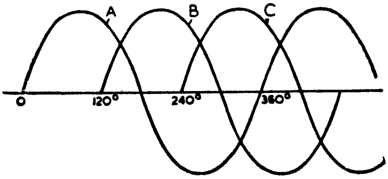


FIG. 65
Three-phase Voltages A, B, C, Displaced One-third of a Cycle (120°) from Each Other.

Even if the reader does not have to use or design three phase transformers, he should be aware of the possibility of interference to telephone circuits which may be caused by certain methods of connexion. The effect of these particular connexions is to cause the circulation of large third harmonic currents in the lines and serious interference can be caused by induction in neighbouring conductors. This matter is discussed later in the chapter.

The three-phase system of distribution is now so frequently used that the general principles are likely to be well known. The voltage of each phase is similar in magnitude and waveform to that of the others, but differs in phase by one-third of a cycle, i.e. 120° (Fig. 65). Although at first sight it might appear necessary to have six cables to supply the three phases, this is not the case; four wires, i.e. one per phase with a common return or *neutral* are sufficient, and in some cases only three need be used. The two basic methods of connexion in general use are known as *delta* (or mesh) and *star* (or Y). See Figs. 66*a* and 66*b*. In addition to these, there is the *interconnected*-star arrangement (Fig. 66*c*), in which each phase is *produced by connecting in series windings in two of the original* phases. Owing to the phase displacement between the two component voltages, the resultant voltage is less than the sum of the two components, so that some loss of winding space occurs. The interconnected-star arrangement has been introduced to overcome the third-harmonic troubles which cause difficulties with the star connexion in certain conditions.

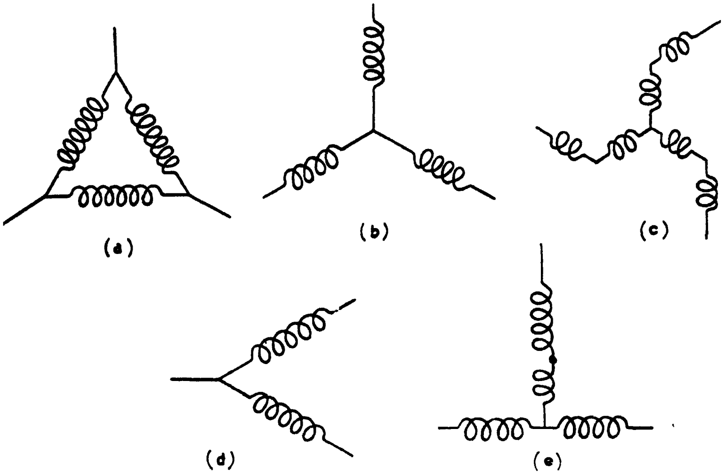


FIG. 66

Various Methods of Three-phase Connexion. (*a*) Delta. (*b*) Star. (*c*) Interconnected Star. (*d*) Open-delta or Vee. (*e*) Tee (Scott).

Open-delta (or *vee*) (Fig. 66*d*) and *tee*-connexions (Fig. 66*e*) are sometimes used for special purposes, and the *Scott* connexion is frequently used for three- to two-phase conversion. Of the various three-phase connexions the star, interconnected-star, and tee can provide neutral points. The neutral wire is not essential even in these arrangements, because in a balanced system no current flows through it. This follows from an examination of Fig. 65, in which it will be seen that the sum of the currents in the three phases is zero at every instant. No neutral connexion is possible in the delta connexion, unless it is obtained artificially by a tapped-reactor arrangement.

A three-phase supply can be transformed by a group of three single-phase transformers with their windings connected either in star or delta. The primaries can be connected in one manner, and the secondaries in the other, or both can be connected similarly. For the present purpose of examining the merits and applications of the various combinations of primary and secondary connexions, it will be assumed that separate single-phase units are used. In practice, however, three-phase transformers are often employed; these are dealt with later in Section 2 of this chapter.

1. PRIMARY/SECONDARY CONNEXIONS

The principal combinations of winding connexions and their advantages, disadvantages, and applications are discussed in the following paragraphs.

(*a*) *Delta/Delta*

The most straightforward method of connexion is both windings in delta. If three single-phase transformers have their primaries connected one to each of the three possible pairs of lines, the secondary voltages will be directly related to the primary line-to-line voltages. Since each line is connected to two coils, the winding currents are less than the line currents. Owing to the phase difference between the two branches, however, each winding takes $1/\sqrt{3}$ times the line current if the load is a balanced one. Any unbalance of currents on the secondary side will cause a corresponding difference between the primary currents. This follows from the fact that each transformer can be regarded as being connected (on both windings) to a separate single-phase circuit. An increased load on the secondary will naturally produce an increased primary current in that phase. The voltages on the primary and secondary terminals of a given line will be more or less in the same time phase, only small differences due to winding reactances being possible.

No neutral point is, of course, available in a delta connected secondary so that it is unsuitable for supplying a lighting load. For this application supplies must be given between a line and neutral which is earthed at one point. The delta/delta arrangement is very little used in England, but it finds more favour in the U.S.A., where the possibility of working (at reduced output) in open-delta if one unit fails is regarded as of some importance.

In delta/delta combinations it is necessary that the transformation ratio of all three units should be the same. If this condition is not satisfied there will be circulating currents round the windings even at no-load. The sum of the voltages in the three secondary windings connected end to end in the same polarity should be zero at every instant and in that case the delta can be completed without a circulating current flowing round it. (Third-harmonic currents are excluded.) This condition of the resultant voltage being zero is only satisfied if the three transformation ratios are equal.

If the three units are to share a balanced load equally and so allow the kVA output of the bank to be equal the sum of the capacities of the three units separately, their impedances must also be equal, otherwise the transformer with lowest impedance will take more than its share of the load.

(*b*) *Star/Star*

In the star connexion, one winding only is connected to each line, the opposite ends of all the windings being joined together. This neutral point may, or may not, be connected to the corresponding point on the alternator. The winding currents are now evidently equal to the line currents, but the voltages across each winding are, in the case of a symmetrical system, only $1/\sqrt{3}$ of the line-to-line voltage. As with the delta/delta arrangement, the primary and secondary line voltages are in phase.

At first sight the star connexion appears at least as simple as the delta, but this is only really true of a four-wire system with a solid neutral connexion on the primary side because otherwise each primary is not connected across two points between which a fixed voltage is maintained, irrespective of the current flowing. In the three-wire supply, two primaries are in series between each pair of line wires; consequently it is not possible to give a four-wire supply from the secondary in spite of the availability of the neutral point, because its potential is not stable. Imagine a load applied between one line and neutral only. A current will flow in the primary of the transformer across the secondary of which the load has been applied, but this current can only flow from the primary line terminals by passing through the primaries of the other two phases. In their case, however, there are no secondary ampere-turns to balance the increased primary ampere-turns and they behave like choke coils. Current can pass only by disturbing seriously the voltage relationships between the primary neutral point and the lines. If the primary line-neutral voltages are unbalanced, a similar state of affairs will appear on the secondary, so that no use can be made of the fourth wire apparently available there.

Unbalanced three-wire loads on the secondary (i.e. connected between lines only) produce similarly unbalanced primary currents but no disturbance of line voltages takes place apart from the small changes due to impedance drops.

The star/star arrangement, since it cannot provide a four-wire supply from a three-wire system, is not in these conditions suitable for supplying single-phase loads such as lighting or heating in residential areas. Its chief application is in supplying power loads on a relatively small scale. Although the stabilization of the neutral could be secured by using a four-wire connexion to the generator on the primary side, this is often not possible owing to the risk of interference with telephone lines due to third-harmonic effects. The possibility of large third-harmonic voltages of 30 per cent or more of the fundamental in the star/star arrangement is in any case a serious disadvantage which limits the application of this connexion. The reason for the high percentage of third harmonic is dealt with in the succeeding paragraph.

Third Harmonic in Star/Star Banks

The magnitude and wave-form of the no-load current of a single-phase transformer have been discussed in Chapters III and X. Recapitulating, the relationship between B and H for a magnetic material such as iron or steel is not even approximately linear for values of B above about 8 000 lines per cm.^{2} B increases less rapidly than H and eventually approaches a saturation value above which it cannot rise however much H is increased. Since the primary back e.m.f. (or induced pressure) is equal to the applied voltage at every instant (apart from a small drop due to the winding resistance) and this *back e.m.f. is produced by the flux variation in the core, it* follows that the flux wave for a sinusoidal input voltage must also be sinusoidal. Owing to the shape of the *B-H* curve, a sinusoidal flux requires a non-sinusoidal magnetizing current, the departure from a sine wave being in the direction of a "peaky" wave, since a disproportionate increase in instantaneous current is required at larger values of H (i.e. larger values of i). The change from a sinusoidal shape to a peaky shape can be regarded as the appearance of additional sinusoidal waves of higher frequency. The frequencies are odd multiples of the fundamental frequency, particularly three times, and of such a phase that the positive peak of the harmonic occurs at the same moment as that of the fundamental. Provided the generator and primary circuit of the transformer are of low impedance, the appearance of harmonics in the magnetizing current of a single-phase power transformer is of little moment, but in certain three-phase arrangements it may be of great importance.

If the magnetizing current wave-forms are plotted from the wave-form of the input voltages and the *B-H* curve of the core material, for each of the three phases, the curves in Fig. 67 will be obtained. The important point which emerges from these curves is the fact that although the fundamentals for the three phases are displaced by 120° from each other, the third harmonics are *in phase.* This is evident from the fact that a complete cycle of third harmonic is completed in the interval between the zero point for one phase and the zero for the next phase, so that the harmonic wave is by this time back to the same point in its cycle as at the start of the fundamental in the first phase. In a three-phase star-connected system then, the third-harmonic currents would be in the same phase in each arm, i.e. at a given instant the direction of third-harmonic current flow would be towards the neutral point in all three phases. If there is a solid neutral connexion to the generator this third-harmonic current flows along it and its magnitude depends on the neutral line and generator impedance. If, however, there is no neutral connexion, there is no path for the harmonic current. (In the case of the fundamental it will be remembered that the phase relationships are such that the condition of the sum of the currents entering the neutral point *being equal to those leaving it is satisfied without disturbance* of the normal operation.) Since there is no path for third-harmonic currents in a three-wire star system, no such current flows in the primaries. The result of this suppression of third harmonic from the magnetizing current which then approximates to a sinusoidal wave-form is to cause its appearance in the induced pressure wave between lines and neutral in both primary and secondary. The non-sinusoidal wave-form of the phase to neutral voltage has a high peak value which imposes undue stresses on the insulation to ground, but provided the secondary load is connected between lines, third-harmonic voltages are not apparent in the output, because they cancel out *between lines*. This is evident from the fact that the harmonic voltage in both phases composing a line-to-line voltage is, say, towards the neutral at a given instant. The resultant line-to-line voltage is then free of third harmonic.

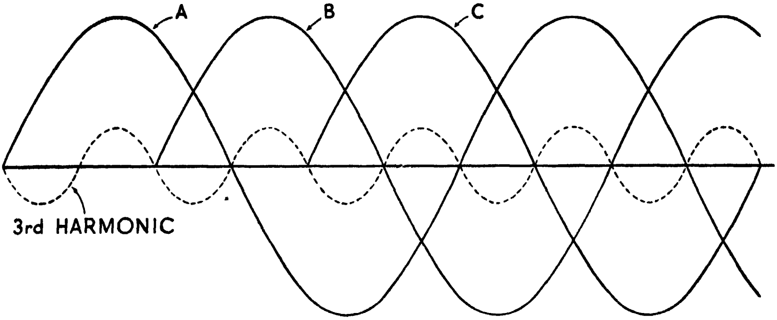


FIG. 67

Third-harmonic Voltages Generated in Each Circuit by Magnetic Distortion are in Phase.

The position with regard to the primary is that if the neutral is not connected to the generator by a low-impedance path it will be at harmonic voltage to earth. The magnitude of this voltage will increase as the flux-density in the core is increased and it may reach 50 per cent or more of the phase voltage in a bank of separate single-phase transformers or a shell-type three-phase unit. The harmonic effects in a three-phase core-type unit are much less pronounced, being usually limited to some 5 per cent. The ill effects of third harmonics described here for star/star connected banks do not accordingly apply *to three-phase core-type units. (The reasons for the satisfactory harmonic performance of the core-type unit are dealt with* later in this chapter in Section 2 on "Construction.") When the harmonic percentage is high the risk of damaging the insulation is quite appreciable, unless the windings have been specially built to withstand stresses at this point, in which case extra expense may be involved. On the other hand, if the neutral is earthed at the transformer the third harmonic voltages appear on the lines and may cause interference in telephone lines running near, by capacity transference. If the generator neutral is also earthed, connecting the transformer neutral to earth too completes a circuit for third-harmonic currents. Third-harmonic voltages on the lines and neutral are considerably lessened by this means and may be reduced to negligible proportions, but large third-harmonic *currents* will flow, and interference may be caused in the telephone circuits running near the power lines by electromagnetic induction.(1) This is more likely to arise from harmonic currents than from the fundamental for two reasons: first, the frequency is higher, which in itself induces greater interference voltages and also produces sounds which are more audible on account of the increased sensitivity of the ear and the telephone equipment generally; and secondly, because the harmonic currents flow in the same direction along all three lines, whereas the fundamental line currents are displaced in phase by 120° and their resultant effect is very small at a distance large compared with the line spacing. The effect of the fundamental is further reduced by transposing the lines at suitable intervals. Transposing the *telephone* wires is, of course, effective in either case.

Even if some interference of this kind can be tolerated, earthing the neutral may be only partially effective in reducing third-harmonic voltages to neutral in the secondary, because it may not be possible to secure a sufficiently low-impedance primary neutral connexion.

Earthing the *secondary* neutral is also apt to introduce troubles, particularly if it is connected to lines of high capacitance. In exceptional circumstances these capacitances may form series resonant circuits with the winding inductances. In these conditions, not only is the voltage stress on the insulation increased further but third-harmonic currents in the *windings are limited only by their resistances and also the iron would reach a high degree of saturation. These conditions lead* to rapid overheating of the transformer and consequent breakdown.

Even if the capacitances of the lines connected to the secondary terminals are lower than those required to give series resonance with the transformer windings, the third-harmonic currents can be considerably intensified.

From these considerations it will be seen that the two principal objections to the star/star connexion lie in the instability of the secondary neutral point when working from a three-wire supply and in the third-harmonic effects. The latter trouble is avoided by using core-type three-phase units.

(c) *Star/Star with Tertiary Delta*

The third-harmonic difficulties experienced with star/star banks of transformers (or shell-type three-phase units) have led to the introduction of a tertiary winding on the same cores as the primaries and secondaries. This third winding (which is not connected conductively to the other two) consists of single coils on each phase limb connected in series to form a closed delta. Since the third-harmonic voltages are in the same phase in each coil, and the polarity of the normal delta connexion is such as to form a short-circuited chain, the third-harmonic currents circulate round the delta and there are no third-harmonic voltages present across the windings. The triple frequency currents flowing round the tertiary delta winding provide the additional ampere-turns at the harmonic frequency necessary to produce a sinusoidal flux in the core. (It will be remembered that in the star-connected primaries only sinusoidal magnetizing currents can flow—the extra ampere-turns of the tertiary produce in combination with those of the primary a resultant magnetizing force of the correct wave-form to produce a sinusoidal flux.)

The tertiary winding may be completely isolated or it can be used to supply a load. In the latter case care must be taken to earth the delta winding either at one terminal or preferably through a neutral earthing compensator. Delta connexions are not usually earthed, and this precaution is consequently apt to be forgotten, but this may result in the winding reaching a dangerously high voltage to earth by capacity coupling if both primary and secondary are high-voltage windings.

(*d*) *Star/Delta*

Owing to the third-harmonic difficulties arising from the use of star/star connexions, the combination of delta primary and star secondary windings forms a very frequently-used transformer. As explained in (*c*), dealing with tertiary delta windings, third-harmonic troubles are eliminated from the star winding by the use of a delta connexion which allows free circulation of third-harmonic currents which supply the ampere-turns equivalent to those required from the suppressed third-harmonics of the magnetizing currents. The primary neutral can now be earthed without introducing interference troubles and the neutral is kept stable.

The star/delta arrangement is best suited for step-down purposes, because star windings are inherently more suitable for high voltages and delta windings for low voltages. This arises from the relative values of currents and voltages in the two methods of connexion. For a given voltage between lines, the coils in a star winding will each carry full line current, but will only have $1/\sqrt{3}$ line voltage across them. Delta coils, on the other hand, carry only $1/\sqrt{3}$ of the line current but support the full line-to-line voltage. The star coils thus have fewer turns of heavier conductor than the delta coils for the same total voltage and current rating on the three phases. In order to make a robust winding in the mechanical sense, a high-voltage coil should have as few turns and be wound with as heavy a conductor as possible; the star connexion is evidently the one to choose. On the other hand, when the voltage is low, the conductor size is in any case large enough to produce a mechanically-strong coil unlikely to be distorted by overload currents, and the difficulty is likely to be that the conductors become too stiff to handle. The lower current of the delta winding is then an advantage. Similar considerations apply to the insulation; on the high-voltage side, the insulation is always a major problem and the lower voltage on a star coil is welcomed. On the low-voltage side, however, insulation thicknesses are governed more by mechanical than electrical considerations, so that the higher voltages in delta arrangements do not affect the space given up to insulation. The chief use of the star/delta connected transformer is thus in step-down applications, particularly at the far end of a high-voltage transmission line for reducing from, say, 66 000 to 11 000 volts. The primary neutral can be earthed to ensure a symmetrical distribution of voltages between lines and ground and to ease insulation stresses between windings and core. The absence of a neutral point on the secondary makes the star/delta connexion unsuitable for final distribution transformers, at any rate in residential areas where single-phase lighting loads with one side earthed have to be supplied. For balanced three-phase loads such as motors, the objection does not apply.

Unlike star/star and delta/delta combinations, the star/delta transformer has a phase difference of approximately 30° between corresponding primary and secondary terminal voltages. The delta side may either lead or lag on the star according to the polarity of the coil ends joined on the star side. This feature makes it impossible to parallel a star/delta transformer with a star/star or delta/delta transformer even if their transformation ratio and other characteristics are similar. The groups which can be operated in parallel are given in Section 3 of this chapter.

(*e*) *Delta/Star*

Third-harmonic pressures are not induced in the windings of delta/star transformers because the primary windings of each phase are independently connected between a pair of lines of the supply and the magnetizing current can, therefore, assume the wave-form necessary to produce a sinusoidal flux variation. The output voltages in the star windings are accordingly sinusoidal and the neutral can be earthed without untoward effects. The arrangement is, therefore, very well suited for supplying groups of unbalanced single-phase loads as in residential areas. A typical rating for such purposes would be 500 kVA transforming from 11 000 volts to 400 volts. H.T. windings would probably be of the sectional type, whilst the l.t. windings would be helical. In smaller sizes, such as 100 kVA, cross-over h.t. coils would be used.

The use of a delta/star transformer for step-down purposes does not follow the principles laid down in the previous section on the star/delta type, in which it was shown that the delta connexion is inherently best suited to low voltages. In practice the highest voltage at which a delta winding can be used with reasonable satisfaction depends on the size of the transformer. When the rating is measured in a few hundreds of kVA, 11 000 volts represents a high figure for this connexion, but 33 000-volt delta connexions are used in 5 000-kVA transformers.

Another application of the delta/star arrangement is at the sending end of a high-voltage transmission line. Being a step-up transformer in this case, the windings are well suited mechanically to the voltages used.

The delta/star arrangement is probably the most commonly used one of all. Its principal disadvantage is the absence of a symmetrical earthing point.

(*f*) *Star/Interconnected Star*

The interconnected star or zig-zag winding was introduced as a substitute for the delta/star connexion in those applications where the primary voltage is too high, relative to the size of the unit, to make a delta winding sufficiently robust mechanically owing to the large amount of insulation required and the small amount of copper. At the very common primary distribution voltage of 11 000 volts the lowest rating for a delta winding is very little under 100 kVA. On the other hand, a star winding, on account of the lower number of turns and lower proportion of insulation, can be used down to 50 kVA, a very useful rating for isolated consumers. Unfortunately the inability of a star-connected primary to deal with single-phase line-to-neutral loads when used in conjunction with a star-connected secondary, and the unsatisfactory third-harmonic performance of the star/star arrangement, preclude its use in many situations.

The star/interconnected-star combination overcomes many of the objections to the star/star arrangement, and enables the robust star winding to be employed for the high-voltage winding.

The interconnected-star winding is made by connecting in series in each phase two coils of opposite polarity placed one on each of two separate transformers (or magnetic circuits in the case of a three-phase type). The resultant output in each phase is thus the vectorial difference between the outputs from two separate phases (Fig. 68). Owing to the reversed polarity of the second half of each winding, third-harmonic voltages are cancelled out from the line-to-neutral voltages in the output of each phase. The secondary neutral can be earthed without any of the troubles which may arise if this is done with a star/star connexion. It should perhaps be emphasized that the third-harmonic pressures are still induced in the half secondary windings and, of course, in the primary, so that the insulation of these coils must be made sufficiently good to withstand the extra peak voltage which results. Furthermore, since the third-harmonic induced voltages are present in the primary, the neutral cannot be earthed without risk of causing interference as in the star/star case. This should be contrasted with the tertiary-delta method of eliminating third harmonic pressures. The mechanism there is fundamentally different in that circulating currents in the delta actually supply the suppressed component of the magnetizing current, so eliminating the harmonic from the flux wave. The induced voltage in every winding is then sinusoidal.

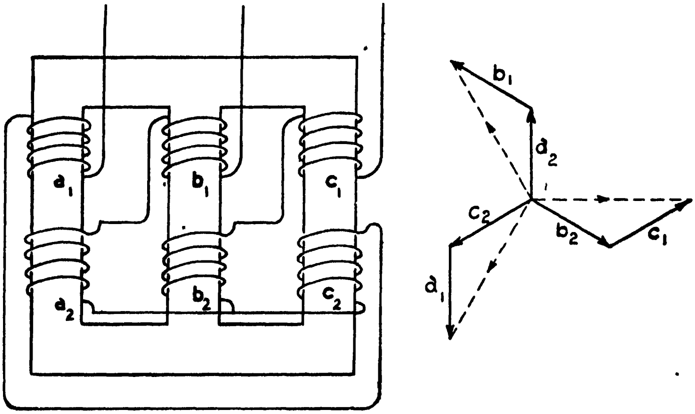


FIG. 68

The Interconnected Star Arrangement and its Vector Diagram.

The interconnected star primary actually goes a stage farther than the normal star, in that the transformer and generator neutrals *must* not be connected together by a fourth wire or via earth, quite apart from interference considerations, because excessive third-harmonic currents would flow which would cause overheating of transformers and alternator. The currents in this case are so much greater than in the plain star arrangement because the interconnexion of the windings makes them effectively non-inductive for the third-harmonic currents which are in the same time-phase in each phase of the supply. The reactance of the transformer is accordingly very low, and large currents flow.

The interconnected-star/star arrangement enables unbalanced loads to be supplied between the lines and neutral on the secondary side. It will be remembered that this is one of the virtues of the delta/star system. If the primary is a plain star it cannot supply a load between secondary line and neutral, because the primaries of the two idle transformers act like choke coils in series with that of the active one and seriously deflect the neutral point. This does not happen with interconnected-star primaries because the secondary-load current now flows through two transformers in series and the corresponding load components of the primary currents also flow through the two primaries in series direct from line to line. The single-phase secondary load is thus supplied by a line-to-line primary current and the unbalancing is only the small amount arising from impedance drops.

The output of each phase of a star/interconnected star group consists of the resultant output of one of the primary-voltage vectors and another primary vector reversed. Since the primary phases differ 120° angularly, the separation between the two secondary components is 180°–120°, i.e. 60°. The resultant of these two will differ by 30° from one of the original vectors. Like the delta/star combination there is thus a phase difference of 30° between input and output voltages of a star/interconnected-star arrangement; the two arrangements can be operated in parallel without difficulty. (See Section 3, Parallel Operation.)

Another important application of the interconnected-star arrangement is in transformers supplying three-phase rectifiers (Chapter XIII). A plain star connexion used in this way causes d.c. magnetization of the cores of three-unit groups or shell-type three-phase transformers, and it is almost essential to prevent this by interconnexion so that the d.c. fluxes neutralize each other. This is not so necessary in three-phase core-type transformers.

(*g*) *Vee/Vee*

The vee or open-delta connexion (Fig. 69) is sometimes used in place of the normal delta, as, for instance, during the initial stages of rural electrification when the load is small but is expected to grow later on, or temporarily owing to the failure of one of the three closed-delta connected units. It might have been thought on superficial consideration that the removal of one transformer would seriously unbalance the system voltages, but this is not so; the extent of the unbalance is only that due to the impedance drops in the transformers. The reason for the maintenance of balance lies in the fact that the voltage

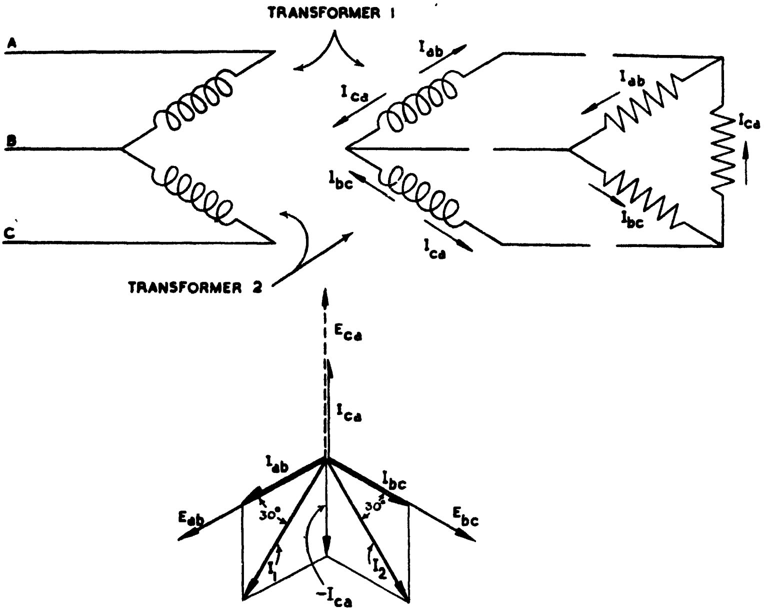


FIG. 69

The Open-delta Connexion of Two Single-phase Units to Supply a Three-phase Load. The Phase-angles of the Secondary Currents are 30° Leading and Lagging Respectively.

vectors of the three phases form a closed equilateral triangle. If the line-to-line voltages E_{ab} and E_{bc} are produced by the secondaries of two transformers whose primaries are connected between the corresponding alternator terminals, the voltage E_{ca} will be the same in magnitude and phase (apart from a small impedance drop) as though it had been transformed by a third transformer connected between these points. The two transformers are acting purely as single-phase transformers, so that just as the alternator maintains the line-to-line voltages on the primary side, irrespective of the fact that one primary has been removed, so the secondary voltages will also appear correctly.

Consider the effect of three individual resistance loads which make up a balanced three-phase load connected between each pair of output terminals of the open-delta bank. The current I_{ab} in the load across E_{ab} will be directly transformed and will appear as a current I_{AB} in lines *A* and *B*. In a similar way, the current I_{bc} will appear as a current I_{BC} in lines *B* and *C*. The load current I_{ca} is, however, supplied by the two secondary windings in series; additional currents I_{CA} will, therefore, flow in the primary lines *A* and *C* to supply this load. The primary line currents I_A, I_B, and I_C are the resultants of the various components; it will be evident from the vector diagrams that they are equal in magnitude and differ by the usual 120° in phase. Actually it makes no difference to the line currents whether the current I_{CA} flows through the two primaries in series or whether it flows through the third primary which would be there in a closed-delta set-up, but this current component has a marked effect as far as the transformers are concerned. Unlike the current components I_{AB} and I_{BC} (corresponding to I_{ab} and I_{bc} in the load), which are in-phase with their respective voltages, I_{CA} is not in phase with either of the individual transformer voltages but only with their resultant. Since the individual voltages differ by 120° in phase, the current I_{CA} leads one voltage by 60° and lags on the other by a similar phase angle. Each transformer is, therefore, in the position of supplying a current component in-phase with its voltage and another equal component 60° leading or lagging on the voltage. The vector difference of these two components, i.e. the actual current supplied by the transformer, is at 30°

phase-angle with the voltage. The internal power-factor of each transformer is for this reason only 86·6 per cent (cos 30°) even when supplying a unity power-factor load, so that the rating of the open-delta group is $\frac{2}{3}$ × 86·6 per cent, i.e. 57·7 per cent of that of a closed-delta group of similar transformers.

The open-delta connexion is more popular in the U.S.A. than in Great Britain, particularly in the matter of temporary use following a breakdown of one unit in a delta-connected group.

The primary and secondary line pressures of a vee/vee group are in phase, so that it can be paralleled with a delta/delta group, but it is not economical to do so because the different transformers do not take an equal share of the load. Those arms in which there are two transformers will not be fully loaded when the remaining arm having only one transformer is giving its full-load current.

(*h*) *Tee/Tee*

The tee-connexion is very little used in three-phase to three-phase transformations, but in the form of the Scott-connexion it is of importance in three-phase to two-phase conversion. The tee arrangement somewhat resembles the star connexion, and the winding voltages being comparable the tee-windings have similarly desirable characteristics for high voltages. Unlike a star connexion, however, the two windings shown horizontally in Fig. 66*c* are both in the same phase and are therefore wound on one transformer. They are, in fact, one winding centre-tapped. This is known as the "main transformer" and the other one as the "teazer." The voltage of the teazer is 0·866 of that of the whole winding of the main transformer, and by tapping it suitably, a neutral point can be obtained.

As in the open-delta arrangement, the kVA rating of the pair is less than the sum of the ratings of the individual transformers.

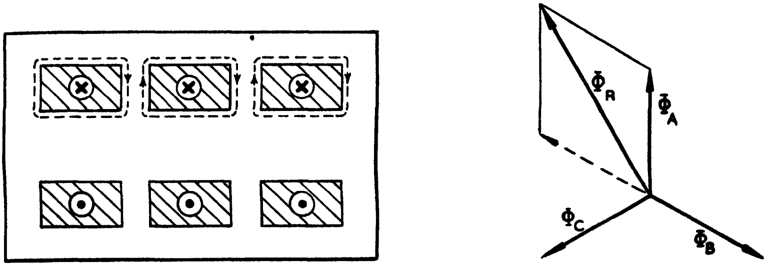
2. CONSTRUCTION

The use of three separate transformers in the way so far assumed makes for flexibility and ease of replacement in the event of faults occurring, but it is somewhat more expensive than the alternative of using a special three-phase transformer in which the three sets of windings are placed on a single multi-limbed core. By arranging the magnetic circuits suitably it is possible to make considerable savings in core material, since some parts of the circuit can be made common to two phases. As in single-phase transformers, the construction can be either shell-type or core-type; the latter is more common, particularly in the United Kingdom. The relative advantages and disadvantages of the two types centre round the harmonic effects, which are liable to be serious in certain star-connected arrangements, particularly if a neutral connexion is not completed.

(*a*) *Shell-type*

If three separate single-phase units are used to transform a three-phase supply, the magnetic circuits for each phase are, of course, completely separate; the shell-type arrangement of core is similar to three separate transformers with their cores laid end to end and in that toc the magnetic circuits are largely independent. It is, however, possible to save the yokes of the middle transformer by making them common with those of the two outer transformers. An appreciable economy results from this reduction in iron content, and the floor space required is also reduced. To make this saving possible it is necessary to make a slight re-arrangement of the magnetic circuit as compared with the simple end-to-end system, because this would result in an excessive flux-density in the intermediate yokes.

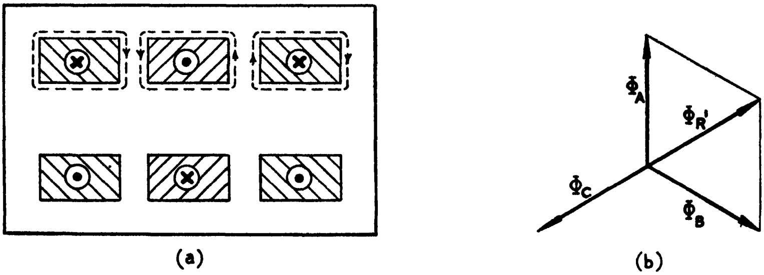
If the three cores from which the shell-type core is built up were placed end to end in the normal phase sequence and the yokes of the middle one were removed, the flux density in what remains of the two central yokes would be the resultant flux-density due to the two adjacent phase coils. Since the common portion is, in effect, the "top" of one transformer and the "bottom" of the next, the resultant intensity is not the *sum* of the fields of the two phases, but the difference (Fig. 70*a*). If the algebraical difference between two phases (say A and B) is examined at various instants it will be seen that it reaches a value greater than that reached by a single phase at any instant. The peak value of B in the intermediate yokes would, therefore, be higher than that in the limbs. This is shown in the vector diagram of the fluxes, Fig. 70*b*, where Φ_R is seen to be greater than either Φ_A or Φ_B. If, however, the winding of the middle transformer is reversed, so that a current in the positive direction produces a flux in the middle limb in the opposite direction to that in the other coils, the fluxes in the two central yokes will add instead of subtract (Fig. 71*a*). This might appear to make matters worse, but remembering that the fluxes differ in phase by 120°, it will be seen that their sum is algebraically less than their difference. The effect of reversing the flux *B* is indicated in Fig. 71*b*. The resultant $\Phi_{R'}$ is then equal in magnitude to either of the component fluxes Φ_A or Φ_B, so that the flux-density in the yoke is equal to that in the side limbs. A weight of iron (and a corresponding space) equal to two yokes is saved



(a) (b)

FIG. 70

The Magnetic Circuits and Flux Vectors of a Three-phase Shell-type Core in which the Positive Directions of Primary Current are the Same in the Three Phases. Note the excessive resultant flux Φ_R in the inner yokes.



(a) (b)

FIG. 71

If the positive direction of current in the middle phase is reversed, the resultant flux $\Phi_{R'}$ in the inner yokes is reduced to that in the side limbs.

by this reversal of phasing, as compared with three separate transformers. The arrangement is not suitable for star-star windings in certain conditions, but in this respect the shell-type three-phase transformer is no different from three separate single-phase ones.

The shell method of construction leads to a more robust job than the core type with its long, rather poorly-supported limbs with little to keep the coils in place. The shell type is, however, at a disadvantage in the matter of ease of replacement of faulty coils, because it is necessary to remove a large proportion of the laminations to obtain access to the coils.

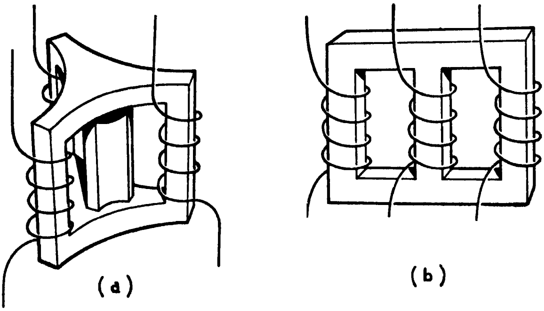


FIG. 72

(*a*) The Magnetic Circuit of a Three-phase Core-type Transformer Arranged to Show Similarity to Four-wire Electric Circuit. (*b*) Practical Arrangement with Central Limb Omitted.

(*b*) *Core-type*

The core-type three-phase transformer, like its shell-type counterpart, allows a considerable saving in iron by making parts of the core common to two or more phases. In this respect it goes farther than the shell arrangement, for active parts of the core, i.e. the limbs, also take part in the sharing. The core-type arrangement can be regarded as the magnetic equivalent of a star-connected electrical circuit working without a neutral wire. Imagine the limbs of three single-phase transformers, complete with their windings, stood vertically at the corners of an equilateral triangle and a common return limb placed at the centre of the triangle (Fig. 72*a*). The four limbs would, of course, be connected together at top and bottom by triangular yokes. Now if the fluxes in the three limbs are equal and sinusoidal, as is the aim in three-phase systems, there will not be any resultant flux in the common centre limb, as can be seen by reference to Fig. 65, which although drawn for voltages, can also be taken to represent the fluxes in the three magnetic circuits. If the centre limb carries no flux, it can be omitted without affecting the working of the transformer, and this is what is done in the three-phase core-type system. To make construction easier and more substantial, the three remaining limbs are arranged not in a triangular form but in a straight line (Fig. 72*b*). This results in some lack of symmetry in the magnetic paths, and in consequence different magnetizing currents in the three phases, but this is not usually of any great moment.

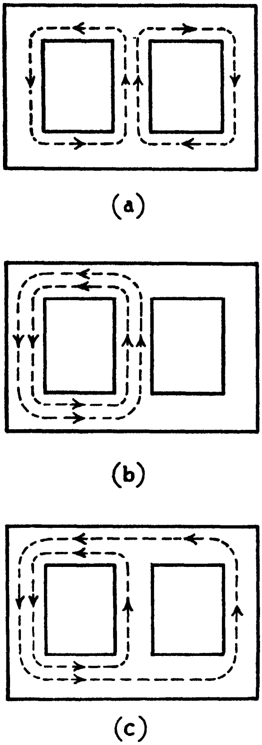


FIG. 73
Flux Distribution at Three Different Points of a Cycle, in a Three-phase Core-type Transformer.

Examining the magnetic circuit in greater detail, it will be seen that in each limb there are two m.m.f.s in series at any moment (Fig. 73). Take the central limb, for example, at the moment when phase *B* flux is at its maximum in a positive direction, and assume that this represents an upward direction. The fluxes in the other two limbs will at this instant be equal to each other and both negative. Their magnitudes will be Φ_{max} sin 30°, i.e. $\Phi_{max}/2$ in a downward direction. The path of the flux is thus up the central limb, dividing equally into the two halves of the yoke and down the outer limbs. As the cycle proceeds the flux in the central limb begins to decrease and it does not divide equally into the sections of the yoke; a greater portion will flow down the left-hand limb which carries phase *A*. A little later still, that in the right-hand limb (phase *C*) will have decreased to zero, whilst phase *A* will have reached $\Phi_{max}/\sqrt{2}$. At this instant the flux in the central limb (phase B) has dropped to $\Phi_{max}/\sqrt{2}$ too. The result is then that the path is up the centre and down the left-hand side limb whilst the right-hand side is unmagnetized (Fig. 73*b*). At a point another 30° later still, i.e. 60° from the first position examined, phase C has begun to increase again and in the positive direction. Its magnitude will be $\Phi_{max}/2$ whilst phase B, although still in the positive direction, has now fallen to $\Phi_{max}/2$ too. Phase A has now reached its peak value Φ_{max}. The distribution is now down the left-hand limb, along the lower yoke, dividing into two equal portions at the centre and right-hand limbs, joining up again in the upper yoke (Fig. 73*c*). The magnetic lengths of the two paths via the centre and right-hand limbs is not equal, and this will cause some disturbance of the relative values of magnetizing current in the three phases. That for phase B will be lower than that in the other two, since the picture is repeated in the reverse direction 180° later. If this is compared with that at the commencement when B was at its peak, it will be seen that the average magnetic length for phase B is less than that for either of the others.

The central limb of the hypothetical triangular arrangement of cores described earlier is not always omitted, although it is never actually placed in the centre. Five-legged core-type transformers are sometimes made, the two outer limbs being unwound. These outer limbs are in effect the return magnetic path which is omitted from the normal three-legged type. The five-legged type provides an independent magnetic circuit for each phase and so behaves like a three-phase shell-type transformer or three separate single-phase ones as regards third-harmonic voltages in star/star circuits. The usual core-type three-phase transformer is of the three-legged type and in the remarks which follow, it is assumed that the three-legged construction is meant whenever a core-type is mentioned.

Third-harmonic Phenomena in Core-type Transformers

It has been seen that certain highly undesirable effects may occur in star/star shell-type transformers owing to the suppression of the third harmonics of the magnetizing currents. The flux wave then becomes flat-tapped, and the e.m.f. generated by it is very peaky owing to the appearance of anything up to 60 per cent third harmonic. This effect is very much less troublesome in three-phase core-type units owing to the magnetic interaction between the phases. Since the third-harmonic components of the magnetizing currents in the three phases are all in the same time phase, the magnetic fluxes generated by these harmonic currents are in the same direction (say downwards at the instant in question) in each of the three limbs. In consequence there is no resultant third-harmonic m.m.f. round any of the closed paths in the core, and the flux has to complete its path through the air from the bottom yoke back to the top yoke (Fig. 74). Some of this flux will find its way through the tank and clamping fittings of an oil-immersed transformer but even so, the reluctance of the return path is so high that the magnitude of these flux components is very small. The fundamental flux, of course, completes its path entirely in iron as already described, because the phase relationships of the m.m.f.s in the various limbs ensure resultant m.m.f.s round the different paths. The third harmonics in the flux waves are in this way kept down to quite low proportions and the third-harmonic outputs in the voltage waves between line and neutral are usually well under 5 per cent. The third-harmonic pressures between line and line are, as in shell-types, in opposition and cancel out.

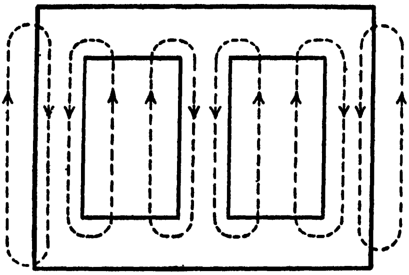


FIG. 74

Paths of the Third-harmonic Flux in a Star/Star Core-type Transformer.

A three-legged core-type three-phase transformer can thus be used in most star/star circuits with isolated neutrals. Care must, however, be exercised when the transformer is feeding long lines, because even the small percentage of third harmonic may be accentuated by series resonance in lines of considerable capacity and overheating due to large harmonic currents may take place.

Unbalanced Loading of Core-type Transformers

The other undesirable feature of the star/star connexion of a bank of single-phase units was shown to be the fact that a single-phase load applied between the neutral and one line on the secondary side causes serious disturbance of the primary neutral potential if it is not connected to the alternator. The flow of current in the secondary of the transformer supplying the particular phase is accompanied by the flow of a corresponding load component in the primary. This current, however, can flow only via the primaries of the other two transformers. Since these are on open-circuit on their secondaries, the increased current can flow only as an increased magnetizing current. The magnetizing current can increase only if the voltage across the winding rises, and that is what happens. The line-to-neutral volts thus increase on the two unloaded phases and drop on the loaded one.

This effect is less serious in a three-legged core-type three-phase unit, because again, as in the case of third-harmonic components, the primary current from the loaded phase in its passage through the other two coils magnetizes the limbs upon which they are wound in the same direction as that in the working limb, so that the return path for the fluxes is back through the air. The reluctance of this path is high and the flux is kept low; the corresponding voltage disturbances are also low.

Three-phase Core-type versus *Single-phase Transformers*

A three-phase core-type unit possesses at least as great advantages over three separate single-phase ones in the matter of material savings as does a three-phase shell-type transformer. The savings are chiefly in core material and in the provision of terminal bushings, only three being required per high-voltage winding instead of six.

In addition, the core-type three-phase transformer has the advantage that it can often be used star/star connected with isolated primary neutral.

3. PARALLEL OPERATION

If two transformers are to work in parallel, it is essential that, when the primaries are in parallel, the instantaneous potentials at corresponding secondary terminals should be equal. In order that two three-phase transformers or groups of single-phase units can operate satisfactorily with their primaries and secondaries in parallel, the conditions applying to the parallel operation of single-phase transformers must be satisfied, but in addition there are other requirements peculiar to three-phase working. That is to say that, in addition to having

(*a*) the same polarity;
(*b*) the same transformation ratio; and
(*c*) the same percentage impedance;

which are required for single-phase working, the transformer must also have

(*d*) the same phase-angle between corresponding primary and secondary terminals;
(*e*) the same direction of phase rotation; and
(*f*) the same kind of three-phase voltage regulation with a balanced load.

These special requirements for three-phase types are now considered in detail.

Primary/Secondary Phase-angle

It has already been explained that when both primary and secondary have the same method of connexion, e.g. star/star, the voltages at corresponding terminals on the two sides are in the same time-phase. As regards this particular requirement it is thus possible to parallel not only a star/star group with another star/star group or a delta/delta group with another delta/delta, but also a delta/delta with a star/star. The various combinations can be divided into two classes; any of those in the same class can be connected in parallel with each other, but not with one in the opposite class. The first class consists of those in which the secondary line-to-line voltages are derived directly from primary line-to-line voltages, and those in which line-to-neutral voltages are derived from line-to-neutral. In each case the corresponding primary and secondary voltages are in phase. These combinations are—

Class 1 {
Delta/Delta
Star/Star
Delta/Interconnected-Star
Vee/Vee
Tee/Tee

The second class are those in which only one winding is connected directly between line and line, the other one being obtained by the addition of two different phases. There is a phase difference of 30° between corresponding input and output terminals in transformers of this group. The combinations here are—

Class 2
- Star/Delta
- Delta/Star
- Star/Interconnected-Star
- Interconnected Star/Star

Owing to the phase angle of 30° between corresponding primary and secondary terminals of transformers in Class 2, it is not possible by any reversal of windings or exchange of phase connections to arrange that the outputs are in phase with those from a transformer having connexions in Class 1.

Phase Rotation

By phase rotation is meant the sequence in which the line voltages pass through their maximum values. Thus, if the time order of succession is A, B, C, the rotation is said to be standard, whereas A, C, B (i.e. C, B, A) would be reversed. The rotation is of course primarily determined by the generator, but the sequence can be altered at any point by interchanging two line wires. It is evident that the apparent phase rotation can be reversed in this way, because it amounts in effect simply to the re-labelling of the lines; if the line which was called B is now called C and vice versa, the sequence will obviously be reversed.

Symmetry of Regulation

This factor is likely to be of importance only when open-delta groups are in use. The effect of applying a balanced three-phase load to an open-delta bank is to produce slightly unbalanced voltages due to the different regulation of three phases. Consequently a vee-connected bank will not share a load correctly with a closed-delta bank. The phases having two transformers in parallel will take less than their full load, when the single transformer connected to the other phase is fully loaded. The kVA rating of the two groups in parallel is therefore less than the sum of the individual ratings.

Phasing-in

The process of finding which terminals of a bank shall be connected to those of another bank to permit parallel operation is known as *Phasing-in.* Although the correct connexions can be deduced from the maker's information, it is always wise to check the intended connexions by a practical voltmeter test before actually completing them. It may also happen that no information on the internal connexions and winding directions is available, and in that case a trial must be made. In order that two banks can operate in parallel, it is essential that the vectors representing the secondary line voltages of the two banks shall coincide. Whether or not this state of affairs will be achieved can be decided by connecting the primaries of both groups to the lines, and connecting together one secondary terminal of one bank to a secondary terminal of the other bank. If the neutrals are available these should be the secondary terminals selected for connecting together. A voltmeter (capable of reading up to twice the line voltage) is now connected in turn between pairs of terminals which it is expected can be connected together. If a zero reading is obtained in each case, it is an indication that the voltage vectors are coincident as required and the connexions can be completed. (A practical precaution should be taken to see that the voltmeter is in order by connecting between a pair of terminals across which a voltage *is* expected.)

If, however, a reading is obtained between some or all of the pairs of terminals, it is an indication that either the wrong terminals have been joined or that the polarity or the phase rotation of the two groups is different. Just which of these conditions is the cause of the voltage difference can be deduced from a study of the readings obtained. If the primary/secondary combination is one of those in Class 2, i.e. one in which there is a 30° phase difference between corresponding primary and secondary terminal voltage, it is always possible to find a method of connexion which will allow parallel operation whatever may be the internal connexions between windings or the directions of the coils. If the transformers are in Class 1, however, such as star/star groups, it may not be possible to find suitable connexions without altering the internal interphase links of one transformer.

The results of a test of two star-connected secondaries with neutrals joined will be taken as an example of the deductions which can be made from sets of voltage readings.

TABLE XI

Readings	Polarity	Phase-rotation
1. Three zero	Same	Same
2. Three line-voltage	Same	Same
3. One zero Two line-voltage	Same	Reversed
4. Three double phase-voltage	Reversed	Same
5. Three phase-voltage	Reversed	Same
6. One double phase-voltage Two phase-voltage	Reversed	Reversed

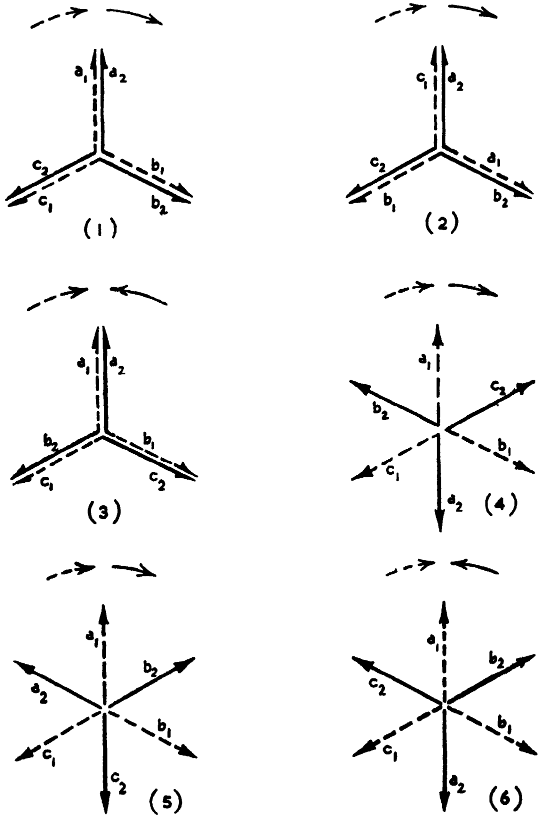


Fig. 75

Vector Diagrams Showing the Relative Phases of the Secondary Voltages for Various Interconnexions between Two Three-phase Transformers.

In Case 1 no alteration is required and the connexions can be completed. In Case 2, although polarity and rotation are the same for both transformers, the wrong pairs have been selected. All three connexions will have to be rotated in sequence either forward or backward, but the readings cannot give any indication which direction will produce the desired result. This can only be found by trial. Case 3 can be corrected by crossing over the links between the two pairs that give voltage readings. Cases 4, 5, 6 will require an internal change of connexions if the transformers have Class 1 primary/secondary connexions, e.g. star/star, in order to correct the polarity, but if the connexions are in Class 2, e.g. delta/star, the polarity can be reversed by changing over two of the primary line connexions. This will also reverse the rotation, which can be corrected by changing round secondary connexions. Fig. 75 shows the vector diagrams for the various cases,

A group of single-phase units to be paralleled with a three-phase transformer is best dealt with one transformer at a time. Both its primary terminals are connected to the appropriate lines and one of the secondary terminals is connected to what is expected to be the corresponding terminal on the three-phase secondary. The remaining terminal of the single-phase unit is then taken through a voltmeter to each of the three-phase transformer's remaining secondary terminals. From the voltage reading obtained it is possible to deduce the phase relationships between the two secondaries and to make connexions accordingly. The other two single-phase transformers are treated in the same manner.

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CHAPTER XIII

Transformers Supplying Rectifiers

ALTHOUGH the effect on a transformer of phase displacements between the output current and voltage has been considered, the output current has so far always been assumed to be sinusoidal in form. There are many cases, however, in which a transformer has to supply a load of which the resistance is not constant but variable over the cycle. The wave-form of the current is then no longer sinusoidal and the r.m.s. value may be increased, so requiring larger section windings. A rectifier-fed circuit is an extreme example of such a load, for its resistance becomes for all practical purposes infinite over part of the cycle whilst it is very low during its conducting period, so that the effective load resistance is only that of the load the rectifier is supplying. In these circumstances, the output which a given transformer frame size is capable of delivering is reduced not only on account of the increased heating resulting from a current of non-sinusoidal wave-form, but also because the windings are often not in use for more than a small fraction of the cycle. The r.m.s. value of the current is then high relative to the d.c. output from the system, requiring larger conductors, and in some arrangements more turns also. The purpose of this chapter is to calculate the increase in the transformer size required for various connexions in terms of the kVA rating that such a transformer would have if supplying a normal resistive load.

Strictly speaking, a rectifier could be said to be any device whose resistances in the forward and reverse directions of supplied voltage are different, so introducing a unidirectional component into the resulting current. The wave-form of the current is not symmetrical about the zero line, so that a d.c. component is present. For the purposes of this discussion the rectifier will first be considered "ideal," that is, with the forward resistance zero and the reverse resistance infinite. This assumption is reasonably valid when applied to mercury-vapour rectifiers, provided that the constant voltage drop of about 15 volts per rectifier in series is subtracted from the output voltage, but requires qualification for "hard" thermionic valve rectifiers and for metal rectifiers.

To avoid confusion, the transformer losses are neglected in what follows and the ratio is also assumed to be unity. The kVA rating established on a one-to-one voltage basis is obviously true for any other ratio, provided that the proportion of the space devoted to insulation is similar.

1. Single-phase Power Supply

When referring to a rectifier circuit as being so-many phase, it is important to distinguish between the number of phases in the supply and the type of circuit, because unfortunately the term "bi-phase" is sometimes used to describe a type of single-phase circuit in which both half-cycles are rectified. This will be described here as *full-wave* as opposed to *half-wave* in which only one half-cycle is used.

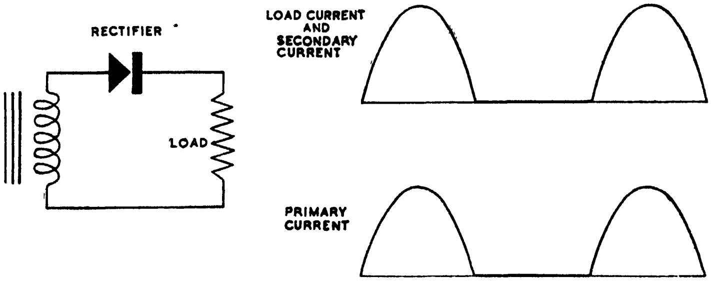

Fig. 76

The Half-wave Rectifier Circuit and Wave-forms of the Current in the Transformer Windings.

Four circuits are in common use for power rectification and these are known as half-wave, centre-tapped secondary, bridge, and voltage doubler. The last three are all full-wave.

Half-wave

This is the simplest possible rectifier circuit; the rectifier, load and transformer secondary are all connected in series (Fig. 76). The action of the ideal rectifier is to allow current to pass normally on one half-wave and to suppress it entirely on the other. The current thus consists of a series of loops of the form of a sine-wave over the angular interval 0 to π separated by intervals of zero output of equal length. If the load is resistive as assumed, the output voltage across this load will be of a similar wave-form. During the conducting period the instantaneous current i will be

$$i = \frac{E_{max} \sin \theta}{R_L} = \frac{\sqrt{2}E \sin \theta}{R_L}$$

where E is the r.m.s. value corresponding to E_{max}. The mean current over one cycle will be

$$I_{mean} = \frac{1}{2\pi}\int_0^{\pi} \frac{\sqrt{2}E \sin \theta}{R_L} d\theta$$

$$= \frac{\sqrt{2}}{\pi}\frac{E}{R_L} \quad . \quad . \quad . \quad . \quad (66)$$

The mean voltage across the resistive load will be

$$V_{mean} = \frac{\sqrt{2}}{\pi} \times E$$

The d.c. power in the load $W_{D.C.}$ is the product of the mean voltage and current, i.e.

$$\frac{\sqrt{2}}{\pi} \times \frac{\sqrt{2}}{\pi}\frac{E^2}{R_L} \quad . \quad . \quad . \quad (67)$$

In order to determine the size of transformer required to supply this rectified output, it is necessary to calculate the r.m.s. values of the currents in the windings because of course it is the r.m.s. value which determines the heating effect. The r.m.s. value of the half-wave current I_s will be

$$\sqrt{\frac{1}{2\pi}\int_0^{\pi}\left(\frac{\sqrt{2}E_s}{R_L}\sin\theta\right)^2 d\theta} = \frac{\sqrt{2}E_s}{2R_L} = \frac{\pi}{2} I_{mean} \quad . \quad (68)$$

Since the transformer has been assumed to have a unity ratio, the primary winding also will have to be capable of carrying this current. But not only have the conductors to be made larger in size than would be necessary for a normal load in the ratio of $\frac{\pi}{2}$ to one, but the turns have also to be increased in the ratio $\frac{\pi}{\sqrt{2}}$ to one. The frame size which will have to be used is therefore one which would normally supply

$$\frac{\pi}{\sqrt{2}} \times \frac{\pi}{2} W_{D.C.} = \frac{\pi^2}{2\sqrt{2}} W_{D.C.} = 3{\cdot}49 W_{D.C.} \qquad (69)$$

The transformer rating for this application is thus said to be 3·49 times the d.c. output. This does not mean, however, that the power absorbed from the mains by the transformer-rectifier combination will be 3·49 times the d.c. power in the load. The reading shown on a wattmeter would be the mean value of the product of current and voltage, i.e.

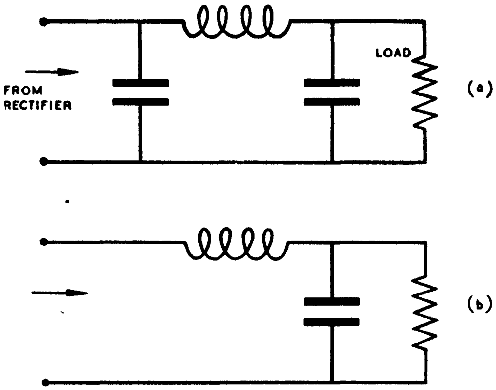

FIG. 77

Capacitor and Choke Filter Circuits for Smoothing the Output from a Rectifier.

$$\frac{1}{2\pi}\int_0^{\pi} \sqrt{2}E_s \sin\theta \times \frac{\sqrt{2}E_s}{R_L} \sin\theta d\theta = \frac{E_s^2}{2R_L}$$

$$= \frac{\pi^2}{4} W_{D.C.} = 2{\cdot}46 W_{D.C.} \quad . \quad . \quad . \quad (70)$$

It has, however, been assumed that there are no losses in the transformer or rectifier so that if $2{\cdot}46 W_{D.C.}$ watts are taken from the mains, they must appear in the load, although the d.c. power is only $W_{D.C.}$ The explanation of this apparent discrepancy is that in a circuit of this kind the remaining watts are in fact delivered to the load in the form of *ripple* volts and current. The ripple voltage contains high percentages of harmonics of the supply frequency.

In most rectifier applications, the presence of a large ripple in the load is most undesirable and *smoothing circuits* are used to separate the d.c. component from the ripple. An inductance may be connected in series with the load and a capacitance in parallel with it. The addition of these smoothing components affects the operating condition of the transformer, often to a marked extent. Two forms of smoothing filter are in common use, known as the choke-filter and capacitor-filter (Fig. 77), depending on which of the components immediately follows the rectifier. The effect of the choke-filter is to reduce the rating of the transformer required whilst the capacitor-filter increases the rating. This arises from the inherent properties of the choke and the capacitor. The former tends to maintain the output current constant by keeping the intake from the rectifier constant. In so doing, of course, it spreads the conducting period over the whole half-cycle and the transformer windings are fully utilized during that time. The capacitor on the other hand, maintains the output current by taking large pulses of charging current from the rectifier whenever the transformer secondary voltage exceeds the potential to which it is charged and then discharging itself through the load (Fig. 78). The duration of the transformer current is thus limited to the short periods near the peak of the output voltage. The windings are, therefore, idle much of the time and the r.m.s. to mean current ratio is high. In practice half-wave rectifiers are seldom used except for very small outputs where the rating is decided largely on the grounds of the mechanical strength of the conductor used rather than its current carrying capabilities. For this reason the effects of the two filters will not be dealt with in a quantitative manner. It is, however, worked out for the full-wave rectifier circuit which follows.

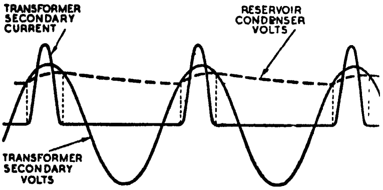
TRANSFORMER SECONDARY CURRENT
RESERVOIR CONDENSER VOLTS
TRANSFORMER SECONDARY VOLTS

FIG. 78

The Peaky Nature of the Secondary Current in a Capacitor Filter Circuit Results in a High r.m.s. Value Relative to the D.C. Output.

A serious disadvantage of the half-wave circuit is that there is a d.c. component in the windings which causes steady magnetization of the core. This cannot be eliminated by any arrangement of windings.

Full-wave: Centre-tapped Secondary

This circuit is in effect two half-wave circuits connected to a common load in such a way that current passes through the load in the same direction during both half-cycles (Fig. 79).

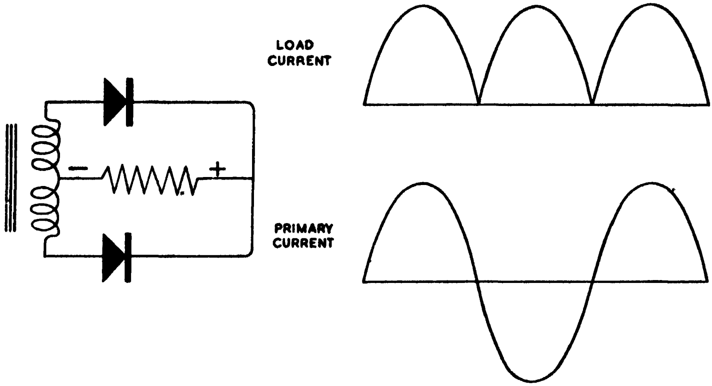

FIG. 79

The Full-wave Centre-tapped Circuit Frequently Used with Thermionic Rectifiers.

If it is assumed that the mean current through the load is the same as in the half-wave case, the mean current in each half-secondary will be only half that in the load. As each half-secondary only works over half the cycle, however, the ratio of r.m.s. to mean current *in that portion of the winding* will remain as before $\pi/2$ to one (equation 68). Related to the mean load current this will be $\pi/4$ to one. Hence

$$I_s = \frac{\pi}{4} I_{mean} = 0{\cdot}786 I_{mean} \qquad . \quad . \quad (71)$$

Since there are two current pulses instead of one, the voltage required *per half-secondary* will be only half that in the first case, but two half-windings are required so that the total secondary voltage bears the same relationship to the d.c. voltage V_{mean} as before.

The secondary rating is therefore—

$$2E_sI_s = 2\frac{\pi}{2\sqrt{2}}V_{mean} \times \frac{\pi}{4}I_{mean} = \frac{\pi^2}{4\sqrt{2}}W_{D.C.}$$

$$= 1{\cdot}745W_{D.C.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

The primary, unlike the half-secondaries, works on both half-cycles and the wave-form (in the perfect transformer which has been assumed) will be sinusoidal. This follows from the fact that the primary ampere-turns always oppose those in the particular half-secondary which is working. The r.m.s. primary current I_p is thus

$$I_p = \sqrt{2} \times \frac{\sqrt{2}E_s}{2R_L} = \sqrt{2}I_s = \frac{\pi}{2\sqrt{2}}I_{mean} = 1{\cdot}11I_{mean} \quad (73)$$

This gives a primary rating E_pI_p of

$$\frac{\pi}{2\sqrt{2}}V_{mean} \times \frac{\pi}{2\sqrt{2}}I_{mean} = \frac{\pi^2}{8}W_{D.C.} = 1{\cdot}23W_{D.C.} \quad (74)$$

The kVA rating for the whole transformer is thus $\frac{1{\cdot}74 + 1{\cdot}23}{2}$ $W_{D.C.}$, i.e. $1{\cdot}48W_{D.C.}$—a much better figure than that for a half-wave arrangement. The fact that current flows during both half-cycles in the full-wave circuit prevents any static magnetization of the core of a shell-type transformer. If a core-type is used, it is necessary to split *each half* of the secondary into two portions, one on each limb.

In arriving at the rating the effect of the smoothing circuits has been neglected, but in practice their effect on the required rating is quite marked. Taking first the choke-feed circuit, it is assumed that the smoothing choke has a very high inductance, so that any fluctuations in the current through the load are effectively smoothed out. This state of affairs is not, of course, attained in practice, but it is found that, provided the inductance does not fall below a certain critical value for the current it is passing, the results closely agree with those which would be obtained with an infinite inductance. Since the load current does not fluctuate, the total secondary current must also remain constant, and what happens is that the wave-form of the current in each half-secondary consists of a rectangular pulse occupying one half-cycle. The effect of the current changes which would tend to result from the falling transformer voltage at the end of the half-cycle is to induce in the inductance an e.m.f. which keeps the current flowing until the voltage of the other half-secondary reaches that across the load. Increases above the mean value as the output of this half-secondary continues to grow are prevented by the back e.m.f. induced in the choke. The result is that the load current snaps from one rectifier to the other every half-cycle. The rectangular current wave having a value of I_{mean} over half the cycle and zero over the other half, will have an r.m.s. value over the cycle of $\frac{1}{\sqrt{2}}$ of the value during the conducting half cycle.

The output voltage V_{mean} is related to the r.m.s. half-secondary voltage by the relationship

$$E_s = 1{\cdot}11\, V_{mean} \qquad (75)$$

The secondary rating is therefore

$$2 \times 1{\cdot}11 \times V_{mean} \times \frac{I_{mean}}{\sqrt{2}} = 1{\cdot}11 \times \sqrt{2} W_{D.C.} = 1{\cdot}57 W_{D.C.} \qquad (76)$$

The r.m.s. value of the primary current is equal to the mean value. This will be evident if it is remembered that the r.m.s. value of a wave is unchanged if the polarity of one half-cycle is reversed. If the square-wave primary current is reversed in this way, it becomes constant at the mean value. The r.m.s. primary voltage is 1·11 times the mean d.c. output voltage, so that the primary rating is 1·11 times the d.c. watts. The rating for the whole transformer is thus $\frac{1{\cdot}57 + 1{\cdot}11}{2} = 1{\cdot}34$. This will be seen to be rather better than that for the same arrangement used without a filter.

When a capacitor-filter is used to smooth the output, the secondary current depends very considerably on the impedance of the transformer and rectifier, both of which have been neglected so far. If they are very small, the peak value of the charging current into the "reservoir" or "integrating" capacitor is very high indeed, but in practice the impedances of these parts of the circuit are sufficiently high to limit the peaks considerably and the r.m.s. value of the current in *each half of the secondary* is usually about 1·2 times the mean current in *the load. This value is representative of small transformers up to a few hundred volt-amperes rating. (The capacitor-filter* circuit is not usually employed for larger powers because it does not utilize the transformer windings as fully as the choke-filter type and hence requires larger frame sizes.) The behaviour of capacitor-filter rectifier circuits has been examined in considerable detail by Schade,(1) who gives charts from which the peak and r.m.s. values of the currents through the rectifier and windings can be calculated, not only for the centre-tapped full-wave circuit but also for the half-wave and for the voltage-doubling circuits. These charts show the ratio of r.m.s. to mean current (in each rectifier) for various values of the quantity $n\omega CR_L$ in which n is 1 for a half-wave, 2 for a full-wave and $\frac{1}{2}$ for a voltage-doubling circuit, C is the reservoir capacitance in farads and R_L the d.c. load resistance. Different curves are shown for various proportions of transformer + rectifier resistance to nR_L. For example, consider a small transformer supplying a full-wave rectifier delivering 60 mA D.C. at 360 volts. The value of R_L is 6 000 ohms and the equivalent resistance of the half-secondary of the transformer referred to the secondary will be about 300 ohms. Taking the rectifier as 150 ohms, the percentage becomes $\frac{300 + 150}{2 \times 6\,000} \times 100$, i.e. approximately $3\frac{1}{2}$ per cent. If the reservoir capacitor is 16 μF, the value of $n\omega CR_L$ is 60, and the corresponding ratio of r.m.s. to mean current is read off as 2·5. The mean current through each half-secondary is 30 mA, so that the r.m.s. current is 75 mA. This figure accords well with the figures found in practice and mentioned above, namely 1·2 × 60 mA = 72 mA.

The ratio of r.m.s. to mean current does not vary much with quite considerable changes in the value of $n\omega CR_L$; in fact, increases in its value have practically no effect at all. Variations in the proportion of internal resistance to $n \times R_L$ have more effect, but even this can vary from 1 to 25 per cent without the ratio of r.m.s. to mean current per half-secondary exceeding the limits 2 to 3 at the particular value of $n\omega CR_L$.

With regard to the value of the a.c. voltage required from the transformer to produce a given d.c. voltage across R_L, this ratio too depends on the proportion of transformer plus *rectifier resistance to load resistance as well as the value of the* reservoir capacitor. For the conditions in the example examined above, Schade's curves give a figure of 80 per cent for the proportion of d.c. voltage to peak a.c. voltage. As a figure of approximately 71 per cent would have meant that the r.m.s. and d.c. voltages were equal, it will be seen that the d.c. output voltage from the circuit is rather greater than the r.m.s. value of the half-secondary a.c. voltage. This result is also well borne out in practice, for it usually happens that the r.m.s. secondary voltage required per half-secondary is rather smaller than the d.c. output voltage. In these conditions the secondary rating is approximately

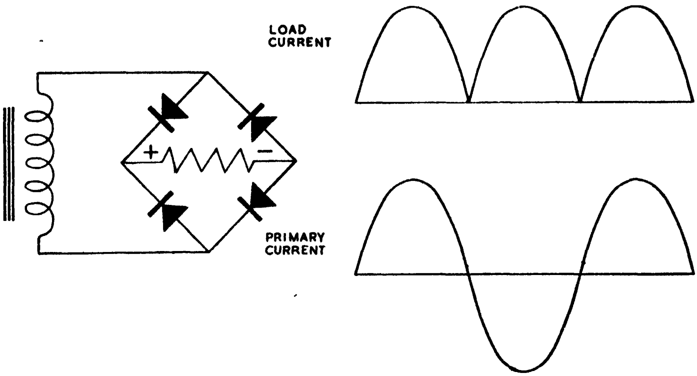

FIG. 80
Full-wave Bridge Circuit Suitable for Metal Rectifiers and Other Cold-cathode Types.

$$2V_{mean} \times 1{\cdot}2I_{mean} = 2{\cdot}4W_{D.C.} \quad . \quad . \quad (77)$$

The primary rating is $\frac{1}{\sqrt{2}}$ of the secondary, i.e. $1{\cdot}7W_{D.C.}$, so that the rating of the transformer would be $2{\cdot}1W_{D.C.}$

Full-wave: Bridge

This circuit uses an untapped secondary feeding a group of four rectifiers, as shown in Fig. 80. The current passes in series through two of the rectifiers during one half-cycle and through the other two on the other half-cycle. As far as the transformer itself goes, the circuit is a good one, making the most use of the windings, but in practice the voltage drop in the two rectifiers in series may be a disadvantage. As the rectifiers have been assumed to have negligible forward resistances, this effect is not brought out in the rating figures.

The whole secondary passes current on both half-cycles and in the absence of smoothing filters, the current is sinusoidal. The r.m.s. voltage and current in the secondary are both related to the corresponding d.c. quantities by the constant $\frac{\pi}{2\sqrt{2}}$. The secondary rating is thus

$$\frac{\pi}{2\sqrt{2}} V_{mean} \times \frac{\pi}{2\sqrt{2}} I_{mean} = \frac{\pi^2}{8} W_{D.C.} = 1{\cdot}23 W_{D.C.} \qquad (78)$$

The primary current is of the same shape and magnitude as the secondary, so that the primary rating and the rating of the whole transformer is $1{\cdot}23 W_{D.C.}$

When used with a choke-filter, this rating figure is slightly improved to a still better figure. The action of the choke is, as in the previous case, to maintain the output current constant so that it is assumed to change over instantaneously from one rectifier path to the other each half-cycle. The secondary current is thus alternating and of a rectangular wave-form. Its r.m.s. value is, of course, equal to I_{mean}. The secondary voltage is still up on the d.c. voltage in the ratio of $\frac{\pi}{2\sqrt{2}}$ to one, so that the secondary rating, and therefore the primary and total rating, is $1{\cdot}11 W_{D.C.}$

A capacitor filter, on the other hand, increases the rating in a similar way to that of the centre-tapped circuit. The same considerations of rectifier and transformer impedance and capacitance value apply, so that the actual rating is greatly dependent on actual conditions. If these are assumed to be similar to those in the centre-tapped arrangement, the secondary rating is better in the proportion of $\frac{1}{\sqrt{2}}$ to one. Since the current pulses occur in the same winding in each half-cycle, instead of alternately in two separate windings, the r.m.s. secondary current is increased in the ratio of $\sqrt{2}$ to one, but as there are only half the number of turns the overall effect is to reduce the required rating by $\frac{\sqrt{2}}{2}$, i.e. $\frac{1}{\sqrt{2}}$ to one. The primary rating is unchanged from the corresponding centre-tapped system with a capacitor filter.

Full-wave: Doublers

The action of the conventional doubler circuit (Fig. 81*a*) is for each capacitor to charge in turn from the transformer secondary; one capacitor charges on one half-cycle and the other on the other half-cycle. The two capacitors are connected in series as regards the discharge circuit so that the output voltage is of the order of twice the input voltage.

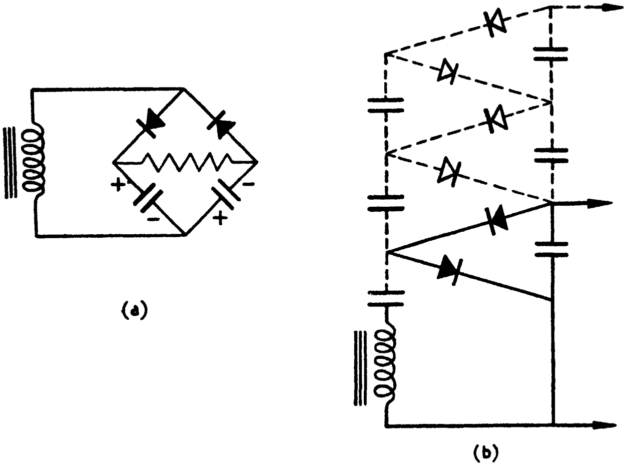

FIG. 81

Voltage-doubler Circuits. (*a*) Conventional Arrangement. (*b*) Cascade Type Capable of Indefinite Extension.

The circuit is inherently of the capacitor filter type and the output voltage and transformer currents are dependent on the size of the capacitors and the rectifier and transformer impedances. The rating of the transformer is similar to that required for a bridge rectifier.

Another type of doubler connexion is shown in Fig. 81*b*. Apart from the feature of having one terminal common with a terminal of the supply, this circuit has the advantage that it can be extended indefinitely by the use of more rectifiers and capacitors in cascade so that very high voltages can be built up. *The current supplying capabilities decrease rapidly as the number of stages is increased, but for some purposes* this may not matter. The chief application is in extra high voltage insulation-testing plants or for research purposes.

2. Three-phase Supply

The behaviour of single-phase rectifier circuits has been analysed when used without filters, as well as in the practical cases of capacity and inductance smoothed arrangements. The unsmoothed cases, however, have been only of academic interest because the ripple voltage on even a full-wave output from a single-phase supply is so high as to make filtering essential for almost any application. A filter is not required for accumulator-charging purposes, but even in that case the analysis is not applicable, because current flows only during the small portion of the cycle during which the transformer e.m.f. exceeds the accumulator voltage. The r.m.s. value of the current is then high relative to its mean value and the transformer rating becomes similar to that required for a capacity-filtered rectifier. These remarks do not, however, apply to many of the three- and six-phase circuits which will now be described, because the instantaneous value of the output voltage never falls to zero, and it is possible to dispense with smoothing circuits completely in some three-phase and many six-phase applications. If twelve phases are employed, the need for filtering becomes even less frequent.

These arrangements of rectifiers used without smoothing filters are frequently adopted in distribution systems giving d.c. from a.c. bulk supplies. This aspect is dealt with in some detail by S. A. Stigant and H. M. Lacey(2) in *The J. & P. Transformer Book*, from which the analyses of ratings and wave-forms in this chapter are reproduced by kind permission of Messrs. Johnson & Phillips, Ltd.

The most important application of rectifiers in low-power work is the supply of d.c. voltages for the anode circuits of thermionic valves. For valves of the receiving class, single-phase arrangements would always be used, but for transmitting apparatus or for industrial oscillators for radio-frequency heating purposes three-phase circuits enable considerable savings in smoothing gear to be effected. For powers greater than a few kilowatts, three-phase is, of course, necessary in any case to avoid serious unbalancing of the lines. In radio-frequency heating apparatus the smoothing filters can often be omitted altogether, particularly if six-phase or twelve-phase rectification is employed, and some of the following arrangements will be found suitable for the purpose.

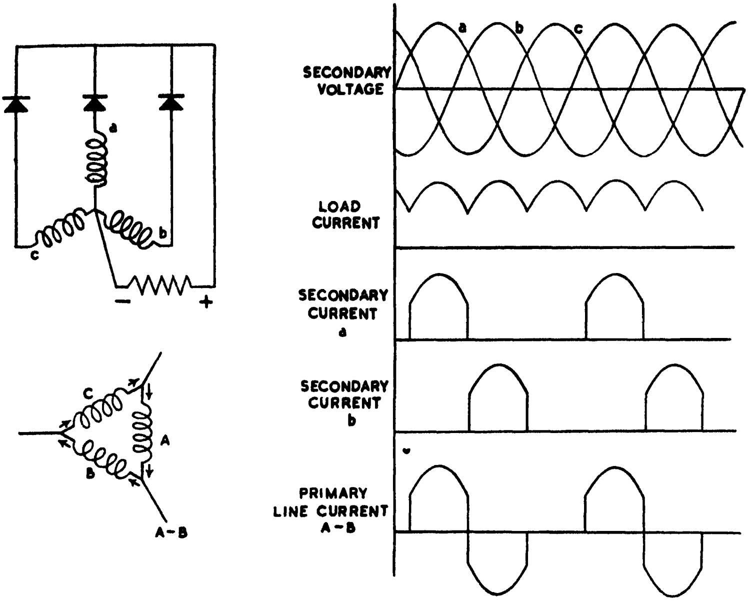

Fig. 82

Star-connected Three-phase Circuit and Wave-forms of Secondary Voltage and Current, and Primary Line Current.

(*a*) *Three-phase Star*

The simplest example of three-phase rectification is shown in Fig. 82. This corresponds to half-wave rectification of a single phase and like that arrangement causes steady magnetization of the core.

In this circuit, as in all polyphase circuits of this kind, only one rectifier is conducting at the same time, according to the phase which is at the moment at its peak positive value. As this phase voltage falls to the point at which it becomes equal to that of the next in sequence, its rectifier ceases to conduct and the rectifier connected to the next phase takes over. Since the arrangement is symmetrical, each phase passes current for one-third of the cycle. The portion of the period over which a phase is conducting is that between $\frac{\pi}{6}$ and $\frac{5\pi}{6}$, i.e. over $\frac{2\pi}{3}$. The mean value of the output voltage V_{mean} is given by—

$$V_{mean} = \frac{1}{\frac{5\pi}{6} - \frac{\pi}{6}} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sqrt{2}E_s \sin \theta d\theta = \frac{3\sqrt{6}}{2\pi} E_s \quad . \quad (79)$$

The load current, $$I_{mean} = \frac{3\sqrt{6}E_s}{2\pi R_L} \quad . \quad . \quad . \quad (80)$$

The r.m.s. current in each phase I_s

$$= \sqrt{\frac{1}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\frac{\sqrt{2}E_s \sin \theta}{R_L}\right)^2 d\theta}$$

$$= \frac{\sqrt{2}E_s}{R_L} \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi}}$$

$$= \frac{2\pi}{3\sqrt{3}} I_{mean} \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi}} = 0{\cdot}587 I_{mean} \quad . \quad (81)$$

The secondary rating (total for all three phases)

$$= 3E_sI_s = 3 \times \frac{2\pi}{3\sqrt{6}} V_{mean} \times 0{\cdot}587 I_{mean}$$

$$= \frac{2\pi}{\sqrt{6}} \times 0{\cdot}587\ W_{D.C.} = 1{\cdot}503 W_{D.C.} \quad . \quad (82)$$

On the primary side, the connexions must be of the delta form, so permitting each phase to take its current directly from the lines without flowing through the windings of other phases as would be the case if a star connexion were used. As only one rectifier is conducting at a time, there are no secondary ampere-turns on the two non-operative phases and the flow of current through these windings would cause a serious unbalancing of the line-to-neutral voltages; a star connexion is, therefore, inadmissible.

The primary current is similar to the secondary and the primary rating is, therefore, the same as that of the secondary. The rating of the transformer is thus $1{\cdot}503 W_{D.C.}$ The primary *line current* of each phase is made up of positive and negative pulses each similar to those in the secondaries. The r.m.s. value of the current is clearly $\sqrt{2} I_s$, i.e. $0{\cdot}829 I_{mean}$.

Owing to the serious difficulty of d.c. magnetization of the core, this circuit is seldom used and details of operation with a filter are not given here.

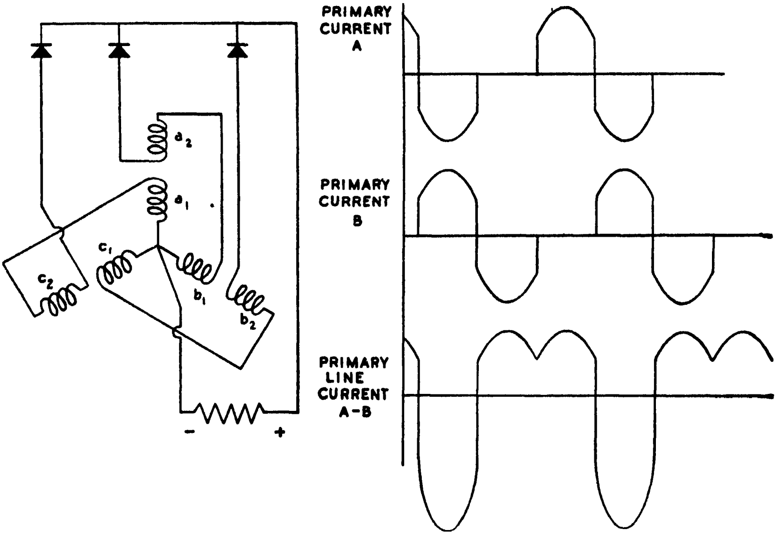

FIG. 83

Interconnected-star Circuit and Wave-forms. This Arrangement Avoids the Steady Magnetization of the Core which Occurs in the Plain Star Circuit. The Load Current (not shown) is Similar to that in Fig. 82. The Primary Line Current is shown for a Delta Primary.

(b) *Three-phase: Interconnected Star*

The difficulty of d.c. magnetization of the core which occurs in the plain star arrangement is avoided in the interconnected-star circuit because each rectifier voltage is obtained from two windings connected in series-opposition and arranged on different limbs (Fig. 83). Each limb, therefore, carries coils passing D.C. in opposite directions round it, and over the cycle there is no resultant d.c. magnetization. Since the two sections of each rectifier winding are in different phases, the resultant voltage obtained by connecting the two in series-opposition is $\sqrt{3}$ times the voltage of each section. In order therefore to obtain the same secondary voltage as in the plain star arrangement both sections must have $1/\sqrt{3}$ times as many turns. Since the secondary current is the same as in the star connexion, the secondary rating is $2/\sqrt{3}$ times as great, i.e. $1{\cdot}735W_{D.C.}$. Since the current to each rectifier is derived from two phases there are secondary ampere-turns in two phases simultaneously and the star primary connexion as well as the delta can be used. If a star arrangement is adopted the primary current in each winding consists of a pulse in the positive direction corresponding to the current in one rectifier followed by a similar pulse in the opposite direction due to the next rectifier. The magnitude of the pulses is not, however, so great as those in the secondaries, because the turns on each half-secondary are $1/\sqrt{3}$ of those on the primary. Each primary pulse is accordingly similar in shape, but only $1/\sqrt{3}$ of the secondary pulse. Since there are two such pulses (one positive and one negative) per cycle, the r.m.s. primary current is $\sqrt{2}I_s/\sqrt{3}$ and the primary rating is lower than that for the plain star connected circuit in the ratio of $\frac{\sqrt{2}}{\sqrt{3}}$, i.e. $1{\cdot}227W_{D.C.}$

The rating of the transformer is thus

$$\frac{1{\cdot}735 + 1{\cdot}227}{2} W_{D.C.} = 1{\cdot}48W_{D.C.} \quad . \quad . \quad (83)$$

If a choke filter is used in conjunction with this circuit and the choke is of adequate inductance, the current wave-form in each rectifier and secondary winding approaches a rectangular form of amplitude equal to the d.c. output and one-third of a cycle in duration.(3) As before, the load current appears to transfer itself instantaneously from one rectifier to the next as the peak voltage is approached in each phase. The secondary currents have clearly an r.m.s. value of $I_{mean}/\sqrt{3}$, whilst the voltage is as before $2E_s/\sqrt{3}$. The secondary rating (for three phases) is therefore

$$3 \times \frac{2}{\sqrt{3}} \times \frac{2\pi}{3\sqrt{6}} V_{mean} \times \frac{1}{\sqrt{3}} I_{mean} = \frac{4\pi}{3\sqrt{6}} W_{D.C.} = 1{\cdot}71W_{D.C.} \quad . \quad . \quad . \quad (84)$$

The r.m.s. value of the primary current bears the same *ratio* to the secondary current as it does in the absence of smoothing and is therefore $\frac{\sqrt{2}}{\sqrt{3}} \times \frac{I_{mean}}{\sqrt{3}}$, i.e.

$$0{\cdot}471 I_{mean} \quad . \quad . \quad . \quad (85)$$

The primary rating is therefore

$$3E_p I_p = 3 \times \frac{2\pi}{3\sqrt{6}} V_{mean} \times \frac{\sqrt{2}}{3} I_{mean} = \frac{2\pi}{3\sqrt{3}} W_{D.C.}$$

$$= 1{\cdot}21 W_{D.C.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

The rating for the whole transformer is

$$\frac{1{\cdot}71 + 1{\cdot}21}{2} W_{D.C.}, \text{ i.e. } 1{\cdot}46 W_{D.C.} \quad . \quad . \quad (87)$$

(*c*) *Six-phase Star*

This circuit is the three-phase counterpart of the centre-tapped secondary form of full-wave single-phase circuits. Instead of each secondary winding being single as in the three-phase star, it is centre-tapped; there are thus six terminals each differing in phase by one-sixth of a cycle (Fig. 84). The conducting period of each rectifier is now only one-sixth of a cycle so that the secondary windings are less satisfactorily utilized, but the fluctuations in output voltage are much less and the output can often be used without smoothing for d.c. main supplies. The primary windings carry no d.c. component, so it is unnecessary to use interconnected secondaries to avoid static magnetization of the core.

The d.c. output voltage is related to the secondary voltage in the following way—

$$V_{mean} = \frac{1}{\frac{2\pi}{6}} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{2} E_s \sin\theta \, d\theta = \frac{3\sqrt{2}}{\pi} E_s \quad . \quad (88)$$

The r.m.s. value of the secondary current I_s is

$$\sqrt{\frac{1}{2\pi} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(\frac{\sqrt{2}E_s \sin\theta}{R_L}\right)^2 d\theta} = \frac{\sqrt{2}E_s}{R_L} \sqrt{\frac{1}{12} + \frac{\sqrt{3}}{8\pi}}$$

$$= \frac{\pi}{3} \sqrt{\frac{1}{12} + \frac{\sqrt{3}}{8\pi}} I_{mean} = 0{\cdot}408 I_{mean} \quad . \quad . \quad (89)$$

The secondary rating (all phases) is given by $6E_sI_s$

$$= 6 \times \frac{\pi}{3\sqrt{2}} V_{mean} \times \frac{\pi}{3}\sqrt{\frac{1}{12} + \frac{\sqrt{3}}{8\pi}}\, I_{mean} = \frac{\pi^2}{3}\sqrt{\frac{1}{6} + \frac{\sqrt{3}}{4\pi}}\, W_{D.C.}$$

$$= 1{\cdot}81 W_{D.C.} \quad . \quad . \quad . \quad . \qquad (90)$$

A delta connexion is essential on the primary side because, as in the three-phase star, there are secondary ampere-turns in only one phase at a time. The primary current consists of a positive and a negative pulse each equal to those in the secondaries, and the r.m.s. value is $\sqrt{2}$ times as great.

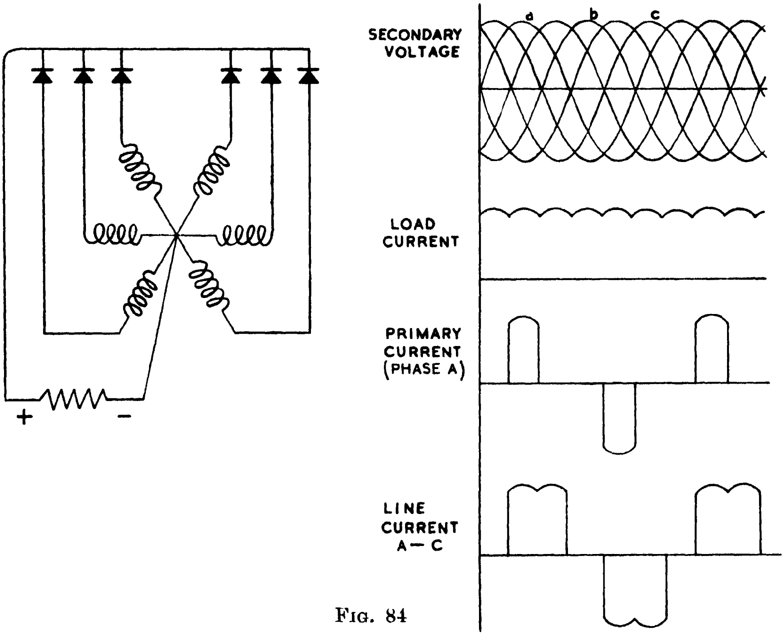

FIG. 84

Six-phase Star Arrangement. The Ripple on the Load Current is much Smaller than that in a Three-phase Star Arrangement.

The primary rating $3E_pI_p$ is therefore

$$3\,\frac{\pi}{3\sqrt{2}} V_{mean} \times \frac{\sqrt{2}\pi}{3}\sqrt{\frac{1}{12} + \frac{\sqrt{3}}{8\pi}}\, I_{mean}$$

$$= \frac{\pi^2}{3}\sqrt{\frac{1}{12} + \frac{\sqrt{3}}{8\pi}}\, W_{D.C.} = 1{\cdot}28 W_{D.C.} \qquad (91)$$

Hence the transformer rating is—

$$\frac{1{\cdot}81 + 1{\cdot}28}{2} W_{D.C.} = 1{\cdot}55 W_{D.C.} \quad . \quad . \quad (92)$$

The effect of a choke-filter smoothing circuit on the r.m.s. value of the secondary current, and hence on the transformer rating, is negligible in this case. This will be readily understood when it is remembered that the wave-form in the unsmoothed arrangement (a portion of a sine wave near its peak) closely resembles a rectangle of the same width as that which is found when a choke-filter is used. The r.m.s. value of the rectangular wave-form is, of course, $I_{mean}/\sqrt{6} = 0{\cdot}408 I_{mean}$. This is, to three significant figures, equal to that already derived (equation 89) for the portion of a sine wave one-sixth of a cycle wide. That is not surprising for even in the three-phase interconnected star circuit the difference in r.m.s. values of the sine-wave and rectangular forms was quite small.

(*d*) *Six-phase: Single Star*

The use of six rectifiers supplied in pairs from a single three-phase star winding corresponds to the bridge circuit for a single phase, and it is often called the three-phase bridge connexion (Fig. 85). Two rectifiers are always in series as in the single-phase bridge. This circuit utilizes the transformer capacity more fully than any of the other circuits described here

Consider the instant at which the phase-to-neutral voltage C is zero. Phases A and B will then have $\frac{\sqrt{3}}{2} E_{max}$ volts positive and negative respectively with respect to the neutral point. Seen by rectifiers 1 and 1′, however, these two voltages have a resultant value of $\sqrt{3}\, E_{max}$ round the load circuit and they conduct. This they continue to do for one-sixth of the cycle by which time phase B has fallen to zero and C has grown to $\frac{\sqrt{3}}{2} E_{max}$. Rectifier 1 continues to conduct, but on the return side the action changes from 1′ to 2′. The resultant e.m.f. round the circuit now increases again, reaching a peak and falling to the initial value again one-third of a cycle from the start. At this point rectifier 1 ceases to conduct and hands over to rectifier 2. The secondary current in phase *A* consists of a double pulse in a positive direction through rectifier 1, a blank interval of one-sixth of a cycle, and then a negative double pulse from rectifier 3′ again followed by a blank interval for the remaining sixth of a cycle. For simplicity it will be assumed that the current pulses are rectangular in form as they would in fact be if a choke-filter circuit were in use. Even operating without it, however, the portions of sine waves so closely resemble rectangular forms that for practical purposes no error results from this assumption. The relationship between the secondary voltage and the mean d.c. voltage is derived in a similar fashion to that in a six-phase star circuit except that it is $\sqrt{3}$ times as great. Although each rectifier conducts for one-third of a cycle, the wave-form passed is not a single pulse, but two sixth-cycle pulses in succession. The factor $\sqrt{3}$ appears because the

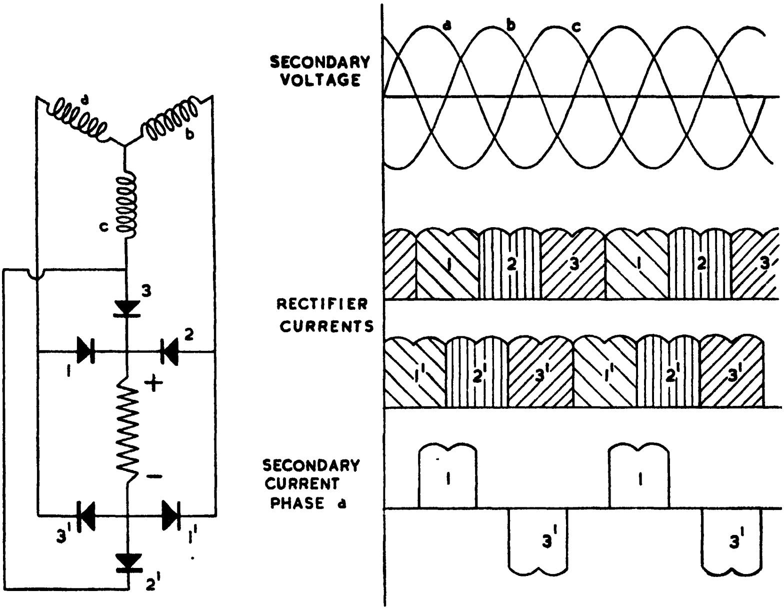

FIG. 85

Six-phase Single-star (or Three-phase Bridge). Like the Single-phase Bridge, it is Most Used for Metal Rectifiers.

d.c. output is no longer taken from the neutral point but is obtained from two phase voltages in series. Hence

$$V_{mean} = \frac{1}{\frac{2\pi}{6}} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{6}E_s \sin\theta d\theta = \frac{3\sqrt{6}}{\pi} E_s \quad . \quad (93)$$

On the assumption of a rectangular current wave-form, the r.m.s. value of the secondary current I_s is

$$\frac{\sqrt{2}I_{mean}}{\sqrt{3}} = 0{\cdot}816 I_{mean} \quad . \quad . \quad . \quad (94)$$

The secondary rating $3E_sI_s$

$$= 3 \times \frac{\pi}{3\sqrt{6}} V_{mean} \times \frac{\sqrt{2}}{\sqrt{3}} I_{mean} = \frac{\pi}{3} W_{D.C.}$$

$$= 1{\cdot}05 W_{D.C.} \quad . \quad . \quad . \quad (95)$$

Since the neutral point is not used, the secondary can also be delta-connected without affecting the performance, although of course the currents and voltages are different. The rating is unaltered, however. The primary can also be either star or delta and the rating is the same as that of the secondary. The rating of the transformer as a whole is thus $1{\cdot}05W_{D.C.}$

(e) *Six-phase: Double Three-phase Series Circuit*

This circuit has no single-phase equivalent, because no phase displacement (other than 180°) is possible in a single-phase system. In a three-phase circuit two separate sets of star windings can be arranged so as to provide a six-phase supply. Unlike the six-phase star circuit, however, the two star points are not connected together. Each three-phase star supplies three rectifiers independently of the other, and the d.c. outputs are connected in series (Fig. 86*a*).

Since the secondaries operate as three-phase star circuits, the secondary rating is the same as that of the simple star, viz. $1{\cdot}50W_{D.C.}$ without smoothing or $1{\cdot}48W_{D.C.}$ in a choke-filter circuit. The primary, on the other hand, operates more effectively than in the three-phase arrangement and passes four current pulses each of one-sixth of a cycle in length. Its rating is, therefore, the same as that of the three-phase bridge circuit primary, i.e. $1{\cdot}05W_{D.C.}$ The result is practically unaffected by the use of a choke-filter. The primary connexion can be star if so desired, provided that a single three-phase unit is used. The transformer rating is thus $\frac{1{\cdot}48 + 1{\cdot}05}{2} W_{D.C.}$, i.e. $1{\cdot}26 W_{D.C.}$ (for choke-filter output).

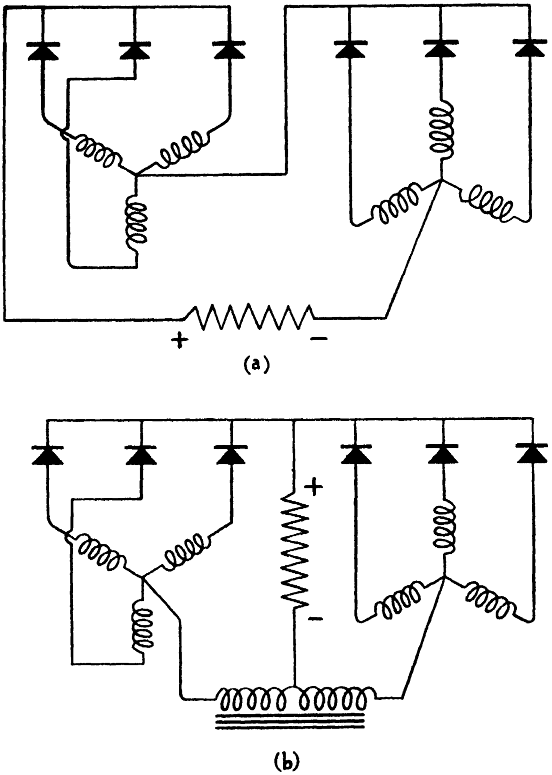

FIG. 86

Six-phase Double-star Circuits. (*a*) Outputs of Two Star Circuits in Series. (*b*) Outputs in Parallel Through a Centre-tapped Reactor.

(*f*) *Six-phase: Double Three-phase with Balance Coil*

Instead of connecting the d.c. sides of the two independent star circuits of section (*e*) in series, they can be paralleled at a point after the smoothing chokes, i.e. each half must have a filter choke. The same result, however, can be obtained at a much lower cost by using one centre-tapped choke instead of two separate ones (Fig. 86*b*). There is then a cancellation of the d.c. component in the flux and the ripple frequency is also doubled. If the choke is short-circuited the arrangement reduces to the six-phase star. The ratings for this circuit are the same as those given for the corresponding series connexion (see (*e*) above).

3. FACTORS GOVERNING THE SELECTION OF A CIRCUIT

Although the rating figures derived in Sections 1 and 2 show clearly that certain arrangements of transformer and rectifier allow smaller transformers to be used than others, this is in fact not the whole story. Not only must the relative costs of rectifiers and transformers be considered (for in some cases the smaller transformer necessitates more rectifiers) but the different kinds of rectifiers, high-vacuum, selenium-disc, pool and hot-cathode mercury-vapour types may each be suited to different circuits.

The thermionic high-vacuum rectifier is for instance essentially a high-voltage type; its impedance is relatively high but it will stand a very high inverse voltage. The centre-tapped full-wave circuit is accordingly more suitable for this rectifier than the bridge. The bridge, however, is usually better for the selenium type, each disc of which can support only a relatively small inverse voltage, and numbers have in any case to be used in series for outputs above about 15 volts, so that the same number of discs would be used for either circuit and the bridge requires a smaller transformer.

Mercury-vapour rectifiers of both pool and hot-cathode types have voltage drops of the order of 15 volts, and this is independent of the load current. They are suitable for high currents at quite high voltages.(4, 5) The pool type is more suitable for applications where short-circuits are liable to happen, because their limiting output is determined chiefly by heating considerations so that momentary heavy overloads are not likely to do much harm. The hot-cathode type, on the other hand, is limited on instantaneous peak-current, for this must not exceed the emission of the filament. If this were attempted the voltage drop would rise sharply and the positive ions would bombard the cathode with greater velocities and destroy the emissive coating. The pool type, however, must be kept always

TABLE XII

Ratings for Transformers for Rectifier Circuits

	Half-wave	Centre-tapped	Bridge	3-phase Star	3-phase Inter-star	6-phase Star	3-phase Bridge		3-phase Balance Coil
							Star sec.	Delta sec.	
(*a*) *Without Filters*									
Sec. voltage . .	2·22	1·11	1·11	0·86	0·99	0·74	0·43	0·74	—
Sec. current . .	1·57	0·79	1·11	0·59	0·59	0·41	0·82	0·47	—
Sec. kVA . . .	3·49	1·75	1·23	1·50	1·74	1·81	1·05	1·05	—
Prim. voltage . .	2·22	1·11	1·11	0·86	0·86	0·74	0·43	0·74	—
Prim. current . .	1·57	1·11	1·11	0·59	0·48	0·58	0·82	0·47	—
Prim. kVA . .	3·49	1·23	1·23	1·50	1·23	1·28	1·05	1·05	—
Mean kVA . .	3·49	1·48	1·23	1·50	1·48	1·55	1·05	1·05	—
(*b*) *With Choke Filters*									
Sec. voltage . .	Not	1·11	1·11	0·86	0·99	0·74	0·43	0·74	0·86
Sec. current . .	used	0·71	1·00	0·58	0·58	0·41	0·82	0·47	0·29
Sec. kVA . .		1·57	1·11	1·48	1·71	1·81	1·05	1·05	1·48
Prim. voltage . .		1·11	1·11	0·86	0·86	0·74	0·43	0·74	0·86
Prim. current . .		1·00	1·00	0·47	0·47	0·58	0·82	0·47	0·41
Prim. kVA . .		1·11	1·11	1·21	1·21	1·28	1·05	1·05	1·05
Mean kVA . .		1·34	1·11	1·35	1·46	1·55	1·05	1·05	1·26

conducting in one anode or another so that the arc is not extinguished but simply passed round from one anode to the next. At least two and preferably three anodes are necessary to ensure this. The otherwise very effective three-phase bridge circuit is thus seen to be unsuitable for the pool-type rectifier because several separate ones would be required, each having only one anode.

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CHAPTER XIV

High-voltage Types

THE term "High-voltage" when used in this chapter must be taken to mean high relative to the kVA rating. A 3 000-kVA transformer, delivering 66 kV or even 132 kV for instance, would not be considered in this category, whilst an X-ray transformer delivering only 100 kV would definitely come within the scope of the chapter. The distinction arises from the fact that in the small transformer rated at only 10 kVA (for a 100-mA output) and probably intermittently at that, special insulation arrangements have to be made, whereas in the case of the 3 000-kVA type the size of the oil-cooling ducts, etc., has to be considerable whatever the voltage and no unusually great care is necessary.

The design of transformers intended to work at high voltages differs from that of normal types chiefly in the methods of insulating the windings. Apart from some small portable transformers where weight saving is of the highest importance, almost all high-voltage types are oil-immersed to improve insulation and prevent corona, in spite of the fact that the oil is not usually necessary in small sizes for cooling purposes. Insulation breakdowns in transformers working within their rated capabilities seldom occur through the sudden puncturing of the insulating material, but follow gradual deterioration at some point caused either by the ingress of moisture or by corona discharges. Oil-immersion is of great value in minimizing the chances of trouble from either of these causes, because it is self-sealing, whereas a wax-impregnated winding may develop minute cracks from repeated heating and cooling, or from rough handling. Once such a crack has been made the so-called "silent discharge," i.e. corona, is likely to take place, with the production of ozone and nitrogen oxides which rapidly attack many insulating materials. Such a process is cumulative and continues until a spark-over occurs and serious damage is done. Rubber deteriorates extremely rapidly when in the neighbourhood of even slight corona discharges and should never be used for insulating leads, etc., in a high-voltage transformer. Polyvinyl-chloride or polythene-covered wires are quite satisfactory in this respect and should always be used where flexible coverings are required. Care must be taken, however, to avoid softening them by high working temperatures.

1. The Use of Immersion Fluids

As already noted, fluid dielectrics offer considerable advantages over solid materials, such as wax, varnish or bitumen, in that they are self-healing if momentarily punctured by voltage surges and are not subject to cracking troubles. The fluids in general use are mineral oils, non-inflammable liquids such as "Pyranol," and inert gases, usually under pressure.

(*a*) *Oil*

Details of the properties and behaviour of transformer oil have been given in Chapter III.

The quality of an oil is judged by the voltage required to cause breakdown between two 13 mm diameter spheres separated by a distance of 4 mm (0·157 in.). The figure called for in B.S.S. 148, is not less than 30 kV r.m.s., although figures up to 70 kV can be obtained on specially dried samples. It should perhaps be reiterated that the goodness or otherwise of a sample of oil, as regards breakdown voltage, is determined much more by its moisture content than by any other factor.(1) Thus an oil giving a figure of 60–70 kV when dry, fails at little over 12 kV if allowed to absorb only 0·1 per cent of water. Finely-divided carbon appears to have little effect on the breakdown voltage, so that flash-overs under the oil, unless very prolonged, do little harm to its insulating properties. Fibrous particles have a bad effect because they tend to line themselves up along the direction of the electrostatic field and so form conducting chains. Here again it is believed that moisture in the form of a film over the fibres is the real cause of deterioration because carefully dried particles of this kind seem to have little effect. Since moisture cannot be completely excluded in practice, however, it is wise to avoid introducing loose fibrous matter more than can be helped.

From these remarks it will be evident that extreme care is necessary to avoid introducing moisture when filling a transformer with oil. First of all the tank, the fittings, and the windings must be carefully dried out, then the oil drum must not be left open to the air unnecessarily and finally the tank must be closed up as quickly as possible after filling. It is desirable that the container should be hermetically sealed, but this involves the problem of expansion of the oil when the temperature rises.

The process of filling is done *in vacuo*. Small transformers can be impregnated with oil in the same way as with molten wax, i.e. in a chamber from which the air can be exhausted, the oil being admitted after a good vacuum has been maintained for a sufficiently long period. The transformer can then be transferred, under the surface of the oil to its own container. Alternatively the windings, placed in position in their cases, can be placed in the vacuum chamber and, after removing the air, oil can be conducted into each container by a separate tube. Larger transformers have to be dealt with by exhausting their own tanks and admitting the oil later by a pipe through the top cover. As the case is usually either cast or fabricated from boiler-plate, it is generally quite capable of withstanding atmospheric pressure, but this requirement should be taken into account when the case is being designed.

The separation required between conductors under oil depends very considerably on the state of the oil and the shape of the conductors, in particular whether they have sharp edges or points. Although less stable than in air, corona does take place from sharp edges under oil(2) and this state of affairs precedes complete breakdown. In the dark a flickering glow is often visible at the points concerned, but even in daylight a disturbance of the liquid is evident before puncture takes place. Some warning is usually obtained in this way before an actual breakdown occurs, but it would of course be very unwise to design on such clearances that even with the oil in a seriously deteriorated condition a state of corona discharge is approached. It is perhaps fortunate that oil in poor condition shows less reduction in breakdown voltage for points relative to spheres than does good oil. In other words, as oil gets old or damp, sharp edges becomes less likely to start breakdowns through the oil.

TABLE XIII

BREAKDOWN VOLTAGES OF IMPURE TRANSFORMER OIL

(Interpolated from data collected by S. Whitehead, *Dielectric Phenomena*, Vol. II)

SPACING (in.)	B.D.V.* kv r.m.s.
0·05	3
0·10	8
0·15	12†
0·20	16
0·25	20
0·30	23
0·35	26
0·40	28

Working on the lines that design clearances should be based on the use of oil in a poor state, Table XIII shows the separation at which breakdown occurs for different voltages in an oil which has a 20 kV figure for a 0·15 in. standard sphere gap. A factor of safety must, of course, be applied to these figures to determine the actual separation necessary between the points in question. In the absence of surges the factor can be of the order of 3 times, but unless precautions are taken in switching off gradually, this may well be insufficient. As the voltage induced at the instant of switching off is proportional to the rapidity of break of the switch used, it is desirable to obtain data on similar circuits. Alternatively, some surge limiting device can be incorporated to protect the transformer.

(*b*) *Compressed Gases*

In those applications where the weight of oil filling cannot be tolerated, e.g. in portable X-ray transformers, an atmosphere of "Freon" is sometimes used. Freon is the name given to a group of halogen derivatives of methane and ethane which give off vapours having a considerably higher dielectric strength than air, and by their use, clearances can be reduced considerably.(3) In most applications the vapour is used with nitrogen at pressures of several atmospheres. Under these conditions the dielectric strength is 2½ to 3 times that of nitrogen at the same pressure. $C_2Cl_2F_4$ and CCl_2F_2 (Freon 12) are generally used when the pressure in the transformer can be maintained at several atmospheres. A portable million-volt X-ray transformer produced by the General Electric Co., of America,(4) for example, uses Freon 12 at 50 lb per in.^{2}

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* These figures are the lowest likely to be encountered for either points or surfaces of large curvature. Oil is unlikely to deteriorate as far as this in small transformers if reasonable care is taken.

† B.S.S. 148 figure for ½ in. spheres is 30 kV r.m.s.

In those applications in which the pressure cannot conveniently be raised above atmospheric CCl_3F is more satisfactory, an increase of 3 to 1 in dielectric strength over that of nitrogen being obtained.

Another compound which has been used recently is sulphur hexafluoride. This is said to have advantages over the Freons, in that it is not decomposed by corona with consequent deposition of conducting films, neither does it cause deterioration of lubricating oils.(5)

The immersion of the whole transformer in oil or other fluid does not usually completely solve all insulation difficulties and it is, therefore, necessary to allow much larger spacing between high-voltage points and the core and other windings than is usually the case; high-voltage transformers consequently have a large window space relative to their core section.

The insulation of a transformer is divided into two groups, *major* insulation between windings or between a winding and core, clamping plates or tank, and *minor* insulation between the parts or sections of the same winding and between turns.

2. PRINCIPAL METHODS OF INSULATION

Several basic principles can be utilized for separating high-potential points from earthy ones or from each other.

(*a*) *The Use of Tapered Windings* (Fig. 87)

Layer windings occupying the length of the core (or half, in the case of two-section windings) are arranged to be earthy at the inner end and can consequently be wound practically to the ends of the former coming in close proximity to the core without danger. As the voltage is built up in each layer the winding length is reduced, the wire being brought further from the core at the end of each layer until the highest voltage point is reached at the outside.

This arrangement is adopted to prevent breakdown from the winding to a low-potential point, such as frame or primary.

It does not reduce the likelihood of breakdown between different points on the winding itself.

(*b*) *Sectionalizing Windings* (Fig. 88)

The aim in winding any high-voltage coil is to ensure that points having a high potential difference are either well separated in space or, if close together, are isolated by a barrier of first-class insulating material. The layer method, (*a*) above, achieves this end by separating successive layers by a turn of paper or similar material, but if the layers are very long, so that the inter-layer insulation becomes highly stressed, it may be preferable to divide the winding into sections in the other direction, i.e. into a number of coils of short axial length but of full radial depth. Interconnexion between the sections is facilitated by winding alternate ones in opposite directions so that two inner and then two outer ends are brought together and joined. Sections are insulated from each other by discs of suitable material. Sectionalization is intended to prevent breakdown between points in the same winding; it does not help to insulate the winding from core or other windings.

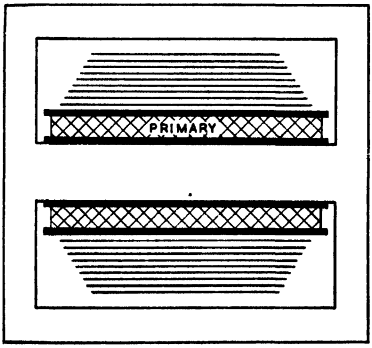


FIG. 87
Tapered Layer Winding. Separation from the Yokes of the Core Increases as the Voltage Builds up Towards the Outer Layers.

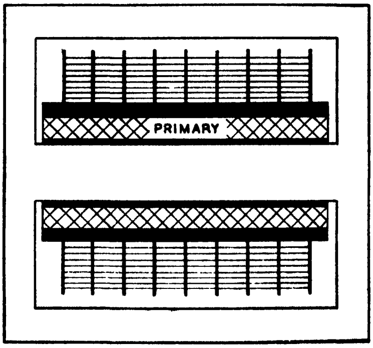


FIG. 88
Sectionalized Winding Preventing Large Potential Differences Between Adjacent Parts of the Coil.

(*c*) *The Combination of Tapering and Sectionalizing* (Fig. 89)

Although sectionalizing does not, of itself, help the insulation of the winding as a whole if each of the sections is wound to the full radial depth, it is possible to reduce their radial depth progressively towards the high potential end. The *internal* diameter of the coil in each section should be *increased* as well as the outside diameter decreased, so as to improve the insulation to the inner winding as well as to the core. A stepped central former, built up from several concentric tubes of different diameters, is a convenient method of achieving this.

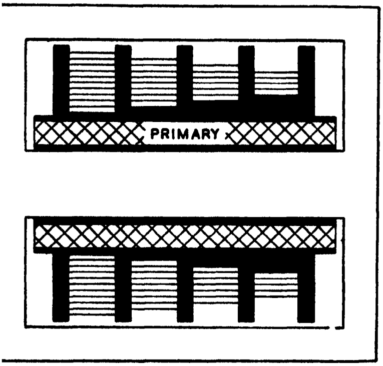


Fig. 89
Sections Decreasing in Radial Depth Towards the High Potential End.

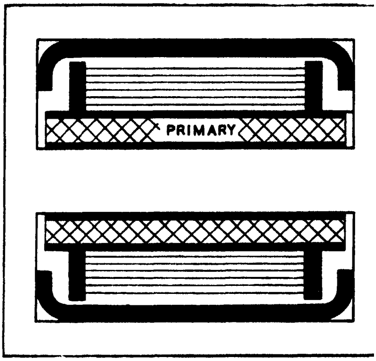


Fig. 90
Core Shields Arranged to Form a Barrier of Solid Dielectric Between High-potential End of Winding and Core.

(*d*) *The Use of Core Shields* (Fig. 90)

If the outside of a coil is at a high potential with respect to the core, breakdown can be prevented either by leaving a sufficiently large part of the window space unoccupied at the outside, or by interposing a solid barrier. Since the danger points are concentrated at two places only round the circumference (only one in a single-phase core type) it is not necessary to surround the high-potential coil with a complete tube. The same result can be achieved by insulating the core at this point by channel-shaped shields of insulating material.

3. Disposition of Windings

The best arrangement of the windings within the space depends on the voltage distribution over the windings. It is assumed in what follows that the primary is in all cases intended for connexion to a relatively low-voltage source (up to 500 volts), one side of which is at or near earth potential. The following voltage distribution possibilities then occur—

(1) Secondary earthy on one side, generating high-voltage at other end.

(2) Secondary earthy in centre, generating high-voltages to earth at each end.

(3) Secondary not itself generating a high-voltage but connected to an external high-voltage supply, e.g. an isolating transformer supplying the filament of a rectifying valve.

(4) Secondary generating a high-voltage between its ends, and also connected to an external point maintained at a high potential. This condition occurs in certain voltage-doubler circuits for high-voltage d.c. supplies.

Some methods of dealing with these cases are detailed below.

(*a*) *Secondary Earthy at One End*

This requirement suggests a form of construction in which the primary is put on first, then a fairly thin tube of insulating material upon which the high-voltage secondary is wound in layer formation with interleaving material of suitable thickness. The length of each layer is progressively reduced so as to increase the spacing between the windings and core sides as the voltage is built up (method (*a*) above). If compactness is aimed at, shields are fitted between the winding and the core at the nearest points (method (*d*)), or alternatively the window size can be made sufficient to prevent breakdown. The weakest point of this construction lies in the large voltages per layer set up in the secondary requiring good inter-layer insulation. Paper-like materials can often with advantage be replaced by high-insulation materials, such as Polythene in strip form, if the impregnating material selected does not adversely affect them. The dielectric strength of Polythene is much greater than that of paper so that thinner strip can be used, and in addition it is less likely to cause corrosion—an important matter where the voltage distributions are such as to favour electrolytic attack of the fine wires. The high stress between layers is avoided if the secondary is broken up into a number of sections, similar to cross-over coils (p. 100) and assembled with insulating discs between. The volts per layer can be reduced as much as desired in this way by using sufficient *sections, but the high-potential end of the winding now requires insulation on the inside as well as on the end and outside*—a requirement best dealt with by using an interwinding insulating tube of stepped wall thickness, increasing towards the high-potential end (method (*c*)). The outer diameter of the other (earthy) end can be made nearly the full depth of the window.

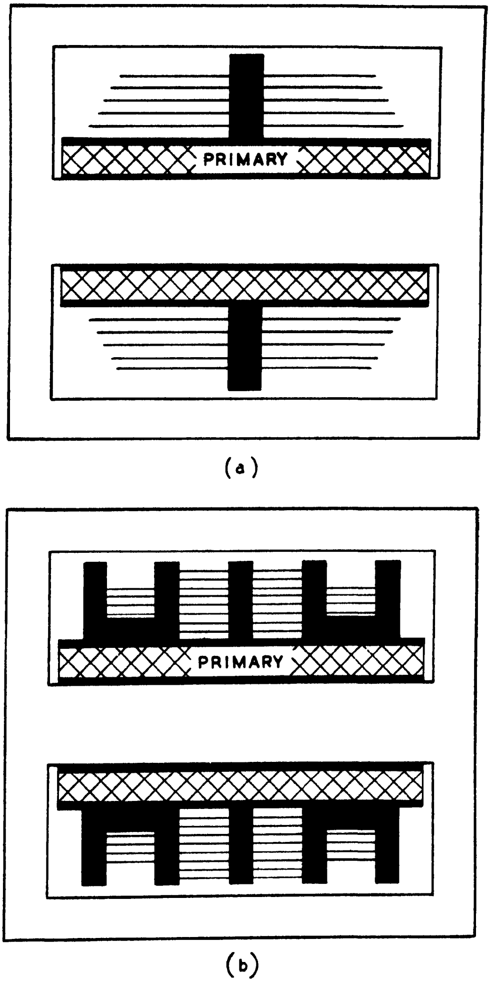


FIG. 91
(*a*) Two-section Layer Winding for Coil Having Centre Point Earthy. (*b*) Alternative Arrangement Using Four Sections to Reduce Volts per Layer.

(*b*) *Secondary Earthy at Centre*

The condition of the high-voltage winding being at a low potential in the middle can be dealt with by methods corresponding to those described in (*a*). A thin interwinding tube can be retained by using only two sections wound in such directions that the two inner ends can be connected together at the centre, and a thick disc of insulation, capable of withstanding the full output voltage, is assembled between the two sections (Fig. 91*a*). The diameter of the disc should be equal *to the full depth of the winding space. The layers of winding can be taken as close to the disc on the inner edges as winding* technique allows, but on the outer edges, they must be progressively withdrawn from the core sides unless, of course, end discs are also provided capable of withstanding at least half the output voltage. This construction suffers (to a lesser extent) from the disadvantages of layer-winding noted in (*a*) and the section arrangement can be used here too provided that a thickened interwinding tube is used (Fig. 91*b*).

(*c*) *Isolating Transformers*

In this case the whole winding has to be well insulated from all other parts, although it does not itself generate a high voltage. These circumstances indicate the use of a thick-walled bobbin of insulating material into which the secondary is wound with inter-layer insulation to suit the particular requirements (Fig. 92). In many cases the output of the winding is only a few volts and the covering of the wire is alone sufficient. Shields can advantageously be used round the core at the points nearest the winding.

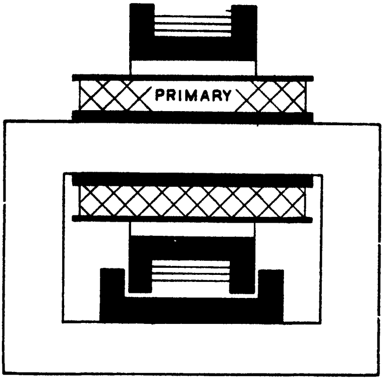


FIG. 92
An Isolating Transformer Having a Low-voltage Secondary Connected to an Externally Generated High Potential.

(*d*) *High-voltage Winding all at High Potential*

The conditions here will be similar to those in (*c*) except that in addition the stresses within the winding itself due to the voltages generated within it must also be dealt with. Since it is necessary on account of the externally applied voltage to use thick insulation between the secondary and other parts, the simple layer method does not offer any great advantages and a sectionalized winding is to be preferred. Further details of construction can be decided only when the magnitudes of the voltages at various points are known.

General

The method advocated in each case has been to make the secondary the outer winding, and this will usually be found to be the more satisfactory, because although on paper it might appear as easy to use a thick tube between core and winding as to provide shields at the outer side, in practice there is considerable difficulty in bringing out the high-potential lead clear of the core, at the small radius. There are conditions, however, where the reversal of the winding order is better, and it may even be worth insulating the whole core and connecting it to the high-potential point. An example is in the cascade-connected sets of transformers used for the generation of a million volts for insulator testing. Here the cores are connected to the mid-points of the 350-kV windings, so reducing the insulation requirements to 175 kV from each end of the winding to core. Since the cores are live, the whole transformers are carried on insulating pedestals.

The use of a single primary winding next to the core centre is likely to result in a relatively high magnetic leakage, i.e. in a high-leakage reactance, with consequent poor regulation. This is particularly likely to be so if the window size is larger than usual relative to the core cross-section—a state of affairs to be expected in high-voltage types.

A poor regulation is, however, not usually a serious difficulty in most of the purposes to which small H.V. transformers are applied, and indeed such characteristics are often definitely advantageous in limiting the short-circuit current flowing in the event of a flash-over in the external circuit. In insulator and cable testing, such flash-overs are, of course, a normal occurrence and the current-limiting effect of the leakage reactance is important.

If a good regulation characteristic is desired it can be secured without greatly increased insulation difficulties by dividing the primary into two sections, one of which is as before next to the core at the inside, whilst the other is also next to the core at the outside of the window space. This is the method used in normal power-distribution transformers having a H.V. winding. Since it would in any case be necessary to provide sufficient spacing or solid insulation between the outside of the H.V. winding and the core, this same insulation will serve to protect the outer half of the primary which, of course, requires only lightly insulating from the core. In the case of the divided primary, however, it will be necessary to make any solid insulation in the form of a complete tube over the H.V. winding, whereas local core shields could otherwise be used.

4. COMBINATION OF HIGH- AND LOW-VOLTAGE SECONDARIES

It often happens that a moderately high voltage (3–7 kV) is required for a cathode-ray tube, as well as a number of other outputs at normal voltages. This is true whether the tube is part of a television receiver or is used for measuring purposes. It is practically always more economical to obtain the high-voltage from a winding on the main transformer than from a separate transformer supplying high-voltage only. Since the *power* consumption of the cathode-ray tube is extremely small, a separate transformer would be working practically on no-load all the time, and since even the smallest wire is capable of carrying more current than is required, the transformer is large and expensive. It is necessary either to use a very large number of turns or to use a large iron section, either of which is costly.

If on the other hand, the winding is applied to another transformer supplying normal loads, the window space has only to be enlarged sufficiently to take the H.V. winding—no enlargement of the primary need be made, since the power consumption is so small. The turns per volt on the main transformer are also likely to be lower than it would be practicable to adopt on a separate unit, so that an all-round improvement is made by this arrangement.

The insulation of the H.V. winding follows the principles laid down earlier. It is usually at a low-potential at one end, and a suitable disposition of turns should be made.

5. SOLID DIELECTRICS

The earlier sections of this chapter have shown that the insulation of high-voltage windings can be carried out in one or other of two basic methods or by a combination of the two.

The points between which a large potential difference exists can either be separated so widely that the path distance between them in the air (or oil if used) is sufficiently great to avoid flash-over, or alternatively a barrier of solid insulation can be interposed. It is principally with the material used for this latter purpose that this section is concerned, although naturally the information is also applicable to the much thinner portions of insulation used between adjacent layers or sections of the coil. Examples of insulating barriers of a substantial type are the shields placed between the outer layer of the winding and the core in Fig. 90, and the disc shown between the two halves of the winding in Fig. 91*a*. In these cases the whole of the voltage generated in the winding is developed across the insulating component and consequently it is most important to ensure that the material is reliable.

Before discussing the suitability of different substances for use in this way, it is desirable to draw attention to the various ways in which failure can take place. The most obvious one is through direct breakdown or puncturing of the dielectric, but in practice it is seldom the cause of trouble. More frequent is failure by "flash-over," i.e. surface discharge over the outside of the insulator. If porcelain or some such non-combustible material forms the insulating barrier, no permanent damage is usually done to it, although of course the winding may be burnt, but if the substance is one which chars readily a "track" of conducting material is formed over the surface and a discharge continues to take place by that route even if the breakdown was initiated by a momentary excess voltage which did not persist. Some materials are much more prone to tracking than others of apparently similar type. A third form of breakdown may occur when the solid dielectric does not fill the gap between the points of differing potential. Owing to the higher dielectric constant of the solid, the intensity of the electric field in the remaining film of air may be sufficiently great to cause corona. Deterioration of the solid by chemical action may then take place very rapidly, resulting in ultimate failure in one of the ways already described. Natural rubber is particularly susceptible to attack in this way, and it should never be used in e.h.t. transformers.

These possibilities are now dealt with in detail.

Direct Breakdown

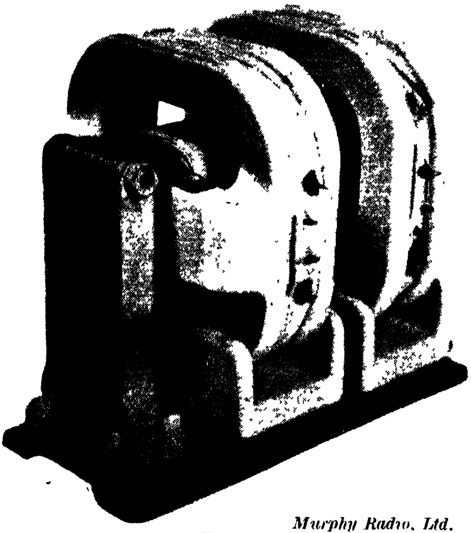
To derive full benefit from an insulating barrier of the type under discussion, the material used must, of course, be sound throughout. Any flaws or cracks reduce the breakdown voltage to a figure similar to that for an equal distance in air. It is in fact largely to avoid the possibility of cracks that a separate insulating material is used; a thickness of the impregnating wax itself would in most cases be satisfactory if it could be relied upon to remain free from cracks. Owing to their mechanical weaknesses, however, impregnating substances cannot be trusted in considerable thicknesses, especially if unsupported by fibrous material, so other insulators have to be introduced. Even when no flaws can be detected, specimens of a given material differ quite widely in the figures obtained in breakdown tests, so a rather large factor of safety has to be allowed when deciding the thickness required for a particular application. When space considerations dictate that the minimum thickness compatible with satisfactory operation shall be used, specimens from the particular batch of material should be tested to the breakdown point.

TABLE XIV

BREAKDOWN VOLTAGES OF INSULATING MATERIALS

	kV per mm
Bakelite (laminated sheet)	10–25
Ebonite	30–100
Fuller board (oiled)	5–30
Glass	25–120
Methyl Methacrylate	20
Mica	50–150
Mycalex	13
Paper (waxed)	45
Paraffin wax	9
Polyvinyl-chloride	25–75
Polystyrene	20–30
Polythene	40
Porcelain (vitreous)	13–18
Rubber	15–45
Urea-formaldehyde	28

Figures for substances in common use are given in Table XIV, but it should be emphasized that they are to be taken only as a guide. Mica is seen to have the highest voltage figure, but it is too expensive to use in large thicknesses. Glass would also appear to be a good material but unfortunately its surface performance is often unsatisfactory. Ebonite gives quite good breakdown figures, is easily worked, and is not expensive; consequently it is one of the best materials for the purpose. It can readily be worked into flat discs, plain or stepped tubes, or bobbins. Some grades can even be bent to shape after moderate heating; this is particularly useful in forming core shields. Waxed paper is also seen to be a good material, so that tubes formed by rolling up many turns of interleaving paper can be used with confidence as inter-winding insulation provided that thorough impregnation *in vacuo* is afterwards carried out. A pile of paper discs can also be employed between sections when the other method of construction is adopted.



Murphy Radio, Ltd.

FIG. 93

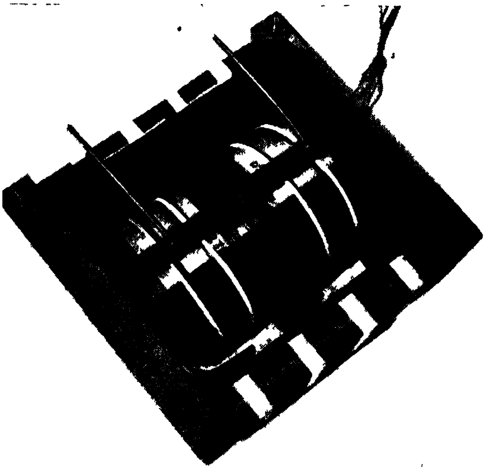
A Porcelain Bobbin Heater Transformer for Operation with 10-kV Between Windings.

Porcelain when suitably prepared is an excellent insulator, but the shapes in which it can be produced are somewhat limited. Dense porcelain for high-voltage purposes is usually formed on a rotating head after the fashion of a potter's wheel so that the shape must be basically circular. Other shapes can be produced by pressing methods but the dielectric strength is then much lower.

The plastic materials naturally offer marked advantages in large-scale production owing to the rapidity with which complicated shapes can be moulded, but in many cases their electrical performance is comparatively poor. This is particularly true of phenol-formaldehyde materials. Polyvinyl chloride and polyethylene have quite satisfactory breakdown figures but their melting points are too low to permit their introduction into many impregnating fluids.

Surface Discharges

Some materials, quite satisfactory in other respects, fail through surface flash-over. The path length in air from one high potential point to the other, measured round the insulating barrier, must of course be sufficiently long to avoid any risk of sparking through the air, but the actual distance over the insulator surface must be greater still. This can usually be ensured by suitable shaping, one method being to turn a series of grooves to increase the path length. Glass is a particularly bad offender in respect of surface breakdowns—a film of moisture is probably the cause of this.

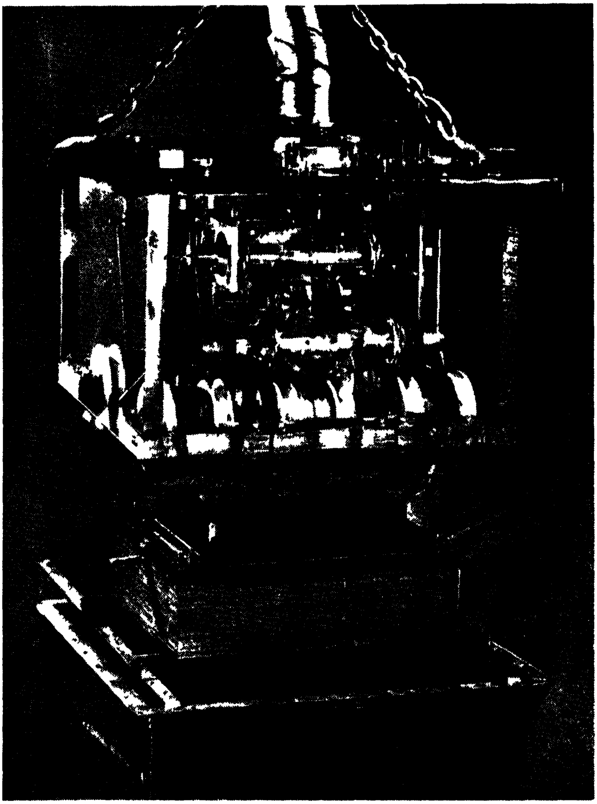


Newton and Wright, Ltd.

FIG. 94

100-kV X-ray Transformer Withdrawn from Tank. The Central Insulating Barrier, Tapered Windings and Core Shields are Clearly Seen.

When assessing the relative merits of different dielectrics, the chemical stability of the material must be taken into account. If the effect of a momentary flash-over is to cause the surface to carbonize through charring, the insulating performance is permanently impaired so that flash-over occurs next time at a lower voltage until quite soon breakdown is continuous. Glass, porcelain, mica and other fireproof materials are not affected in this way, so that permanent damage does *not usually occur. Phenol-formaldehyde materials,* on the other hand, are particularly bad as regards "tracking," and they are best avoided. If they have to be used, the surface should be covered by wax-dipping. Ebonite will track if subjected to frequent flash-overs but not nearly so readily as the phenol-based preparations.



Newton & Wright, Ltd

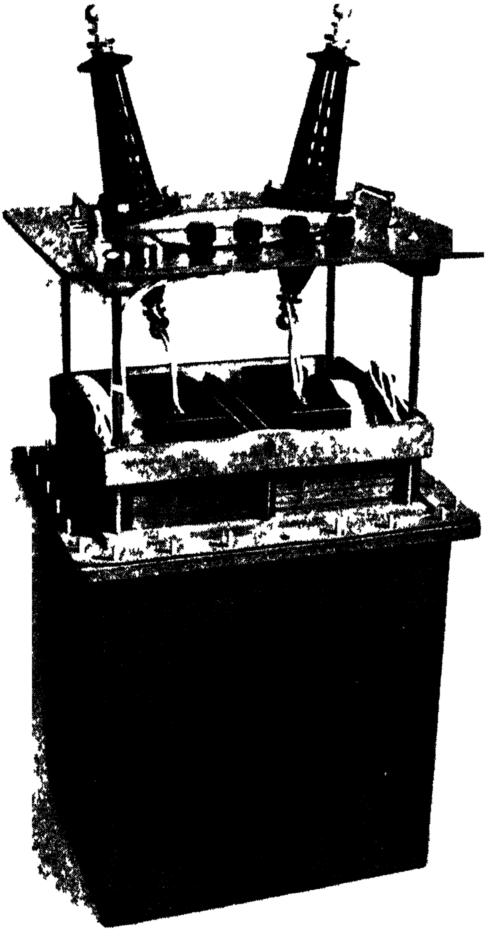
FIG. 95

An Exploded View of a Complete X-ray High-voltage Generator Employing the Transformer Illustrated in Fig. 94.

As a general guide, the distance between points of opposite polarity measured over the insulator surface should not be less than five times the distance through the body of the material.

Compound Dielectrics

When two or more dielectrics of different S.I.C. values are placed in a previously uniform electric field, the voltage distribution is changed so that the potential gradient is greater in the material of lower S.I.C. Since every solid has an S.I.C.



Murphy Radio, Ltd.

FIG. 96

Twin Isolating Transformers for Filaments of Valves in Full-wave Rectifier Circuit. Each Transformer is Rated at 4 volts 15 amp and is Insulated for 40 kV.

value greater than unity, it follows that if it is introduced into an air gap, the remaining layer of air is more highly stressed than before. It may, therefore, quite easily happen that although a solid barrier is quite capable of withstanding the whole of the potential difference, ionization of the air will take place if gaps are left in series with the barrier. Even if this produces no ill effects on the insulating material, the presence near the transformer of large numbers of ions may initiate discharges elsewhere.

One of the places in which corona troubles of this kind may occur, is in the gap between the winding and the core shield in Figs. 90 or 92. Adequate separation of the high-potential points will, however, prevent even the augmented voltage gradient from becoming excessive. Another place where trouble can be expected is in the lead-out insulators. Fortunately the material normally used is porcelain, which is sufficiently inert chemically not to be affected. Oil-filled porcelain bushings are frequently used on high-power transformers, so avoiding the presence of air films. An alternative method would be to deposit a metallic film on to the surfaces of the porcelain following the technique of the silvered-ceramic capacitor. The problem of stress concentration is, of course, particularly acute in capacitors made up with separate metallic plates, and the deposition of a conducting metallic film actually on the dielectric surface avoids the difficulty.

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CHAPTER XV

Auto-transformers

An auto-transformer is a transformer having only a single winding, part of which is common to both primary and secondary circuits. A transformer of the normal type in which the only connexion between primary and secondary windings is an inductive one is sometimes called a *double-wound* transformer to distinguish it from an auto-transformer in which there is a direct metallic connexion between the input and output sides. In a double-wound type all the power transfer between primary and secondary is through the medium of electromagnetic induction, but in an auto-transformer some of the power is transferred directly. The direct connexion between input and output sides of an auto-transformer may, in some cases, be a serious disadvantage. On the other hand, the efficiency of an auto-transformer is higher than that of a normal type of similar dimensions or, alternatively, the same efficiency can be achieved in a smaller overall size. This is particularly noticeable if the input and output voltages are in a ratio of not more than two to one (up or down).

Auto-transformers are frequently used to enable equipment intended for operation on one mains voltage, e.g. 240–250, to be used on a supply where the voltage is somewhat different, e.g. 220 volts. In this case the metallic connexion between input and output is no disadvantage and the saving in cost as compared with a double-wound type is very considerable. Another application is in motor starting where a slightly reduced voltage is required for a few moments.

1. GENERAL PRINCIPLES

An auto-transformer can be either a step-up or a step-down arrangement. Its operation can best be examined by first considering a double-wound transformer suitable for the input and output voltages and currents required. It will be remembered that, neglecting the exciting current, the primary and secondary ampere-turns are equal and opposite, so that at any instant the primary and secondary currents circulate round the core in opposite directions. Furthermore the e.m.f. induced in a given number of turns will be the same in both windings, provided that the magnetic leakage is negligible. It accordingly follows that if the primary is tapped at a distance from the low potential end equal to the number of turns on the secondary (if a step-down type, or vice versa if step-up) then an e.m.f. equal to that of the secondary will be induced in it and, if current is drawn from this tapping, the effective value of the current in the common portion will be the difference between input and output currents. It will thus be understood that not only is the secondary winding completely dispensed with, but part of the primary winding can be reduced in section owing to the smaller current it has to carry. The extent to which the transformer can be reduced in size depends on the ratio of input and output voltages, being greatest when the two voltages are nearly equal. Under these conditions, the proportion of turns common to input and output is highest, and at the same time the primary and secondary currents must also be nearly equal, so that their difference is small. It is thus possible to reduce the cross-section of an increasing proportion of the turns to a progressively smaller size as the ratio approaches unity.

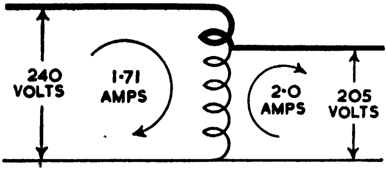


FIG. 97

A Step-down Auto-transformer. The Series Winding is Shown as a Heavy Line.

As an example, take the case of an auto-transformer to supply 2 amperes at 205 volts from a 240-volt supply (Fig. 97). For simplicity let the losses be neglected, and assume that there are 10 turns per volt in each winding.

The output is 2 × 205 = 410 volt-amperes.

$$\therefore \text{ Input current} = \frac{410}{240} = 1{\cdot}71 \text{ amp.}$$

The common winding will thus carry only the difference between input and output currents, i.e. 0·29 amp. The remaining part of the winding (known as the *series winding*) carries the full input current of 1·71 amp. As compared with a double-wound arrangement, the saving amounts to the complete removal of the secondary of 2 050 turns, all of which would have had to have been capable of carrying 2 amp (say SWG 20) and the reduction in size of an equal number of primary turns from a wire suitable for 1·71 amp (i.e. SWG 20) to one for 0·29 amp only (SWG 29). This is, of course, a case where the auto-arrangement shows markedly to advantage because the ratio is quite close to unity. At the other extreme consider a valve-heater transformer supplying 10 amp at 4 volts from a 240-volt supply. The output is 40 VA, so that the input current is $\frac{40}{230} = 0{\cdot}17$ amp (again neglecting impedance drops and exciting current). As a double-wound type, the windings would thus be 2 400 turns (on the basis of 10 turns per volt) of 32 gauge wire for the primary and a secondary of 40 turns of 14 gauge. Auto-connected the secondary, as such, would be removed, but the bottom 40 turns of the primary instead of having to carry only 0·17 amp would have to carry $10 - 0{\cdot}17 = 9{\cdot}83$ amp. For practical purposes this would be of the same section as the original secondary. The remaining part of the winding (now the series winding) would be unchanged, so that the net saving amounts to the removal of 40 turns of SWG 32 wire—a negligible gain which would be far outweighed by the disadvantage of a direct connexion to the supply.

An interesting case is that of a step-down ratio of 2 : 1. Consider an output of ½ amp at 115 volts from a 230-volt supply. The output is 57½ VA, so that the primary current (at 100 per cent efficiency) would be $\frac{57\frac{1}{2}}{230} = \frac{1}{4}$ amp. Double-wound, the primary would thus be 2 300 turns of, say, 30 gauge wire for ¼ amp, whilst the secondary would be 1 150 turns of 26 gauge for ½ amp. As an auto-transformer half the turns would form the common winding carrying $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ amp, the other half forming the series winding also carrying ¼ amp. The winding would thus be of the same gauge throughout and is also identical in turns and gauge with that which would have formed the primary of a double-wound type. The saving thus amounts to the removal of the whole of the secondary, without alteration to the primary. If any ratio greater than 2 : 1 is selected, the common winding has to carry a *larger* current than the primary of a corresponding double-wound transformer, so that the removal of the secondary is partly off-set by the increased size of the common winding.

All the examples so far selected have been of the same class, so as to show the effect of different ratios on the saving effected by the auto-transformer principle, without confusing the issue, and the class happens to have been step-down. The conclusions are, however, equally valid for step-up ratios. Take an example similar to the first one examined and consider its behaviour when used in reverse, i.e. to deliver 2 amp at 240 volts from a 205-volt supply (Fig. 98). The output is 240 × 2 = 480 VA. As a double-wound arrangement the primary current would be $\frac{480}{205} = 2{\cdot}34$ amp, so that the primary winding would consist of 2 050 turns of SWG 19. The secondary would be 2 400 turns of SWG 20. In a step-up auto-transformer the *primary* can be considered as the winding which is discarded, the input being connected to a tapping on the secondary. In this case the whole of the 2 050 turns of SWG 19 are no longer required and 2 050 turns of the secondary only have to carry 0·34 amp and so can be wound with 28 gauge. The remaining 350 turns of SWG 20 form the series winding. The extent of the saving is thus similar to that in the corresponding step-down case.

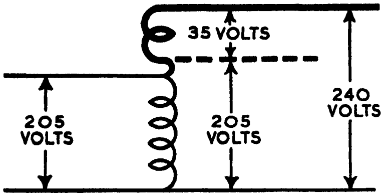


Fig. 98

A Step-up Auto-transformer. The Size of the Transformer Need be no Larger than that of a Double-wound Type Supplying the Same Current at only 35 volts.

The step-up auto-transformer lends itself readily to an alternative method of examination which shows the comparison of double-wound and auto-transformers in a somewhat different light. Hitherto the comparison has been between the sizes of the two kinds when delivering the same output; it is now proposed to leave the windings and frame size untouched and compare the *VA outputs* obtainable in the two methods of connexion. From this point of view, an auto-transformer can be regarded as a perfectly normal double-wound type in which the output is connected in series with the supply. In the example last quoted, for instance, the auto-transformer consisted of a common winding of 2 050 turns of SWG 28 and a series winding of 350 turns of SWG 20. Regarding the common winding as the primary and the series winding as the secondary of a double-wound transformer, the rating would be 35 volts 2 amp, i.e. 70 VA. The identical transformer used auto-connected delivers 240 volts 2 amp, i.e. 480 VA. The rating has thus been increased nearly seven times.

Besides enabling a numerical basis to be given to the comparison, this conception of the operation of an auto-transformer gives a clearer physical picture of the method of power transfer than that first described. When so regarded, it is clear that a large proportion of the output power is drawn directly from the supply without transformation and that the series winding only supplies the "boost." The rating of the series winding is evidently equal to the rise in voltage, at a current rating equal to the output current. As the ratio of output volts to input volts approaches unity, the voltage required from the series winding decreases, but the total power delivered falls much less rapidly. The ratio of auto-connected to double-wound rating thus increases as the input to output voltage ratio becomes nearly equal to unity.

2. ADVANTAGES AND DISADVANTAGES

The advantages of an auto-transformer have been stressed in the section above; namely, a larger output for a given sized frame, or a higher efficiency in a given frame if the rating is not increased. In practice it is probable that a compromise would be made such that there is some improvement in efficiency and the size of the unit for a given rating is reduced. The voltage regulation of an auto-transformer expressed as a percentage is lower than that of a double-wound type in proportion to the ratio of voltage change to the voltage of the higher pressure side. Taking the same transformer and connecting it first as a double-wound and then as an auto arrangement, it is evident that the *actual* voltage drop in the series winding is the same in each case, but the *percentage* of the output voltage is much lower when this includes the input volts as well as those induced in the series winding.

The exciting current of an auto-transformer *expressed as a percentage of rated current* is also much lower than that of a double-wound transformer of the same size and rated output.

The disadvantages are the direct connexion between input and output and the possibility of excessively high voltages appearing on the output side should the common winding become open-circuited. The direct connexion between input and output rules out the use of auto-transformers in many circuits in communications and similar equipment because high d.c. potentials may exist between the two sides. Again, in portable apparatus which may be connected to the mains through a reversible plug and socket, the potential between one side of the output and earth may be quite high in one direction of plugging-in although the potential difference between the two output terminals may be quite moderate. This makes the use of auto-transformers out of the question in many domestic appliances. The low-percentage impedance of an auto-transformer may be an advantage in some applications, but in power-supply systems it is not usually so regarded because the short-circuit current is, of course, correspondingly greater. In order to safeguard auto-transformers from the mechanical stresses which would occur under short-circuit conditions, external reactances are often included to limit the current to values which can be safely withstood.

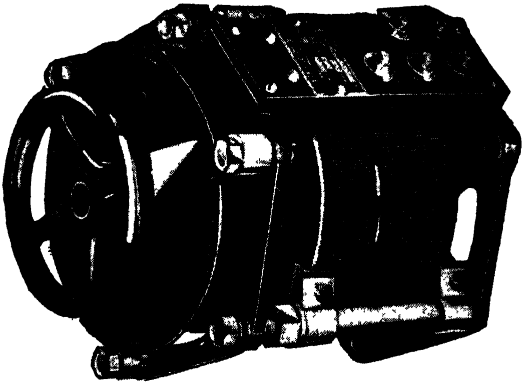
CHAPTER XVI

Variable-ratio Transformers

In laboratory work particularly, but also sometimes in power distribution, it is necessary to be able to vary the voltage of a circuit continuously instead of in the steps normally provided by a tapped transformer. For some purposes it would, of course, be satisfactory to use a variable inductance or choke coil in series with the circuit as a means of adjusting the current to the desired value, but often a source with good voltage regulation is necessary. Several devices of different kinds are available, each having its own sphere of usefulness. For output voltages ranging from zero up to slightly over line voltage at ratings of a few kilowatts only, the *continuously variable auto-transformer* is very convenient. For larger outputs the *induction regulator* and the *moving-coil voltage regulator* are both available. These types are described in sections below.

1. THE VARIABLE-RATIO AUTO-TRANSFORMER

The simplest device for regulating the voltage applied to a load, is the variable auto-transformer. One of the best-known types is the "Variac,"(1, 2) illustrated in Fig. 99. Strictly speaking, the output voltage is not continuously variable, but for practical purposes the steps are sufficiently small to be neglected. The core consists of a deep stack of ring-shaped laminations. In the case of 2-kVA size (rated to deliver up to 8 amp at any voltage up to 230 volts) the lamination stack is approximately 5 in. outside diameter and 3¼ in. deep. The radial width of the stampings is about 1¼ in. Upon this core is arranged a uniformly distributed winding, each turn passing through the central opening and returning over the outside. The winding thus resembles a toroid, except that it is rectangular in section instead of circular. It is usually arranged in a single layer, although a form of pile winding can be used, so enabling a larger number of turns to be applied. The insulation is removed from a circular track around the upper horizontal face of the winding and a carbon brush carried on a rotating arm makes contact with any desired turn on the winding. The winding is tapped at a few points to suit different input voltages and by applying the input between the correct tapping and the end of the winding a constant number of volts per turn are obtained. The voltage between the brush and the common end of the winding can be adjusted to any desired value between zero and the maximum by rotating the arm so as to make contact at the right point.



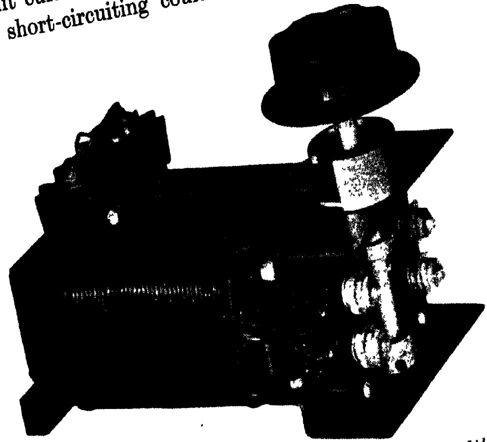
Zenith Electric Co., Ltd.

FIG. 99

A "Variac" Continuously Variable Auto-transformer Capable of Delivering 8 amp at Any Voltage from Zero up to 230.

If only a short range of voltage control is required around the nominal supply voltage, a modified arrangement can be used, employing a more conventional method of construction (Fig. 100). A shell-type core is used in which the window shape is very long and narrow. The winding can then be completed on a former in the normal way and the stampings of T- and U-form assembled into it. The difficulties of making the toroidal type of winding are thus avoided. The winding is composed of a number of layers, only those turns to which contact may be required being in the outermost layer. A track is bared across these turns and a sliding carbon brush bears on them at any required point.

The use of a carbon brush rather than a metal one is an important feature of the "Variac" system. Only by the use of a high-resistance material of this kind is it possible to use a brush sufficiently wide to bridge adjacent turns without the short-circuit current being excessive. If a narrow brush were used, the short-circuiting could be avoided, but the circuit would be opened between each turn. The carbon is also good from the mechanical point of view, as it enables a firm contact to be made without causing serious wear of the conductors.



Zenith Electric Co., Ltd.

FIG. 100

A "Variac" Transformer for Continuous Variation of Voltage Over a Moderate Range Above or Below the Input Voltage.

"Variacs" for 230 volts 50 cycles input are made in sizes ranging from 580 VA up to 5 kVA. The current rating is the same at all output voltages except at the ends of the range where a rather larger current is allowable. Three-phase groups with a linked control are available.

2. THE INDUCTION REGULATOR

For larger outputs than the "Variac" type of arrangement can handle the Induction Regulator is used. The arrangement resembles an induction motor in which the stator cannot revolve freely of its own accord, although it can be rotated through a fraction of a turn by a hand-wheel control and worm reduction gearing (Fig. 101). The latter is irreversible, so preventing any torque produced by the rotor from causing rotation. The stator windings are connected to the output side, whilst the rotor windings are connected through flexible leads or slip-rings to the supply. The windings are of the distributed type normally used in a.c. motors.

The rotor winding corresponds to the primary of a normal transformer, whilst the stator forms the secondary. When the two windings are parallel to each other, the coupling between them is greatest and the maximum voltage is induced in the stator. If the rotor is turned so as to lie at an angle to the stator, the mutual inductance is reduced and the output voltage falls. When the angle is 90°, the output voltage is zero. When used for small adjustments of the line voltage, the secondary is connected in series with the input, so allowing the secondary voltage to be added to or subtracted from the input voltage. This corresponds to rotation of the secondary on either side of the zero output position. In this zero position (at right angles to the primary) the impedance of the secondary would be the cause of quite a serious voltage drop when full-load current is flowing, so that a third short-circuited winding is added to the rotor in a position at right angles to the primary. No currents are induced in the short-circuited winding by the primary, since there is no coupling between them, but in the zero position this third winding is tightly coupled to the secondary and so reduces its impedance to a reasonably low

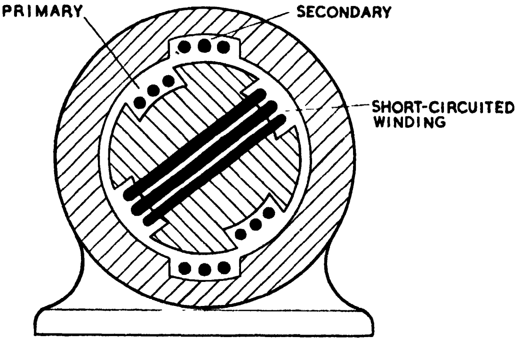


Fig. 101

The Single-phase Induction Regulator, Showing the Primary, Secondary, and Short-circuited Windings.

value. In this way the voltage drop due to the current in the load circuit is kept down to a sufficiently small value.

A *three-phase induction regulator* is somewhat different in principle. The field produced by a three-phase magnet system, in which the three coils are separated angularly in space by angles equal to the phase displacement of the currents in them, is constant in magnitude and rotates at a uniform angular velocity. The effect of turning the rotor coils through an angle is, therefore, to alter the phase of the e.m.f.s induced in them but not their magnitude. In order to produce an output voltage variation, it is accordingly essential to connect the secondary in series with the supply, so producing a resultant voltage, the magnitude of which is changed as the phase relationship between the components changes (Fig. 102). The range of voltage outputs obtainable is between the arithmetical difference and sum of the primary and secondary voltages.

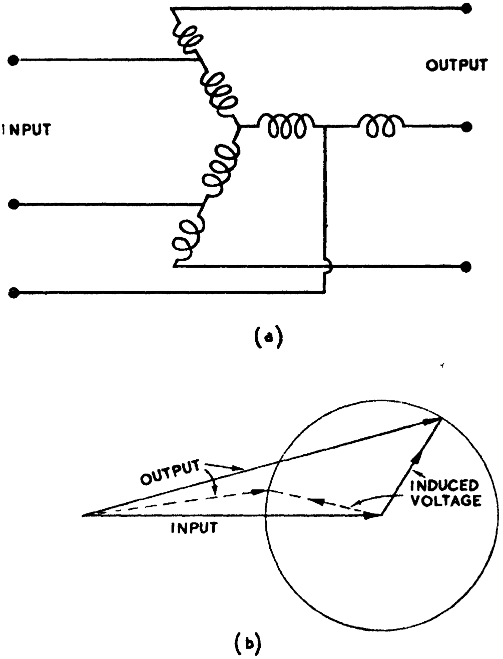


Fig. 102

Three-phase Induction Regulator. (*a*) Circuit Diagram. (*b*) Vector Diagram showing Derivation of Output Voltage at Two Different Phase Settings of Induced Voltage.

3. The Constant-current Transformer

For certain special purposes, such as street lighting, an arrangement is sometimes required for maintaining a constant current through a circuit of varying impedance. The lamps are all connected in series, unwanted ones being short-circuited. In such cases the constant current transformer can be used. Circuits of this kind are more frequently used in the United States than in England.

A shell-type core is adopted, having a long narrow window space at one end of which is the primary winding connected to the constant *voltage* supply. The secondary is free to move up and down the window because it is carried from a partially counter-balanced beam or on chains passing over pulleys to a counterweight (Fig. 103). The flux-density at which the core is worked is kept high, so that the flux leakage is considerable. Under these conditions the leakage reactance of the transformer is very high when the secondary is at its highest position (i.e. farthest from the primary). By suitable design it can be arranged that the leakage reactance in this position is such that the current in the secondary when it is short-circuited is equal to the desired constant value of current. At the *lowest*

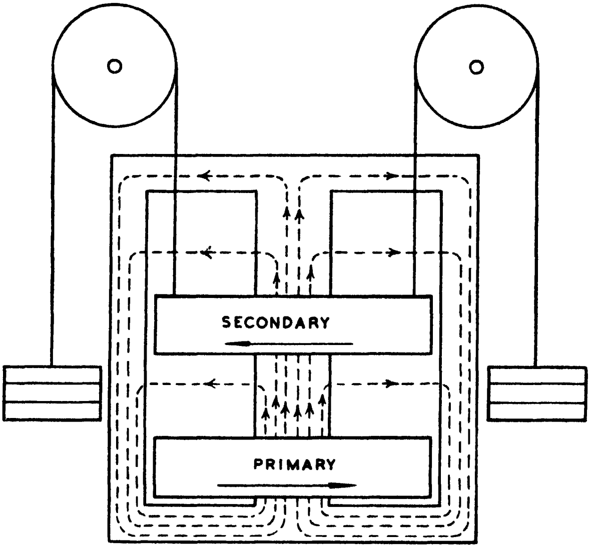


Fig. 103

Constant-current Transformer for Street-lighting. The Coil is Counterbalanced and Moves under the Influence of Forces Due to the Interaction of the Secondary Current with the Magnetic Field.

position the terminal voltage when passing the rated current must equal the maximum desired voltage.

Now from Lenz's Law it can be seen that there is a force between the two coils tending to push them apart. Lenz's Law indicates that the direction of a current induced by any change in the magnetic conditions of a system is such as to bring into play forces tending to restore the system to its initial state. Consider the instant at which the primary current is passing through zero; the flux linkage with the secondary will be zero. The primary current then begins to grow, producing a flux linkage with the secondary; a current is thus induced and its direction is such as to react with the flux to drive the coils apart because only by so doing can the initial state of no flux linkage be approached.

The mechanism of the effect can be examined in a more detailed fashion by observing the instantaneous directions of the currents and fluxes and applying Fleming's rules. Imagine that the direction of the primary current at a particular instant is anti-clockwise seen from above. so producing a flux which is upwards through the central limb of the core. Owing to the high flux-density and the shape of the magnetic circuit some of this flux will leak from the central limb to the outer limbs following a horizontal path above the primary windings. Now the direction of the secondary current will be practically opposite in phase to that in the primary (see Chapter III), so that it is clockwise seen from above at the instant selected. By Fleming's left-hand rule, the mechanical force on the conductors due to the interaction of the secondary current with the primary leakage flux is in an upward direction. Similarly during the next half-cycle the flux and the currents are both reversed, leaving the force upwards as before. The result is thus a tendency for the coils to separate when a secondary current is flowing. At a particular current the force will be balanced by the residual weight of the moving system, and the coil will assume a position in which it is in equilibrium, but if the load impedance is reduced, so causing the current in the secondary to increase, the mechanical force becomes greater and the coil is driven upwards until the increased magnetic leakage restores the current to its original value. The operation is thus seen to be to maintain a constant current in the secondary irrespective of the impedance of the load, within limits imposed by the allowable travel of the coil. The highest output voltage is, of course, given when the coil is at its lowest level; any further increase in load-impedance results in a drop in current, the terminal voltage remaining practically constant as in a normal transformer.

Over the range in which regulation is occurring the power-factor of the primary varies very considerably, being practically equal to that of the load being supplied when working at maximum output voltage, and very low and lagging when the secondary is short-circuited. Since the secondary current is practically constant over the working range, the primary current is also constant apart from the effects of variations in the exciting component.

The communications engineer is unlikely to have occasion to deal with constant-current transformers for lighting service, but he may find devices of this kind useful in laboratory and experimental work. An alternative method of securing a constant-current supply for a *resistive* load from a constant voltage source is to connect the variable load across the inductance (or the capacitance) of a series-tuned circuit which has been adjusted to be in resonance with the supply frequency. If the resistance of the inductance is neglected, the current through the load is independent of its resistance.(3, 4)

4. The Moving-coil Voltage Regulator

The words "moving-coil" usually bring to mind the arrangement of a pivoted coil so frequently used in d.c. voltmeters and ammeters; again the constant-current lighting transformer described in Section 4 of this chapter and the induction regulator are also "moving-coil" devices, but in this section the term "moving-coil regulator" is applied to a particular arrangement for continuous voltage variation developed by Messrs. Ferranti, Ltd.(5)

The principle upon which the device operates is shown in Fig. 104. Two coils wound in opposite directions upon a long closed core are connected in series across the supply. The output is taken between the lower end of the pair and the junction of the two coils; the output voltage is, of course, a function of the ratio of the impedances of the two sections. By reducing the impedance of the upper coil the voltage could be raised, and it could be reduced by lowering the impedance of the lower coil. In the Ferranti moving-coil regulator this alteration in the impedances of the coils is brought about by sliding over them a third coil which is short-circuited on itself. There are no external connexions to the moving coil, a fact of considerable importance in simplifying the mechanism. The moving coil, which surrounds the other two within the other limbs of the core, can be raised or lowered by a screw rotated either by hand or in the automatic types of regulator by a simple motor. In this way a continuously variable output is obtainable between certain limits depending on the tightness of coupling which can be secured between the moving coil and the particular fixed coil concerned. In general, however, this simple arrangement does not allow the output voltage to be reduced quite to zero or to be raised quite to the input voltage. If the full variation up to one or both extremes is necessary, additions can be made to allow it.

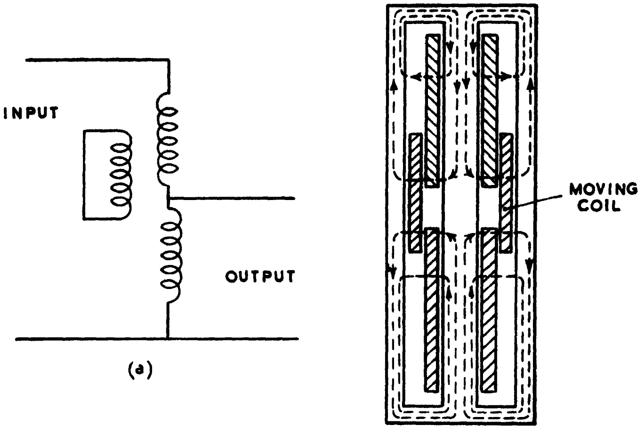


Fig. 104 (b)

The Principle of the Ferranti Moving-coil Regulator.

These additions consist of "buck" and "boost" coils in some cases and auxiliary auto-transformers in others. By combinations of these, any desired range of variation up to or above the input voltage can readily be obtained.

The magnetizing current is independent of the position of the moving coil and, unlike that of a normal transformer, is practically sinusoidal. This is because the magnetic path for each of the fixed coils is completed through the air between inner and outer limbs. Another interesting fact is that the current in the moving coil is proportional to the load current, being zero at no-load. This arises from the fact that the two fixed coils are wound in opposite directions.

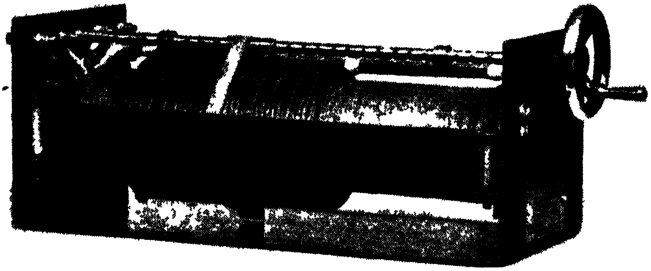


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FIG 105

A Small Moving-coil Regulator with Cover Removed showing Windings and Outer Limbs of Core.

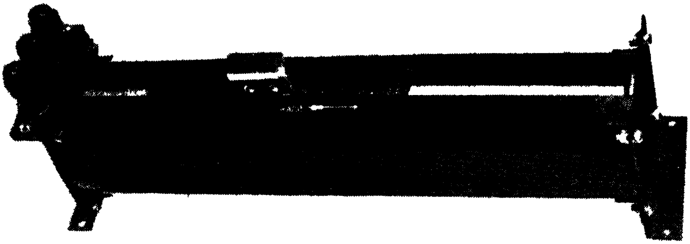

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FIG. 106

A Small Moving-coil Regulator for Laboratory Use.

Typical low-power moving-coil regulators are illustrated in Figs. 105 and 106.

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CHAPTER XVII

Transformers Depending on Magnetic Saturation

THE whole technique of transformer design as applied to normal types is based on the assumption that the lack of linearity in the *B-H* characteristic of the core material has no appreciable effect on the working of the device. Whilst it would be incorrect to suggest that this non-linearity is glossed over in the usual treatment, it is true that the efforts of the designer are usually directed to the choice of such conditions of operation that the *B-H* characteristics have little effect on the output. With considerable satisfaction, the designer is able to say that in large transformers the exciting current can often be neglected in full-load calculations, and if forced in certain cases to take cognizance of its existence he assumes it to be sinusoidal. There is, however, a class of special transformers which depend on the non-linearity of the core material for certain features of their operation. There are, for instance, the stabilizing types which give a practically constant output (± 1 to 2 per cent) when supplied with input voltages varying over a range of the order of ± 15 per cent. The earlier arrangements of this kind produced output voltage wave-forms which were far from sinusoidal, but this trouble has now been largely overcome by the use of suitable filters and these stabilizing transformers are in common use, particularly for the power supplies to measuring instruments employing thermionic valves.

Then again the non-linear characteristics of the core are put to good use in the "peaking" transformers which deliver short steep pulses for special purposes such as flashing tubes for stroboscopic illumination and for the "firing" of grid-controlled mercury-vapour rectifiers. In the latter application the pulse has to be produced at varying points in the anode-voltage cycle, and it is possible to arrange the peaking transformer so that a continuously variable timing is available.

Another way in which the non-linearity is used is in frequency multiplying circuits in which a particular harmonic such as the third is accentuated and selected from the network by suitable filters.

The methods of operation of these various types of transformers naturally vary considerably and the various sections of this chapter deal with the details of each separately. It is, however, perhaps desirable to recall that the use of a high flux-density in the core of a transformer connected to a low-impedance source of power such as the supply mains does not itself produce appreciable distortion of the output voltage wave. In order to produce large percentages of harmonics in the output voltage, it is accordingly necessary to insert an impedance in the primary circuit so that the considerable magnetizing current distortion which is present is reflected into the induced voltage wave through the voltage drop in the impedance.

It is seldom possible to calculate the results to be expected from non-linear transformers and much of the design is based on empirical methods.

1. STABILIZING TRANSFORMERS

If a transformer with a closed iron core has its primary connected in series with a high resistance to a source of alternating current of variable voltage it will be found that the voltage across the secondary measured by a rectifier instrument will at first rise nearly linearly with the input voltage. Soon, however, it will be noticed that the secondary voltage is rising less rapidly than the input voltage, and eventually a practically constant output voltage would be reached. Examination of a *B-H* curve for the core material shows clearly that this behaviour is to be expected. The induced e.m.f. in the secondary is at any instant proportional to the rate of change of linkages of the flux with the coil turns, i.e. to $-T_s\left(\frac{d\phi}{dt}\right)$ and the average e.m.f. over a half-cycle between Φ_{max} and $-\Phi_{max}$ is equal to $T_s \times 4\Phi_{max}$. Similarly over the next half-cycle, the average value of the e.m.f. will be $-4T_s\Phi_{max}$. Now, since Φ_{max} $(= B \times a)$ approaches a limiting value with increasing values of H, the change of flux linkages also approaches a limiting value, so that the average value over a half-cycle does not increase appreciably once the current exceeds a certain figure. The reading of a rectifier-type meter which is proportional to the half-cycle average value accordingly tends to a limiting value too. Examined by an oscilloscope, the voltage wave would be seen to become more and more peaky as the input voltage is raised even after saturation is reached at the peaks of flux, but the voltage peaks would become narrower. The reason for the progressive increase in peak voltage is that the flux always reaches the same maximum value irrespective of the current, owing to saturation, and this value is reached in a shorter and shorter time interval as the current is raised. The corresponding instantaneous induced e.m.f. also increases indefinitely, but its duration is reduced.

This arrangement evidently provides a basis for an automatic output-stabilized transformer. The essential differences between it and the normal circuit are first of all the use of a high resistance in the primary circuit, ensuring that the current is sinusoidal in wave-form and of a magnitude proportional to the input voltage, and secondly that the flux-density is very considerably higher. The result achieved is to stabilize the half-cycle *average* voltage, but the wave-form of the output is very far from sinusoidal and is unacceptable for many purposes. If a constant r.m.s. value is required or a better wave-form, modifications have to be made. (The r.m.s. output voltage, although increasing less rapidly than the input, continues to increase as the wave-form becomes more peaky.) The residual variation can be compensated by connecting in series opposition a smaller voltage which is obtained from a normal transformer giving an output voltage proportional to the input. This arrangement forms the simplest type of stabilized output transformer system which is practicable.

The Simple Compensated Stabilizer

The series resistance, which forms an essential part of the system described above, is very wasteful of power and the heat generated in it is apt to be an embarrassment in a compact unit. A non-dissipative reactance can, however, be substituted without affecting the result, provided that the impedance remains constant over the cycle and over any range of input applied to it within the rated capacity of the system. A reactance which satisfies this condition is an iron-cored inductance of adequate cross-section and with an air-gap in the iron circuit. Since the compensating voltage required is usually only a fairly small fraction of the total, this inductance can also be fitted with a secondary winding and used as the compensating transformer. The system is thus composed of two transformers having their primaries in series and their secondaries in series opposition.⁽¹⁾ One has a core of large cross-sectional area with an air gap and the other is of small sectional area and without a gap (Fig. 107). The individual output voltages of the two when connected in series to a variable voltage supply are shown in Fig. 108. The r.m.s. output of the saturating transformer rises rapidly at first and then at a much lower rate, whilst that of the gapped transformer rises linearly over the full voltage range. By choosing a suitable secondary voltage for the latter it can be arranged that the difference between the outputs of the two transformers is practically the same at any input voltage exceeding a minimum determined by the position of the knee of the saturating-transformer output curve.

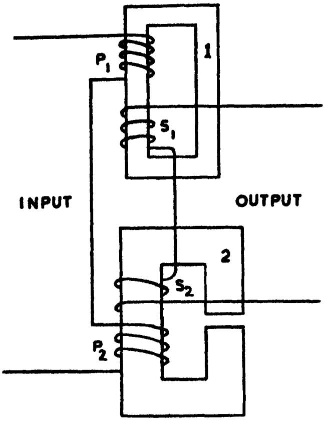


FIG. 107

The Basic Principle of the Saturated-Core Stabilizer. Upper Core 1 of Small Section, Lower Core 2 of Ample Section to Avoid Saturation and with Air-gap.

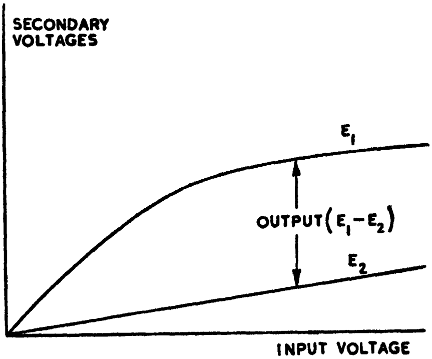
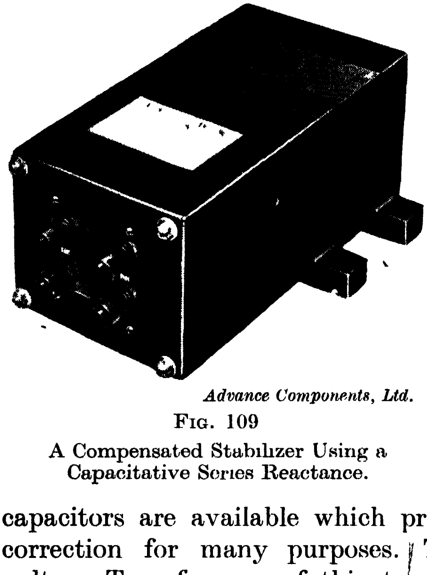


FIG. 108

The Output Voltage is the Difference Between the Output from Core 1 which Tends to a Limiting Value as the Iron Saturates and that from Core 2 which continues to Rise in Proportion to the Input Voltage.

This arrangement suffers from the disadvantage that the output wave-form is far from sinusoidal. It is in fact necessary to adjust the compensating voltage to different values according to whether the r.m.s. value or the peak value is required to remain constant with varying input voltages. The losses in the constant-impedance transformer are also rather high, and it is accordingly replaced in most modern stabilizers by a capacitor. The fact that the voltage drop across the capacitor is in phase opposition to that across the inductance involves certain complications and this arrangement is treated below.



Advance Components, Ltd.

FIG. 109

A Compensated Stabilizer Using a Capacitative Series Reactance.

Circuits Incorporating Capacitors

Although the capacitative-impedance type has been approached through the compensated inductive-impedance type, it is not essential to use compensation. Simple uncompensated stabilizers using capacitors are available which provide a sufficient degree of correction for many purposes. The "Advance" Constant-voltage Transformers of this type for instance reduce a ± 15 per cent voltage variation of the supply to about ± 1 per cent. A typical stabilizer of this type is illustrated in Fig. 109.

If a closer degree of stabilization is required, the compensated type of transformer can be used⁽²⁾ and the fluctuations are then reduced to as little as ½ per cent for supply voltage variations of even 40 per cent. Either a constant r.m.s. or a constant mean (per half-cycle) can be provided by the manufacturers.

The output-voltage variation *with secondary load* (i.e. voltage regulation) is stated to be between 4 per cent and 7 per cent, except for the smallest sizes. A considerably improved figure is obtained from the types fitted with harmonic filters. The efficiency is in the region of 70 per cent to 80 per cent, and the input power-factor on a resistive load is leading. Frequency variations of the supply affect the output voltage approximately in the same proportions.

The use of a capacitor as the series impedance in a magnetic-saturation stabilizer introduces the peculiar property of "jumping" into a stabilizing state at a certain voltage as the input voltage is gradually raised, and remaining in till a much lower voltage as the input is reduced again. For example, a stabilizer intended to work over the range 190–270 volts input might jump into operation at a rising voltage of 160, but once started it would be possible to reduce the voltage to as low as 70 before it would "drop out."

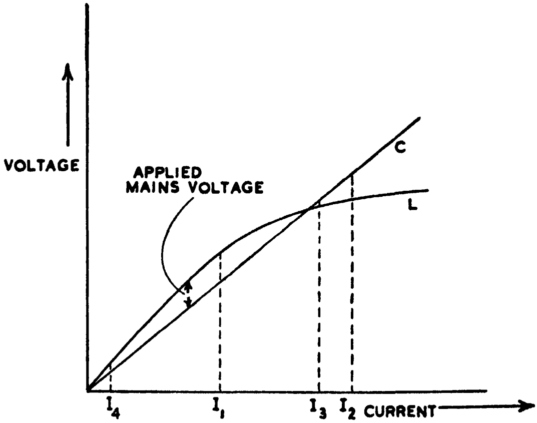


Fig. 110

The Principle of the Condenser-type Stabilizer. The Current "Jumps" from a Value I_1 to I_2 as the Input Voltage is Gradually Increased, but Follows the Curves well below I_2 if the Voltage Falls Again.

This behaviour is explained in Fig. 110, which shows the current-voltage characteristics of the capacitor and the saturating reactor plotted to the same scale. Since the voltage across the capacitor lags 90° behind the current through it, whilst that across the inductance would, neglecting losses, lead the current by 90°, the two voltages are at any moment in opposition. (The currents in the two elements are always equal and in phase, since they are connected in series.) The difference between the two voltages at a particular current value is equal to the applied voltage, so that if the input is gradually increased from zero, the current will increase at first more or less proportionately to the voltage and then more rapidly, until the input voltage reaches a value equal to the maximum separation of the two curves. A further increase in input can then be accepted only at a current above the crossing point of the capacitor and reactor curves. The current accordingly jumps from the value I_1 to I_2 and then increases again steadily with further rise in input voltage.

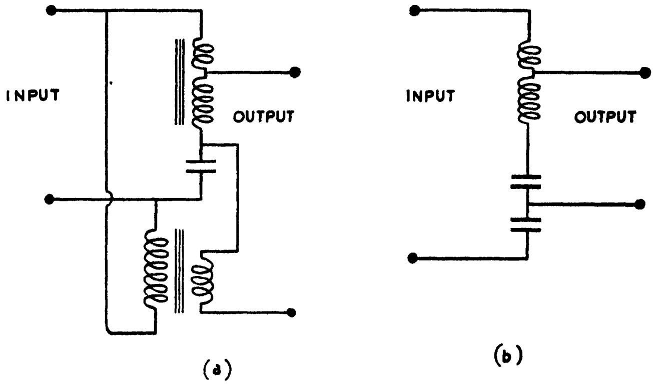
If the input voltage now falls, the current falls also and operating conditions pass into the region below I_2 which was missed out when the input was rising. According to this explanation it should be possible to reduce the input voltage to zero at the cross-over point still leaving a finite current flowing, but this is the result of neglecting the losses in the saturating reactor. If these losses are taken into account, the two voltages are not in direct opposition and the conditions cannot be accurately represented on a two-dimensional diagram. The effect of the resistive losses is, however, to prevent the exact cancellation of the two voltages and at a point somewhat above the intersection (I_3) the current suddenly falls to a value at which the voltage difference is again of the same value, i.e. I_4.

If the circuit were linear, the arrangement would, of course, be in series resonance at all values of applied voltage, but owing to the saturating effect in the core the inductance value of the reactor varies with the voltage across it, so that resonance could be said to occur only at the intersection point. Nevertheless, as Fig. 110 shows, the capacitor and reactor voltages are considerably higher than the input voltage, so that proper attention must be given to the insulation of these components. As most general-purpose stabilizers are arranged to provide an output voltage equal to the mean of the extreme input figures, the output is conveniently taken from a tapping on the reactor, since the output is normally less than the whole reactor voltage. The reactor then works as an auto-transformer at considerably higher efficiency than would be obtainable from a double-wound type. Apart from its lower efficiency, however, there is no objection to the use of a separate output winding instead of a tapping on a single winding.

As Fig. 110 shows, the output voltage (proportion to V_L) continues to rise as the input voltage (equal to the difference between V_C and V_L) increases, although much less rapidly. A small compensating voltage, rising in direct proportion to the input, must accordingly still be connected in opposition to the voltage across the inductance in order to keep the output constant. This can be derived from a step-down transformer of the normal type (Fig. 111 (*a*)) or, as in Fig. 111 (*b*), by dividing the capacitative element into two unequal sections by adding a larger capacitor in series and modifying the value of the original one slightly so as to bring the resultant capacitance back to its original value.(3) Remembering that the voltage across the capacitor is in the opposite phase to that across the inductance, the desired compensation is obtained by taking the output from the points shown

Reducing Harmonics

For some purposes the harmonic content of the simple stabilizers is objectionable and consequently filters are often fitted to reduce the harmonic percentages in the output.(4, 5) Apart from the removal of interference caused by high harmonics, the approach to a sinusoidal wave-form removes difficulties due to the differing adjustments necessary to secure constant r.m.s. or constant peak-voltage output. In measuring apparatus employing thermionic valves this is quite important, for the temperature of the valve heaters depends on the r.m.s. voltage, whereas the output from the rectifier supplying the valve anode voltage is largely dependent on the peak value of the applied voltage.



(a) (b)

FIG. 111

Methods of Obtaining the Small Reversed Voltage for Compensation.
(*a*) From a Separate Transformer. (*b*) from a Capacitative Tap.

The filters used to reduce the harmonic content can be series-tuned circuits connected across the load, a separate circuit being required for each harmonic it is desired to remove. Alternatively a rejector can be connected in series with the output.

Working Flux-densities

From the point of view of constancy of output over a wide range of input voltages the best results are obtained by raising B as high as possible. On the other hand, the temperature rise of the iron becomes unmanageable and the magnetizing current also tends to cause difficulties in the primary winding, if too high a value is used. The rate of loss of heat from the core can be increased by inserting cooling fins between some of the stampings, so adding to the area in contact with the atmosphere, whilst the cross-sectional area of the primary conductor is made considerably larger than that of a normal transformer of similar output. A reasonable figure for the flux-density is about 17 000 lines per cm^2 at the upper end of the input voltage range.

The design of a stabilizer is largely empirical and the variations between different samples of supposedly similar iron would in any case make accurate calculations useless. The performance of these special types, unlike that of a normal transformer, depends greatly on the magnetic characteristics of the core material, and it is usually necessary to provide in any design intended for quantity production some adjustment, such as a series of closely spaced tappings, which will allow each individual stabilizer to be set up correctly. The starting point in the design is the capacitor. The voltage across the capacitor at no-load is approximately equal to that of the supply plus that across the inductance. Whilst this latter voltage is not necessarily that of the output, since a step-up or step-down *can* be introduced, it reduces the efficiency to do so, so that a desirable arrangement leads to a capacitor voltage of about 500 volts r.m.s. for a 230-volt stabilizer.

It is perhaps worth noting that special precautions have to be taken in the production of paper dielectric capacitors for a.c. operation on voltages above 300 r.m.s. The life would be relatively short if the voltage per unit exceeded this figure, however thick the paper dielectric was made (within economical limits). This is in contrast to the behaviour of such a capacitor on D.C. where an indefinitely long life is obtained provided that a paper of suitable thickness is used. It appears that the alternating electric field brings about a disintegration of the paper dielectric if the critical voltage is exceeded, so that it is necessary to connect two or more capacitors in series. A similar result is achieved in a smaller space by winding an isolated foil between two dielectrics which thus work in series. The figure of 500 volts r.m.s. required for the usual stabilizer is thus obtained by two series-connected elements. Increases in voltage above 500 cause a disproportionate increase in cost, since three units would be required.

The capacitance value adopted is decided by the maximum output which it is desired to take from the device. As a rough guide the current through the capacitor should be about three times the maximum output current desired. Since the voltage applied to the capacitor is approximately twice that across the load, the capacitative impedance should be two-thirds of that of the maximum load. For example, if the stabilizer is to supply up to 75 watts at 240 volts, the output current on a resistive load would be about $\frac{1}{3}$ amp. The capacitor should then be such as to pass 1 amp at 500 volts, i.e. about 6 μF.

The transformer is next designed to have a primary reactance equal to that of the capacitor at the supply frequency and to work at a flux-density of some 15–17 kilo-lines per cm.^{2} Final adjustments are usually made by trial.

2. "PEAKING" TRANSFORMERS

Although it is possible to obtain stroboscopic effects from a neon or other gaseous-discharge tube fed from a sinusoidal supply of the correct frequency, the image tends to be blurred instead of sharp because the period of illumination is long enough to allow some movement to be visible. The duration of the flash can conveniently be reduced by the use of a transformer which produces sharp peaks of secondary voltage.

The output from any transformer working at a high flux-density and with an impedance in its primary circuit tends to be peaky, as noted in Section 1 of this chapter. This effect is due to the approach of the iron to saturation, so that over a large part of each half-cycle there is little flux change because the maximum flux is reached early.(6) The flux *change* from a high positive value to a high negative value and vice versa accordingly occurs in a short period of time at the middle and end of each cycle and an output peak is produced followed by a quiescent interval. This effect can be accentuated by the use of nickel alloys in the magnetic circuit;(7) at low intensities their permeability is high, but they are easily saturated so that a strongly flattened flux wave is obtained. The need for an external primary impedance can be avoided by making use of leakage, intentionally increased by a magnetic shunt.

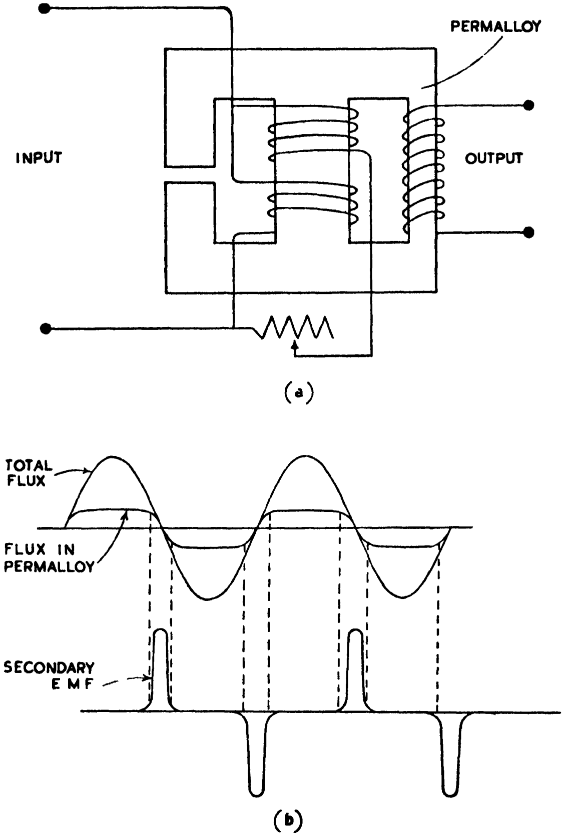


FIG. 112

(a) "Peaking" Transformer with Composite Core of Silicon-steel (Large Section) and Permalloy (Small Section). (b) Saturation in the Permalloy is Reached at an Early Point in the Cycle. An Induced Voltage Appears only During the Short Periods of Flux Change.

A typical transformer incorporating these features is shown in Fig. 112. Two primary windings are used to allow some measure of control over the timing of the pulse. One primary is connected directly to the supply whilst the other is connected through a resistance or capacitor so that the phase of the current through it leads that in the first coil by 90°. By varying the relative magnitudes and directions of the two component fluxes, the phase of the resultant flux can be altered over an appreciable angle with little change of magnitude. The secondary is wound on a nickel-alloy core, such as Permalloy, and the flux takes this path at the beginning of its cycle, so inducing a voltage in the winding. The Permalloy is, however, soon fully magnetized, and the reluctance of this path becomes high so that the flux is diverted across the shunt. No further flux changes, therefore, occur in the secondary until the end of the half-cycle, when a quick reversal takes place and a secondary voltage peak is produced.

An important application of peaking transformers is in the control of rectifiers. The adjustment of the d.c. output voltage from mercury-vapour rectifiers used for supplying the anode voltage of high-power oscillator valves in radio transmitters and industrial heating plant, is being carried out to an increasing extent by means of grid control. A grid-controlled mercury-vapour rectifier is similar in principle (although not in practical arrangement) to a triode valve. By applying a negative voltage of sufficient magnitude to the grid, the anode current is prevented from flowing even when the anode is at a positive potential relative to the cathode. During the other half-cycle when the anode is negative, no current would flow in any case. Unlike its counterpart in a hard-vacuum triode, however, the grid cannot regain control if the anode current is once allowed to start, because the positive ions which are attracted to the grid form a space charge which prevents the grid from stopping the anode current however far negative its potential may be raised. Not until the end of the half-cycle when the anode potential becomes negative can the grid re-establish its control. A very convenient system of output-voltage control can be set up by giving the grid a constant negative potential sufficient to overcome the maximum anode potential and arranging to superimpose a short positive pulse at any desired point in the cycle. Once started by the pulse, the current will continue to flow for the rest of the half-cycle. The need for a steep front to the pulse arises from the variability of the rectifier characteristics with temperature and during life. By using a sufficiently large steep pulse, the instant of starting is substantially unaffected by wide variations in the "control ratio" of the grid.

An installation of grid-controlled mercury-vapour rectifiers using magnetically produced pulses for voltage control has been described by J. C. Read.(8) In this equipment the variation of the firing point of the rectifiers is effected by the variation of a d.c. magnetization of the core superimposed on the a.c. one.

The output is taken from across a resistance connected in series with the winding of the saturated core. Assume that the direction of the d.c. polarization is positive and trace out the current changes starting from a voltage zero following a positive half-cycle. The a.c. voltage begins to go negative and since the core is saturated, the inductance is practically zero and the current is accordingly determined by the series resistance. The current thus commences along a sinusoidal curve on the negative half-cycle. When the a.c. ampere-turns reach such a figure that the resultant a.c. and d.c. magnetization is less than that producing saturation, the coil becomes inductive, and as is the habit of inductances, maintains the current at a constant value. The wave-form thus departs from the sinusoidal shape and follows a horizontal line. The current in the now inductive circuit lags behind the applied voltage, but eventually the resultant flux again reaches the saturation value, the inductance disappears and the current returns abruptly to the sinusoidal shape which is followed for the rest of the positive half-cycle of voltage. The steepness of the current wave-front at the return of saturation can be made very great by the choice of a material which saturates suddenly. If the d.c. ampere-turns are now reduced, the coil remains inductive longer and the steep portion occurs later in the cycle. The firing point varies from 0° to 160°, with ratios of D.C. to A.C. (peak) ampere-turns ranging from 0·7 to practically zero. One important advantage possessed by the d.c. polarization method of control is the ease with which it can be arranged to give automatic high-speed shutting down in the case of overloads on the d.c. circuit. If the fault is only a momentary one, the voltage is re-established at once, so that only a fraction of a second is lost. This form of protection is provided by arranging to pass a fraction of the d.c. output current through a separate coil on the pulse transformer. If the current becomes excessive either the pulse is shifted to such a point in the cycle that the anode current is completely arrested or else the pulse does not occur at all.(9)

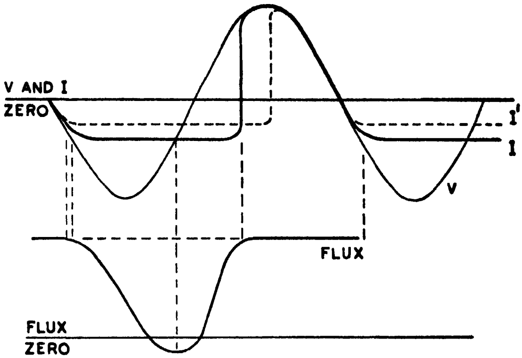


Fig. 113

If an A.C. Field is Applied to an Iron-cored Reactor already Saturated by Steady Magnetization a Back E.M.F. is Induced when the Instantaneous A.C. Magnetization Becomes Sufficiently Great to Overcome the Saturating Field. The Dotted Line shows the Voltage Wave-form for Another Value of D.C. Note the Phase Shift of the Steep Portion.

3. Static Frequency-multipliers

The production of odd-order harmonics such as the third, has been discussed at some length in Chapter X. It was explained that owing to the shape of the *B-H* curve for the ferro-magnetic materials used for transformer cores, the magnetizing current of a transformer departs considerably from the sinusoidal wave-form. If the primary or the source impedance is considerable, the voltage drop will cause the magnetizing current distortion to be reflected in the output voltage wave-form. If it is desired to make use of these harmonics, a high value of flux-density is used in the core, so accentuating the wave-form distortion. The harmonics generated in this way can be extracted from the fundamental by tuned filters.

An instance of the production of large percentages of third harmonic is in the use of a shell-type three-phase transformer (or three separate single-phase transformers) with star-connected windings. This phenomenon was described in some detail in Chapter XII and it was seen that in a star-connected primary circuit the magnetizing current is maintained in a sinusoidal shape by the high impedance offered to the harmonics generated in one phase by the impedance of the windings of the remaining phases. A sinusoidal magnetizing current produces a distorted flux and hence a distorted induced voltage which is rich in third harmonic. If delta-connected secondaries are fitted to the cores, the fundamental voltage induced round the three windings will be zero, but the third harmonics in each are in phase and additive. If, therefore, one corner of the delta is left open, third-harmonic voltages are available between the open ends. Power can be drawn from the winding at the triple frequency and the conversion reaches quite a high efficiency. Owing to the considerable reactance of the secondaries at the third harmonic, it is advisable to connect in series a capacitor of such a capacitance as will neutralize the inductive reactance. Brailsford(10) and later Cantwell(11) give particulars of frequency-tripling transformers which reach efficiencies approaching 80 per cent, although at a rather low primary power factor. This can be improved by working at lower flux-densities, although the transformer then becomes larger for a given output. Cantwell shows that the iron is best utilized for tripling purposes when peak flux-densities of 20 000 lines per cm^2 or slightly more are used.

Circuits of this kind are used for a number of purposes, usually of a low-power nature. For instance, they form a very convenient way of deriving a 150-cycle supply for control purposes using voice-frequency telephone circuits. Another application is the supply to portable machine tools and woodworking machinery using induction motors. The highest speed obtainable from an induction motor running from a 50-cycle supply is rather less than 3 000 r.p.m., and this is often inconveniently low for the purpose since it makes portable tools large and heavy or, in the case of wood-working machinery, is too low for satisfactory work on direct-driven cutters. By using a frequency tripler, the speeds can be raised three times with little loss on the electrical side.

Larger converters are quite practicable and may become commercially useful. Brailsford gives design figures for a 550-kW frequency-changer from which he expects to obtain a full-load efficiency of 94 per cent.

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CHAPTER XVIII

Vibrator Transformers

READERS will hardly have read 325 pages of "induced e.m.f.s," "terminal voltages," etc., without becoming very clearly aware of the great advantages which A.C. (through the agency of the static transformer) possesses in the ease with which the voltage may be changed. They will therefore not be surprised that numerous attempts have been made to produce systems for transforming D.C. from one voltage to another without the use of rotating plant. A reasonably practicable system has been evolved in the course of some years for low powers, by converting D.C. to A.C., transforming to a different voltage and, if necessary, rectifying to produce D.C. again. This method was developed for the provision of a d.c. supply for the anode circuits of valves in portable-radio equipment and it has subsequently been used for a number of other somewhat similar purposes. The essential component in the d.c. to a.c. conversion is a changeover switch operated by a vibrating reed maintained in continuous oscillation by a magnet system; this device is generally known as a "vibrator." The use of a vibrating contact-breaker in conjunction with a step-up transformer with an open magnetic circuit, has been well known for many years in the form of the "spark-coil." This arrangement had, however, a low efficiency and it was not usually capable of operation for long periods unattended. A further disadvantage was the very peaky wave-form generated in the secondary. These disadvantages have been overcome in the modern vibrator systems by reversing the magnetic field instead of merely reducing it to zero. This has allowed a transformer with a closed magnetic circuit to be used, so greatly increasing the efficiency as compared with the open-cored type. At the same time the careful design and rigid control of the materials used in the vibrator have largely overcome the lack of reliability. As it is realized that the contacts have a finite life, however, the vibrator is arranged as a separate unit with plug and socket connexions to facilitate rapid replacement. The vibrator is used in conjunction with a transformer having a centre-tapped primary so connected that the d.c. voltage is applied first to one half and then to the other half of the primary, so magnetizing the core first in one direction and then in the other (Fig. 114). The result of this switching is to produce an alternating e.m.f. of rectangular wave-form in the secondary, and this output is used either as A.C. or if desired applied to a rectifier to produce D.C. The rectifier is in some cases a mechanical one, taking the form of additional contacts attached to the vibrator reed. A vibrator of this type incorporating rectifying contacts as well as the primary contacts is usually known as a "synchronous" vibrator.

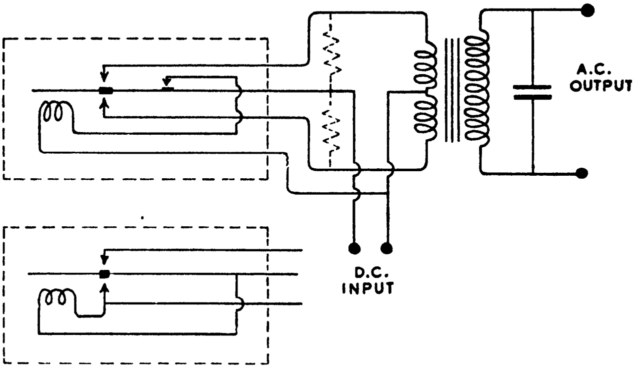

Fig. 114

The Circuit Arrangement of a Non-synchronous Vibrator. The Vibrator Itself is Enclosed in the Dotted Rectangle. The Upper Vibrator is of the Separately-driven Type; the Lower One is Shunt-driven.

From this brief description of the vibrator system it might appear that any transformer of suitable voltage ratings would be satisfactory, but this is not the case. The life of the vibrator contacts is very largely dependent on the choice of suitable values for the circuit constants, notably those of the transformer, and it is absolutely essential that the circuit should be designed as a whole and not as separate units. For that reason some information is given in the following section on the vibrator itself, so that a proper understanding of the whole system can be attained.

1. THE VIBRATOR

The most frequently used type of vibrator is that intended for the production of anode circuit supplies for radio equipment and drawing its power from a 6-, 12-, or 24-volt accumulator. Other types are produced to operate from 220-volt d.c. mains to supply an a.c.-type radio set. Illustrations of a low-voltage type are shown in Fig. 115. The magnet which drives the reed is clearly seen at the top; it is slightly displaced from the centre line of the assembly and hence from the end of the reed when in its position of rest. When the magnet is energized, it attracts the piece of soft iron with which the reed is loaded at its free end. The movement of the reed, however, disconnects the magnet circuit and the reed flies on past the pole piece. It is brought to rest by the springiness of the reed and eventually returned to its original position. In this way the reed is kept in continuous vibration. The magnet can be connected either through a contact of its own (the independent- or separate-drive type) or it may be connected in series with one of the half-primary windings (the shunt-driven type). In the latter case no special contact is required for the magnet, since it is automatically short-circuited by the closing of the main contacts.

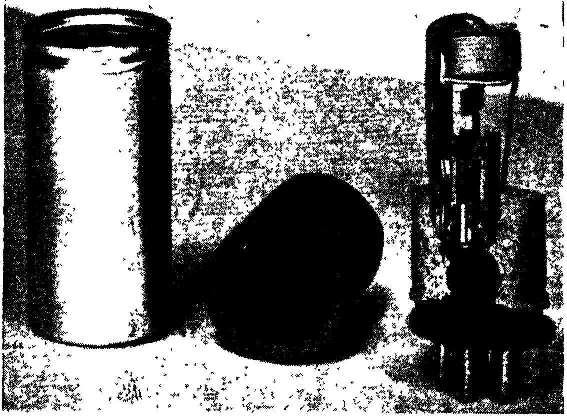

FIG. 115

A Typical Vibrator with Its Case, Showing the Sound-absorbent Lining.

The moving contacts are carried on subsidiary springs attached to the reed and butt against either one or other of the fixed contacts. The spacing is arranged in such a way as to reduce the transit time between the opening of one half of the primary and the closing of the other half to as small a proportion of the cycle as possible. In most vibrators the transit time is of the order of one-tenth of the cycle. To avoid the effects of stray magnetic fields of mains frequency, the reed frequency is usually removed as far as practicable from 50 cycles and its harmonics; the usual values are 85 and 115 cycles per second.

The normal vibrator contacts are rated to work up to some 4-5 amperes at either 6 or 12 volts, the driving coil being, of course, suitably wound for the particular voltage. Heavy-duty vibrators are also made for considerably larger currents at these voltages.

2. The Buffer Capacitor

If the circuit were completely non-inductive, as seen from the primary side, there would be very little sparking at the vibrator contacts, since the potential difference at the moment of separation would be only that of the battery. In practice, of course, the magnetizing current of the transformer cannot be neglected and the leakage reactance may also be appreciable, so that, as the contacts separate, a high e.m.f. is induced in the circuit by its self-inductance; this would cause such serious sparking that the contacts would soon be ruined if special precautions were not taken. Even apart from the effect of sparking on the contacts, however, there would be the difficulty of the serious interference which would be caused to the radio-receiving equipment by the radiation of "noise" from the contacts and associated wiring. The method of minimizing these troubles is to connect a capacitor of suitable value across either primary or secondary windings. In a general way the effect of the capacitor is sometimes regarded as bringing the current and voltage into phase again so that the circuit behaves as a non-inductive one. Under these conditions the primary current would be a minimum and if no oscilloscope is available for the method of adjustment described later, reasonably satisfactory results can usually be obtained by choosing a capacitor of such a value that the no-load current is at its lowest. Since the wave-form of the induced e.m.f. is a rectangular one and can be regarded as being made up of a fundamental together with a large number of harmonics of considerable amplitude, a correcting arrangement of this kind would only be effective at one frequency. It is, therefore, evident that the circuit cannot actually be completely compensated by the buffer capacitor and the choice of its value is a matter for experiment in each application.

A rough-and-ready method of adjustment has already been mentioned, namely that of adjusting to the minimum no-load current from the battery. A better method is to choose the capacitance value by examination of the secondary wave-form on an oscilloscope whilst the normal load is being supplied. Owing to the short interval which occurs at the end of each half-cycle while the reed transfers the battery supply from one half-primary to the other, there would be, in a non-inductive circuit, a short period of zero current. In the inductive circuit of the transformer the attempt to interrupt the current in this way gives rise to very high voltage peaks in the secondary and also in the primary, where the effect is apparent as sparking at the contacts as they separate. The capacitor (if connected across the primary) allows the current in the primary to go on flowing after the contacts have separated and it does so by discharging through the winding. The rate of change of current is accordingly greatly reduced and the e.m.f. induced in the windings is correspondingly smaller. The capacitor charge is, of course, lost to the primary and the potential falls rapidly. Oscillations would doubtless occur in the circuit formed by the capacitor and the primary inductance, but they are cut short after a fraction of the cycle by the closing of the circuit from the battery to the other half of the secondary. *When adjusting the capacitance value the object to be achieved is to make the capacitor potential fall at such a rate that there is no discontinuity at the moment of completion of the circuit to the second half of the winding.* In this way the least radio interference is produced and contact burning is reduced to a minimum. From the latter point of view the capacitance value is a compromise between the best performance at break and at make. At the break a large value would give the lowest potential difference between the separating contacts since it would tend to maintain the voltage across the coil at a value equal to that of the battery, but this necessarily results in too slow a fall and a sudden drop occurs when the contacts make on the other side. The closing of these contacts when there is a potential difference between them causes sparking and interference, so that a compromise capacitance value has to be chosen.

From this discussion it will be evident that the broad picture of the capacitor functioning as a phase correcting device is rather a misleading one, since it takes no account of the fact that a different capacitor would be required if nothing else changed but the contact efficiency (i.e. fraction of the cycle during which the contacts are closed).

It is usual in practice to make the capacitance value on the high side for a newly-fitted vibrator because in the course of continued operation the transit time of the reed tends to become a larger fraction of the cycle. This is to be expected owing to wear of the contacts, which increases their separation, and the tendency of their supports to spread somewhat under the continuous hammering they receive. If the optimum value of capacitor had been chosen when the new vibrator was first plugged in, the rate of fall of voltage at "break" would be too rapid and "overclosure" would result. The "closure" of the wave-form is the vertical height of the sloping portion of the wave expressed as a fraction of the total peak-to-peak height(1) (Fig. 116). Underclosure accordingly means that the frequency of the capacitance-inductance oscillation is not rapid enough to bring the voltage across the primary to a value equal to the battery voltage by the time the contacts close on the second half-cycle. Overclosure, on the other hand, means that the frequency of the oscillations is high enough to allow the circuit to pass the first quarter-cycle and the voltage begins to fall again before the contact closes. Overclosure produces sharp peaks of voltage and is a much more serious cause of interference and transformer breakdown than underclosure. For this reason the capacitor should be chosen to suit a worn vibrator with increased open-circuit intervals and some underclosure tolerated when new.

To simplify the explanation, the buffer capacitor has been assumed to be connected across the primary, but in most cases the capacitance needed to meet the requirements would be prohibitively large. It is usual, therefore, to connect the capacitor across the secondary and in that case the value required is reduced by a factor equal to the square of the step-up turns ratio in accordance with the rules for the value of impedances referred from one winding to another. The voltage applied to the capacitor is, of course, much higher under these conditions, but even the lowest rated capacitor would be much understressed on the primary so that there is a marked gain in cost and size if it is transferred to the secondary. Owing to the effects of magnetic leakage, the performance of a secondary buffer capacitor is not quite so satisfactory as one in the primary circuit, the effect of the leakage reactance being of course greatest at the high-harmonic frequencies. When conditions are particularly stringent, as in heavy-duty vibrators or when working at the higher battery voltages (e.g. 24 volts), the manufacturers recommend that part of the capacitance should be connected on the primary side.

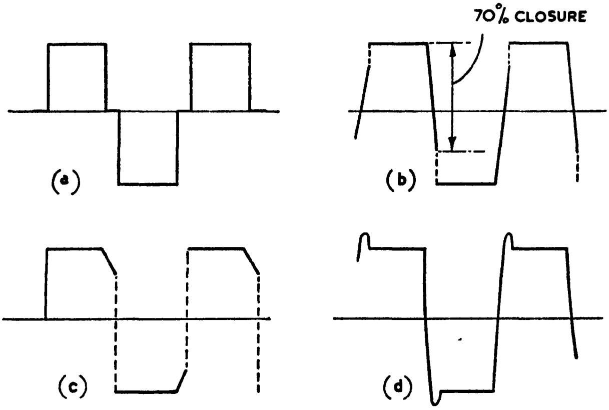

FIG. 116

Output Wave-forms from a Vibrator Circuit. (*a*) Ideal. (*b*) With Correctly Adjusted Buffer Capacitor. (*c*) Underclosure, i.e. Capacitor too Large. (*d*) Overclosure, i.e. Capacitor too Small.

3. THE TRANSFORMER

As already noted, the transformer for a vibrator power supply needs to be specially designed for the purpose. One obvious difference between a vibrator transformer and a normal one is the use of a centre-tapped primary necessitating a redistribution of winding space between the primary and secondary if a maximum efficiency is to be secured. Actually, instead of allowing practically equal spaces, the primary should be given $\sqrt{2}$ times as much as the secondary (assumed to be untapped). If, however, the secondary is also centre-tapped as when arranged for supplying a "synchronous" vibrator or a thermionic valve rectifier, this does not apply and equal spaces should be allowed. In practice it is often not possible to follow these proportions at all closely, because the low-voltage primary consists of quite a small number of turns of thick wire and the space allowed is frequently determined by such considerations as the need for restricting the winding to an integral number of layers. To reduce the wire size below that required to fill a whole number of layers would not allow the space to be used for the other winding, whilst an increase in size would take up an inordinate amount of extra space.

Another difference between a vibrator transformer and an ordinary one of similar output capabilities is its larger size. This is a result of the need for reducing the magnetizing current to a lower percentage than is usual in power transformers. It will be remembered that the reduction of the destructive sparking at the contacts to reasonably small dimensions depends on correct choice of the buffer capacitor size. If the circuit (as seen from the primary side) were completely non-inductive no current would flow during the period in which the reed is changing from contact on one side to contact on the other. If the circuit is inductive, as it always is in a transformer arrangement, energy is stored in the inductance and the current cannot be instantaneously interrupted without the release of the energy in the form of sparking. A properly chosen buffer capacitor is able to withdraw this energy from the circuit at the instant of break and return it later when required. This action is dependent on many factors such as contact timing, transformer inductance, etc., some of which change as the vibrator ages and others which, although constant in a particular unit, vary appreciably between different samples in a production run. Owing to the serious contact and other troubles which follow incorrect buffer capacitor action, the only safe way to handle the problem is to reduce the magnetizing current to as small a proportion of the load current as possible. A failure to adjust correctly will then result in a smaller proportionate energy unbalance judged as a fraction of the total load. Reducing the percentage magnetizing current involves an increase in primary inductance, and without a loss of efficiency this can only be brought about by the use of a larger core or a higher permeability material. The latter alternative would be possible only through the use of nickel alloys and these are prohibitively expensive. The only practicable method is accordingly to use more turns or a larger core section, either of which allows a lower flux-density.

The highest flux-density used in vibrator transformers is about 8 000 lines per cm^2, but as the input voltage variation which has to be tolerated is quite considerable the flux-density under *average* conditions is lower still.

The design of the transformer must not be based on transformer considerations only but on the system as a whole. It must take into account the fact that the vibrator drive itself takes something of the order of 1 watt, and this is in effect a no-load loss. There is accordingly no point in making the no-load losses of the transformer much smaller than this figure, since it would result in an increase in the full-load losses, and produce poor regulation. This is, of course, simply an example of the normal compromise which has to be made between a large number of turns giving low iron losses but high copper losses, and fewer turns with the corresponding increase in iron losses and decrease in copper losses.

When calculating the voltage drops in the windings, etc., the effect of the transit time of the contacts must not be forgotten. Owing to this open-circuit condition over a fraction of the cycle, the "flat top" value of the voltage or current waves is greater than their "half-cycle" mean value. The extent of the rise of peak value above the mean value depends on the shape of the wave-form assumed. If the fraction of the cycle during which the contacts are closed is k, we can write

$$I_{max} = I_{mean} \div \beta$$

where $$\beta = f(k).$$

If a simple rectangular wave-form with zero values at the change-overs (Fig. 116*a*) is assumed, then $\beta = k$. If the wave-form is assumed to be the ideal one obtained with 100 per cent closure, i.e. similar to (*b*) in Fig. 116, but with the changeover consisting of a single straight line from a flat-topped peak on the one side to that on the other, the value of β can readily be seen to be $\frac{1 + k}{2}$.

Since the winding resistances of vibrator transformers are usually kept quite low in order to obtain a high efficiency, the value assumed for β is not of very great importance. A reasonably accurate estimate of its value is, however, necessary in the later stages of a transformer calculation, in order to convert the output voltage from a peak value to the mean or d.c. value, if rectified, or to an r.m.s. value if retained as A.C. In this conversion, the accuracy of the final result is, of course, directly dependent on the value of β used, whereas in the voltage-drop calculations only a small correction is affected.

When two sets of contacts are involved as in a synchronous vibrator system, the values of k for the two sets are not usually the same. In a synchronous arrangement k is usually lower for the rectifying contacts than for the primary ones, so this lower value is used in the calculations.

As a matter of theoretical interest, it should be noted that unlike the normal condition with sinusoidal supply, the wave-forms of flux and induced e.m.f. are not the same. A moment's reflexion will show that the flux must take such a wave-form that its *rate of change* is proportional to the induced e.m.f. and it is only in the special case of sine and cosine waves that the two wave-forms are the same. Since the magnetizing current is proportional to the flux and the total primary current wave-form is somewhat modified by this component, the current and voltage wave-forms are not identical. In practice, however, there is little need to use a different value of β for the voltage-drop calculation.

The operation of this factor will be seen in the example which follows—

EXAMPLE

A transformer for a synchronous vibrator is wound on a square stack of 1-*in. No-waste laminations* (*Table XVII*, *p.* 471). *The secondary, consisting of* 2 600 *turns of SWG* 37 *enamelled wire centre-tapped, is the inner* *winding and the resistance of the whole winding is* 228 *ohms. The primary is also centre-tapped and consists of* 64 *turns of two paralleled strands of SWG* 21 *enamelled wire. The resistance of the whole winding is* 0·16 *ohm. The effective voltage applied to the half primaries, after allowing for drops in leads, noise-suppression chokes, etc., is* 5·85 *volts and the contact resistance is* 0·10 *ohm. Calculate the d.c. output voltage when supplying* 50 *mA.*

Assume the value of k for the rectifying contacts to be 0·7; the corresponding value of β for 100 per cent closure is then 0·85.

The peak input voltage to the transformer half-primary is 5·85 volts, so that the peak induced voltage per half-secondary would be

$$5{\cdot}85 \times \frac{1\,300}{32} = 238 \text{ volts.}$$

From this must be deducted resistance drops in the windings. This entails estimating the peak currents in the windings. Using a figure of 0·85 for β in the secondary we get a peak current of 50/0·85, i.e. 59 mA. Since the primary circuit is actually broken completely by the primary contacts, β must be taken as being equal to k, namely 0·70. (This remark is not applicable to the secondary since the buffer capacitor is permanently connected across it, and supplies current during the open-circuit periods of the secondary contacts.) The peak value of the load component of the primary current is then

$$\frac{1\,300}{32} \times \frac{0{\cdot}05}{0{\cdot}70} = 2{\cdot}9 \text{ amp.}$$

To this must be added the exciting current component. The peak flux-density is best calculated from first principles, as in Chapter III, p. 53. The input voltage of 5·85 is the peak value, so this must be multiplied by β to obtain the mean value. Although a figure of 0·7 was taken for the primary *current*, the primary *voltage* will have the same wave-form as the secondary voltage since the same flux links both. The value of β for the primary voltage wave is then 0·85. The flux changes from $+\Phi_{max}$ to $-\Phi_{max}$ in a half-period so that the rate of change of flux with time is $4\Phi_{max}f$ lines per second, giving an average e.m.f. of

$$4\Phi_{max}fT_p \times 10^{-8} \text{ volts.}$$

The peak flux-density is then

$$\frac{5{\cdot}85 \times 0{\cdot}85}{4 \times 0{\cdot}91 \times 2{\cdot}54^2 \times 115 \times 32} = \text{approx. 6 kilolines per cm.}^2$$

The iron loss in 0·014-in. 4 per cent silicon steel for a sinusoidal voltage at $B_{max} = 6$ kilolines per cm², is approximately 0·3 watt per lb and, as the weight of the core is 1½ lb, the total iron loss would be 0·45 watt. For a vibrator-driven circuit, the loss is larger and, in addition, the magnetizing current, although in quadrature, will make some addition to the resultant current. A reasonable estimate for the total peak primary current would thus be 3·2 amp.

The total resistance of the primary circuit per side including the contact is (0·08 + 0·10) ohm, giving a peak voltage drop of 3·2 × 0·18 volt, i.e. 0·58 volt. Referred to the secondary, this becomes

$$0{\cdot}58 \times \frac{1\,300}{32},$$

i.e. 24 volts. The secondary voltage drop is 0·059 × 114, i.e. 7 volts, making a total drop of 31 volts. The peak output voltage is then 238 − 31 = 207 volts. Using a value of β of 0·85, the *mean* output voltage is 207 × 0·85 = 176 volts. This is, of course, the voltage to be expected across a resistive load without a reservoir capacitor. The addition of a 16 μF capacitor would raise the output voltage to a figure approaching the peak value of the secondary voltage, i.e. 207 volts.

Measurements made on a transformer gave results in good agreement with calculation as will be seen from the figures below, but it should be emphasized that many approximations have been made, particularly in the selection of the value of β. If the voltage drops were a larger fraction of the induced voltage, errors would have a more serious effect on the result. It is interesting to note, too, that the largest single voltage drop is due to the primary contact resistance of the vibrator, which is by nature a particularly uncertain quantity. Great accuracy cannot, therefore, be expected in the calculated output voltage. Another consequence of the high drop (when referred to the secondary) is that the use of a larger transformer with lower winding resistances would not greatly increase the overall efficiency of the convertor.

	CALCULATED	MEASURED
Output voltage, without reservoir capacitor	176	173
Output voltage, with 16 μF capacitor .	207	204
Input current (mean)	2·25	2·33

4. SPECIAL VIBRATOR CIRCUITS

Although the normal type of vibrator circuit eliminates sparking at the contacts almost completely if correctly adjusted, there is always a chance that changes in contact timing, output loading, etc., will give rise to trouble in that respect. A circuit has been developed by O. Kiltie,(2) in which the circuit is broken at or near zero current, thus avoiding sparking and allowing the output obtainable from a given sized vibrator to be considerably increased. The essential feature of the arrangement is the introduction of a large capacitor in series with the primary circuit and arranged so that it is charged and discharged by the vibrator contacts through the transformer primary. Since the charging current will fall practically to zero after a suitable time interval, it is possible to break the circuit without sparking.

A number of other features result; the wave-form is more nearly sinusoidal (this may or may not be advantageous), no harm is done to the windings if the contacts stick; short-circuits on the output side do not matter. The last advantage makes the system particularly suitable for the running of gaseous-discharge tubes since the short-circuit current is limited.

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(2) O. KILTIE: "New Type of D.C. to A.C. Vibrator Inverter." *A.I.E.E. Trans.* 59 (Apr., 1940), pp. 245–247.

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CHAPTER XIX

Instrument Transformers

IN all the cases so far discussed a transformer has been employed as a means of delivering power to a load at a pressure different from that at which it was supplied, without the use of rotating machinery. The application now to be considered is that of providing, often at a distance, a close copy of a voltage or current in a circuit for the purposes of measurement. In d.c. circuits, a low resistance can be inserted in any conductor through which is passing the current to be measured and the voltage drop in millivolts indicated on a meter located some way away on a switchboard. In a.c. work the same result can be secured by the use of a *current transformer*, with the additional advantage that the meter circuit is completely isolated from the conductor which may be at a potential of many thousands of volts relative to earth. Just as in the d.c. case, the actual current flowing through the meter is usually much lower than that in the main circuit. The important point is that the secondary or meter current is always related to the main current by a constant and accurately known factor. The meter scale is then calibrated to read the current in the main circuit, i.e. the true meter current is multiplied by the current transformer ratio.

In addition to the current transformer, there is the *voltage or potential transformer*, which enables voltage readings to be taken safely at a distance from the points under observation. This is a most important feature when it is realized that readings of hundreds of kilovolts are involved when controlling power transmission networks. A voltage transformer closely resembles a small power transformer except that the insulation requirements are often much greater than would usually be applicable to power transformers of similar size. Current and voltage transformers are described as instrument transformers, from the use to which they are frequently put, but actually large numbers are also used in power-circuit protective systems for dealing automatically with faults. By connecting the secondaries of two current transformers in opposition, so that under normal conditions there is no output, a condition in which the balance is upset by some fault such as a breakdown to earth can be detected immediately and the circuit isolated.

When used for measuring purposes, it is important that instrument transformers should not only give a true copy of the original voltage or current on a known scale of reduction but should also deliver an output voltage or current which is in phase with the corresponding voltage or current in the circuit being investigated. This requirement arises from the fact that the instrument transformers are often used not only for measurements of voltage and current but also in combination for power measurements. If the magnitudes of the voltages applied by the transformers to the pressure and current coils of a watt-meter are correct, but there is a phase shift between one or both of them and the corresponding quantities in the main circuit, an error will arise in the apparent power-factor and hence in the power reading. This error is least on unity power-factor loads and increases rapidly with the phase angle between voltage and current in the load. Let the phase angle between voltage and current in the load be ψ and the combined phase-angle errors of the instrument transformers be α. Then the watt-meter reading instead of being $VI \cos \psi$ will be $VI \cos (\psi - \alpha)$. But $VI \cos (\psi - \alpha) = VI (\cos \psi \cos \alpha + \sin \psi \sin \alpha)$. If α is very small then $\cos \alpha = 1$ and $\sin \alpha = \alpha$ (circular measure). The reading then differs from the true value by the second term $VI \sin \psi \sin \alpha$. If ψ is small (i.e. near unity power-factor), the error term becomes vanishingly small. As ψ increases, however, the error becomes greater. It will thus be understood that a small phase error is an important requirement of an instrument transformer, particularly if it is required for use in circuits of low power-factor.

The phase-angle is regarded as positive when the output current leads the reversed primary current(1). The corresponding convention is accepted for voltage transformers in most countries.

The "load" applied to the secondary winding of an instrument transformer is described as the *burden* rather than load; the latter term is reserved for the circuit being measured.

The burden often consists of several meters or other protective devices, connected in parallel to a voltage transformer or in series to a current transformer, and may be up to 200 VA for voltage transformers and 40 VA for current transformers. Since the burden is likely to be inductive rather than capacitative, the convention is to regard the phase-angle as *positive* when the voltage across the burden *leads* the current through it.

1. Voltage Transformers

A voltage transformer closely resembles a small power transformer, although the size may actually be quite considerable if the primary voltage is very high. Voltage transformers for use on 132 kV, for instance, may stand 13 ft high and weigh several tons, although the volt-ampere output may be only some 200 or so. This large size is a result of the insulation difficulties on the primary side and between windings. It is, of course, essential that there should be no avoidable risk of breakdown between the high-voltage side and the secondary, because this would nullify the safety advantages which result from the use of an instrument transformer. A voltage transformer is subjected to lightning surges in just the same way as a power transformer and the stresses tend to be greatest across the end turns of the winding. The "non-resonating" method of construction, which is usually fully applied only to power transformers working at the very highest voltages, can often be adopted with advantages in size reduction in potential transformers at lower voltages (Chapter XI).

The secondary voltage is always arranged to be a standard value irrespective of the nominal primary voltage. The standard voltage in Great Britain and many other countries is 110, but in the U.S.A. a voltage of 115 is adopted. The ratio is quoted in the form *primary voltage/secondary voltage*, e.g. 11 000/110.

Errors and Their Correction

If a potential transformer were a "perfect transformer" the ratio of primary to secondary voltages would always equal the turns ratio and the primary and secondary induced e.m.f.s would be in phase. In an actual transformer, however, there are certain departures from the ideal which introduce ratio and phase-angle errors; the windings have resistance, the core requires a finite magnetizing force to produce the working flux, there are losses in the core, and there is a leakage of flux between primary and secondary, the results of which can be depicted as leakage reactances in series with the input and output. The result of the core defects is to cause a current to flow in the primary even when no secondary burden is connected and the consequent voltage drop in the primary impedance makes the secondary voltage lower than it should be. To correct this, turns can be added to the secondary or removed from the primary, and complete correction can be obtained in this respect. When a secondary burden is applied, there will be a voltage drop in the secondary impedance and a further drop in the primary due to the load component of the primary current. *At a given burden* these drops can be corrected by a further adjustment of the turns ratio, but at any other burden there will, of course, be over- or under-correction. The power-factor of the burden is important in its effect on the ratio error. The burden is usually inductive and the leakage reactances then cause a drop in output (i.e. an increase in ratio), but if the burden is capacitative the effect tends to be reversed. The ratio error corresponds to the voltage regulation of a power transformer which can only be arranged to give its nominal voltage at a particular load.

The phase-angle error too is due to the finite resistances and reactances of the windings and the no-load primary current, but it cannot be corrected by turns ratio modifications. It can only be minimized (but not removed) by attention to the dimensions of coils and core. The resistances are kept down by making the turns as short as possible and of sufficiently large cross-section, whilst the leakage resistance is reduced by suitable positioning of the windings. At normal power frequencies, interleaving of primary and secondary coils is not usually necessary. The no-load current is made as small as possible by the use of high-grade materials, working at a lower flux-density than in power transformers, and by using cores with a short magnetic path.

In spite of the possibilities of errors due to the departures from the ideal mentioned earlier, a voltage transformer can be, and usually is, a highly accurate instrument. A factor which contributes largely to this state of affairs is the fact that the input conditions usually vary only slightly during operation. In power distribution systems, the voltage is maintained within a few per cent of the nominal figure, whilst the frequency is even more closely standardized. In consequence the transformer is always used at or near the conditions under which it has been calibrated. The only variable factor of importance under these conditions is the burden, and a variation in ratio between zero and full output of only 1 per cent is fairly easily attained, whilst a figure of 0·2 per cent can be reached by first-class transformers. The phase-angle error need not exceed twenty minutes and is often much less.

Transformers used in testing and laboratory work are, of course, likely to be operated under much wider ranges of primary voltages but, even at input voltages considerably removed from the nominal, errors are seldom important. In any case, a correction can always be applied in laboratory work without great inconvenience. The extent of the ratio and phase-angle error variations with *voltage* is greater at low-burdens than at full-rated outputs, but in both cases the ratio tends to increase relatively to the nominal ratio as the voltage is reduced. This means that the proportion of no-load current to load-component current in the primary increases at lower voltages—a result which would not be expected from power transformer practice, where a reduction in primary voltage causes a disproportionately large fall in exciting current. The reason for this difference lies in the much lower flux-densities used in instrument transformers at rated voltage conditions. When the flux-density is still further reduced at low inputs, the inductance tends to fall; this result occurs also in audio-frequency communication transformers (Chapter XX). The effect of *frequency* on the ratio is, as might be expected, in the direction of an increase in ratio above the nominal at lower frequencies, due to the increase in magnetizing current which is the predominating component in the exciting current. Frequency variations have a greater effect on the phase-angle than on the ratio.

Since a voltage transformer so closely resembles a power transformer, the theory is similar in a qualitative way, although the magnitudes of the various quantities may be considerably different. The vector diagram given in Fig. 117 exaggerates the magnitudes of the various impedance drops; it will be seen to be similar to that of a power transformer. Starting from the flux as the zero reference line, the induced e.m.f. in the secondary E_s lags 90° behind. The primary induced voltage would also lag 90°, but for clarity it is shown reversed ($-E_p$) as a component of the applied voltage. The secondary current I_s lags behind E_s by an angle ψ_B determined by the phase-angle of the secondary circuit as a whole (assumed to be inductive). This current gives rise to resistive and inductive drops which have to be subtracted from E_s to give V_s the secondary terminal voltage. The primary current I_p, made up of the exciting component I_0 and the load component I_B', gives rise to resistive and inductive drops which are added to $-E_p$ to give the primary voltage V_p.

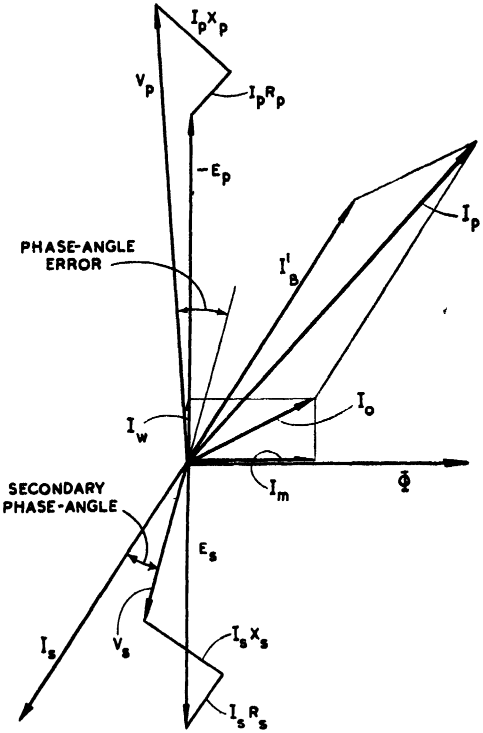

FIG. 117
Vector Diagram for a Voltage Transformer (Errors Greatly Exaggerated).

Quantitative Relationships

The equivalent circuit of a voltage transformer is again similar to that of a power transformer. Since accuracy is of greater importance in measurements than in power work, it is, however, essential to include elements representing the leakage inductances. The primary voltages vary over a very wide range, but the secondary voltage is always either 110 or some other standard figure close to it. It is, therefore, desirable to use quantities referred to the secondary throughout so that comparisons can easily be made between different transformers. The circuit shown in Fig. 118 is drawn in this way; the burden is represented by the impedance Z_B. The *turns ratio* of the transformer (T_p/T_s) has been represented by n so that the quantity R_p, the primary resistance, appears in the diagram as R_p' ($= R_p/n^2$), and the primary leakage inductance as L_{lp}' ($= L_{lp}/n^2$). The primary terminal voltage V_p appears in the diagram as V_p' ($= V_p/n$), whilst the primary current I_p is referred to the secondary as I_p' ($= I_p \times n$). This current I_p' is made up of I_o' and I_B where I_o' is the exciting current referred to the secondary and $I_B = - I_s$.

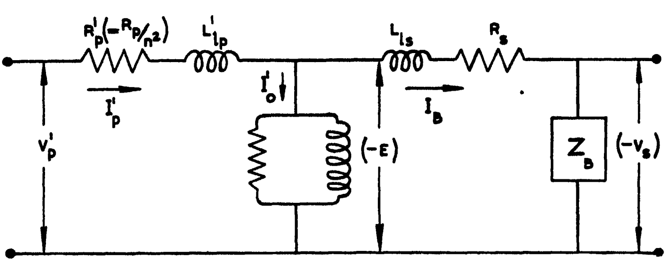

FIG. 118

Equivalent Circuit of a Voltage Transformer Referred to the Secondary. Z_B Represents the Instrument Burden.

The corresponding vector diagram is shown in Fig. 119. In order that the vector quantities can be readily associated with the equivalent circuit of Fig. 118, the secondary voltage is shown reversed as $- V_s$, and the induced e.m.f. referred to the secondary is shown as $- E$. (No suffix is required because $E_p' = E_s = E$.) By showing the reversed vectors $- V_s$ and $- E$ in this way, it is immediately evident that V_p' exceeds $- E$ by the voltage drop in the primary due to the flow of load component and exciting currents, whilst $- V_s$ is less than $- E$ by the secondary voltage drop due to the flow of I_B.

Expressed algebraically these relationships take the form—

$$V_p' = - E + (I_o' + I_B)(R_p' + jX_{lp}')$$

and

$$- E = - V_s + I_B(R_s + jX_{ls})$$

Hence $V_p' = -V_s + I_B(R_s + jX_{ls}) + (I_o' + I_B)(R_p' + jX_{lp}')$

$$\therefore \quad V_p = nV_p'$$

$$= n\{-V_s + I_B[(R_s + R_p') + j(X_{ls} + X_{lp}')] + I_o'[R_p' + jX_{lp}']\}$$

$$V_p/(-V_s) = n\left\{1 + \frac{I_B}{(-V_s)}[(R_s + R_p/n^2) + j(X_{ls} + X_{lp}/n^2)] + \frac{I_o}{n(-V_s)}[R_p + jX_{lp}]\right\} \quad . \quad . \quad . \quad (96)$$

The actual voltage ratio is thus seen to be equal to the turns ratio modified by terms representing the effective impedance drop and the primary excitation drop (seen from the secondary) expressed as a fraction of the secondary voltage.

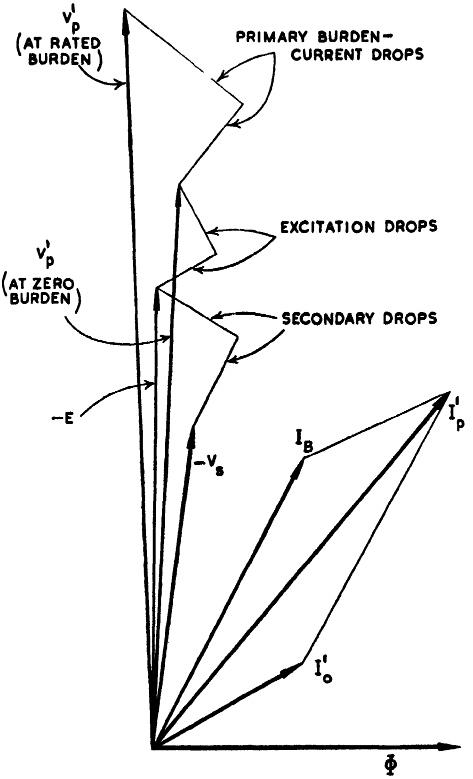

FIG. 119

Vector Diagram Corresponding to the Circuit of Fig. 118. All quantities are Referred to the Secondary.

*The Möllinger and Gewecke Diagram**

The performance of a voltage transformer can be predicted very conveniently by graphical methods making use of data readily obtained from "open circuit" and "short-circuit" measurements. First consider a normal vector diagram of the type shown in Fig. 117, but with all quantities referred to the secondary. If the flux is taken as lying in the reference direction, the induced e.m.f.s will lag 90° behind (Fig. 119); the vertical direction therefore represents the reversed primary e.m.f. referred to the secondary.

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* For fuller particulars see B. Hague: *Instrument Transformers*, upon which this and the corresponding section of Current Transformers is based, by kind permission of the author.

The referred excitation current I_0' is represented by a vector lagging on the primary voltage. The primary excitation resistance and reactance drops are therefore represented by vectors parallel and perpendicular respectively to the exciting current vector. These quantities are accordingly added vectorially to the reversed primary e.m.f., together with similar vectors due to the transferred load-component drop in the primary impedance. The resultant vector V_p' therefore represents the primary voltage. The secondary drops due to the burden current are subtracted from the induced e.m.f. so giving the secondary voltage V_s. In an ideal transformer, all the impedance drops would have been zero, leaving the transferred primary and the reversed secondary voltages equal in magnitude and in-phase with each other. In an actual case, the lack of equality in the lengths of the vectors V_p' and V_s represents the ratio error and the angle between them represents the phase-angle error. Important though these deviations from the ideal may be, they are much too small to allow useful results to be obtained by graphical methods using the vector diagrams in the normal form, but by making modifications first proposed by Möllinger and Gewecke,⁽²⁾ the performance of a voltage transformer can readily be predicted under different conditions of burden.

The vector diagram of Fig. 119 is incapable of accurate interpretation because the lengths of the vectors representing impedance drops are small compared with the terminal voltages. To enable the significant quantities to be measured with reasonable accuracy, the origin is transferred to the point *O′* (Fig. 120) and the direction of the secondary terminal voltage vector V_s is assumed to be vertical, an approximation which is quite justified in view of the small phase displacement between E_s and V_s. The lower part of the diagram consisting of two practically vertical lines representing V_s and V_p' is omitted, and the part above the origin can then be drawn to a sufficiently large scale for useful accuracy by graphical methods. From *O′*, vectors representing the transferred *resistance and reactance drops in the primary due to the excitation* current are drawn to a scale representing percentages of V_s. The resultant impedance drop is then represented by $O'G_0$ and the referred primary voltage V_p' would be indicated by the vector OG_0. With the secondary on open circuit, the percentage ratio error (relative to the turns ratio n) is represented approximately by the distance $O'H_0$ by which the transferred primary voltage exceeds the reversed secondary voltage. The phase-angle between primary and secondary reversed voltages is represented by the distance $O'J_0$ in circular measure. If the units of the vertical scale are percentages of V_s, then units of equal length horizontally represent hundredths of radians of phase-angle error.

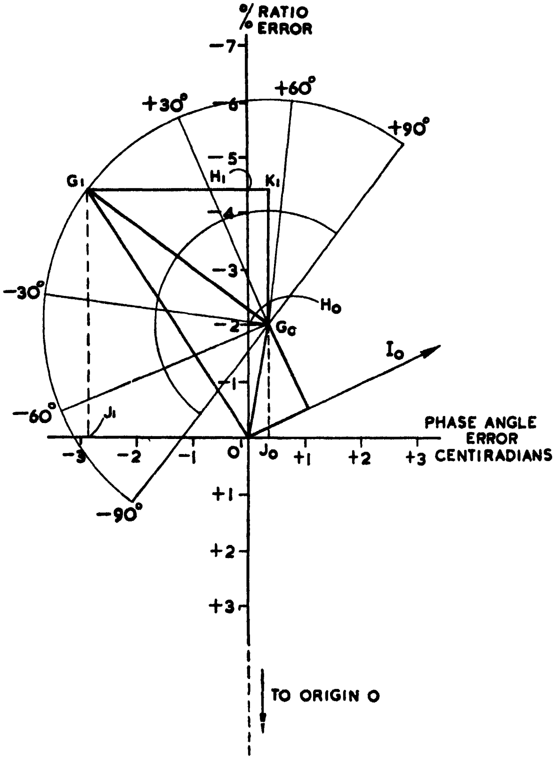

FIG. 120

The Mollinger and Gewecke Diagram for a Voltage Transformer.

So far the errors have been examined on the assumption of a zero secondary burden. If a non-inductive burden is now connected so that a current I_B passes and is in phase with V_s, there will be a further resistance drop $I_B(R_s + R_p')$ represented by a vertical line G_0K_1 and a reactance drop $I_B(X_{ls} + X_{lp}')$ represented horizontally by K_1G_1. The line G_0G_1 then represents the total impedance drop due to the burden current and the corresponding component of the primary current. The percentage ratio error is then $O'H_1$ and the phase-angle error is $O'J_1$. If reference is now made to equation 96 (p. 346) it will be seen that the primary voltage is equal to n times the reversed secondary voltage plus terms representing the impedance drops in the windings. The lines G_0G_1 and $O'G_0$ in Fig. 120 represent these impedance drops graphically.

To deal with burdens of the same magnitude, but different phase-angle, the line G_0G_1 is rotated about G_0 to the required angle and the ratio and phase errors can be read off from the projections on the axes. If the magnitude of the burden is altered, the length of G_0G_1 is altered, as indicated by the arcs drawn around G_0 as a centre. In this way the errors occurring with any combination of burden magnitude and power-factor can at once be predicted with a high degree of accuracy.

Applying the M and G Diagram

In order to be able to draw a Möllinger and Gewecke diagram for an actual transformer it is necessary to know certain particulars. The information required is the turns ratio, primary and secondary resistances and reactances, no-load current and phase-angle. The winding resistances are determined by one of the normal methods, such as a bridge. A measurement is made of the input voltage and current necessary to pass the maximum rated secondary current round the short-circuited secondary winding. The total equivalent impedance is calculated from the input voltage and current; the value is then referred to the secondary. The equivalent resistance of the whole transformer is next calculated and the equivalent reactance obtained by subtraction in quadrature from the impedance. The assumption is then made that the reactance is equally distributed between primary and secondary, thus enabling the no-load reactance drop to be calculated from the no-load current. The point G_0 on the diagram is now plotted, since the no-load resistance drop is readily calculated. The equivalent resistance and reactance have already been found, so G_0G_1 can be drawn for any desired burden.

It will be observed that the phase error falls to zero when the burden has a certain value of phase-angle. This burden is inductive and the ratio error is a maximum under these conditions. For burdens of smaller phase-angle, and for resistive or capacitative ones, the phase error is negative. The ratio error is regarded as negative when the ratio is higher than the nominal, i.e. when the meter attached to the secondary reads low. If the turns ratio is equal to the nominal ratio, the ratio error is always negative except when the burden is highly capacitative. It is, therefore, desirable to reduce the turns ratio slightly to compensate, so that the ratio error is zero at a burden which is likely to be used.

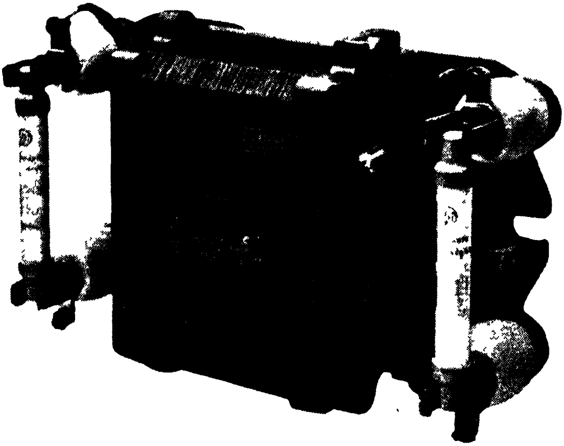

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FIG. 121

A Voltage Transformer Suitable for the Lower Voltages.

Construction

Voltage transformers for fairly low primary voltages (up to say 4 kV) are usually of the dry type. A core-type of construction is used, with windings arranged concentrically on one or both limbs. The secondary is usually next to the core and is carefully insulated from the outer winding by a tube of high dielectric strength. A typical transformer is shown in Fig. 121.

For higher primary voltages, oil filling is often used, but the present tendency is to use porcelain-insulated dry types for indoor applications on account of the fire risk with the oil-immersed construction.

Special cascade arrangements are adopted for voltages above 100 kV.

2. Current Transformers

A current transformer is used to measure currents in a circuit which, on account of its dangerous potential or for other reasons, cannot be brought to the measuring point. Current transformers are also used with protective gear for safeguarding equipment when a fault occurs. The instruments or other devices are always connected in series to the secondary; if it is necessary to disconnect the burden, it is essential to *short-circuit* the secondary to prevent dangerous voltages or damage to the transformer. A secondary current of 5 amperes has been standardized internationally, although for special purposes where very long leads are necessary a current of 1 amp may be adopted. These currents are those which flow in the secondary when the rated primary current is flowing. The nominal current ratio is expressed as the fraction Rated primary current/Rated secondary current, e.g. 750/5.

The accuracy obtainable from a current transformer tends to be lower than that from a voltage transformer, because in addition to possible variations in the magnitude and power-factor of the burden, the nature of the duty is such that there are wide variations in the primary current.

Theory of Operation

The instrument burden connected to the secondary, together with the winding resistance and reactance, form a circuit through which a secondary current I_s will flow and this current will lag behind the secondary induced voltage E_s by an angle ψ_B determined by the burden (assumed inductive), as in Fig. 122. E_s is induced by a flux in the core, and lags 90° behind it. This flux is produced by a magnetizing current component in the primary current, in phase with the flux. There is also a component in the primary supplying the core losses and these two together form the exciting current I_o. The load component I_B' of the primary current is opposite in phase to I_s and of such a magnitude that the primary and secondary ampere-turns are equal and opposite, i.e.

$$I_B'T_p = -I_sT_s$$

or

$$I_B' = -I_s/n$$

The vectorial addition of the load and exciting components of the primary current gives I_p. The actual ratio of the transformer is $-I_p/I_s$, whilst its phase-angle is the angle by which the secondary current leads the reversed primary current vector.

For another value of primary current, the value of E_p changes, so altering I_o and hence disturbing all the vectors concerned. The ratio and phase angle therefore vary with the primary current.

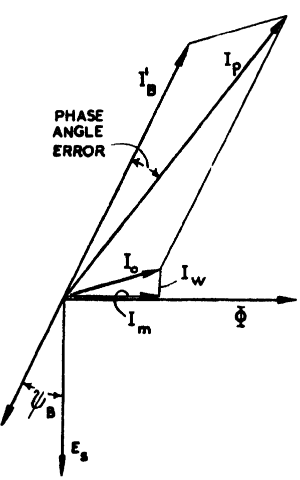

FIG. 122
Vector Diagram for a Current Transformer.

Graphical Determination of Performance

Just as with voltage transformers, highly accurate assessments of ratio and phase errors can be made by graphical methods, provided that the vector diagram of Fig. 122 is suitably adjusted. The method described below is due to Möllinger and Gewecke.⁽³⁾

First of all, imagine the vector diagram of Fig. 122 redrawn on such a scale that the unit is the rated secondary current I_s (i.e. normally 5 amp). This result is achieved by dividing all the quantities by I_s. The zero reference direction is also chosen for convenience to be such that I_s is vertically downwards from the origin. I_B'/I_s $\left(= \frac{1}{n}\right)$ is then drawn vertically upwards from the origin, its length being OO'. E_s leads I_s by the phase-angle of the secondary circuit; this angle is determined by the burden and secondary winding characteristics. The flux then leads the induced e.m.f. (E_s) by a further 90°. The magnetizing component of the primary current is in phase with the flux, so a distance I_m/I_s is marked along the flux vector and since the loss component I_w leads the flux by 90°, a distance I_w/I_s is marked off perpendicularly to the flux. It is convenient to transfer these components I_m/I_s and I_w/I_s to *O′* separately, as will be apparent later. In the particular case illustrated, the resultant total primary current obtained by adding I_m/I_s and I_w/I_s to I_B'/I_s is the vector *O′G*. The distance by which the projection of *OG* on the vertical axis reaches above *O′* is then a measure of the ratio error and the projection on the horizontal axis indicates the phase error expressed in circular measure. This assumes that the phase error is only a few degrees, as is always the case.

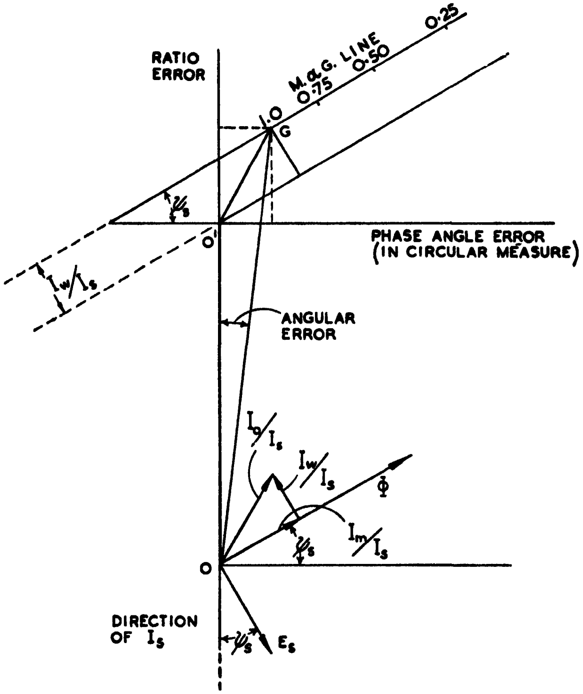

FIG. 123
The Vector Diagram of Fig. 122 Arranged for the Graphical Determination of Ratio and Phase-angle Errors.

The diagram has been drawn for the full-rated current; in order to show the performance at other currents it is necessary to know something of the behaviour of the core under different magnetization conditions. These results are obtained from an "open-circuit" test. The transformer secondary is connected to a voltage-measuring device which takes no current from it (e.g. a potentiometer) and the exciting current and power supplied to the primary are measured for various values of secondary voltage. Curves are then plotted of magnetizing and loss-component currents against secondary voltage. At the low values of flux-density employed, it will be found that the loss component curve is practically a straight line, and since $V_s = I_s Z_B$ and Z_B is assumed constant, the quantity I_w/I_s is independent of V_s. The loss component to be added to the load component of the primary current is, therefore, independent of the fraction of full current which is passing, so that a line known as the Möllinger and Gewecke line can be drawn distant I_w/I_s from *O′* and inclined at an angle equal to that of the flux vector. Distances representing the quantity I_m/I_s are next marked off along the "M and G" line for different fractions of rated current and these points represent the positions of *G* for those current values. The shape of the I_m, E_s curve is such that I_m/I_s becomes greater for small currents, so that the errors of ratio and phase become greater at the lower current readings.

For graphical purposes the portion of the diagram below *O′* can, as in the voltage transformer diagram, be omitted and the upper portion drawn to a sufficiently large scale to allow accurate working. If only the upper part is shown, there is no point in actually dividing the quantities by I_s since this only affects the size of the scale divisions. The procedure then becomes as follows—

Draw a vertical line through the origin (which corresponds to *O′* in the previous diagram) and set off from it an angle representing the phase angle of the secondary circuit. The direction so obtained is that of the secondary induced voltage vector. At right angles to the E_s vector and passing through the origin, draw the flux vector. Parallel to the flux vector and distant from it by a length representing I_w for full-rated current draw the "M and G" line. Any convenient scale can be adopted for I_w. The points representing various fractions of full current are now marked off along the "M and G" line, using the same scale as for I_w. The value of I_m for full-rated current is first marked and points are also marked for fractions of full current equivalent to the values of I_m corresponding to these currents, divided by the fraction in question. Thus, at half-current multiply the value of I_m for that current by two and mark off along the "M and G" line. One-per-cent ratio error is 50 milliamp on a 5-amp secondary current, so that the vertical scale of percentage ratio errors can now be marked using the scale adopted already for I_w. On the horizontal axis, a similar distance represents $\frac{1}{100}$ radian of phase error.

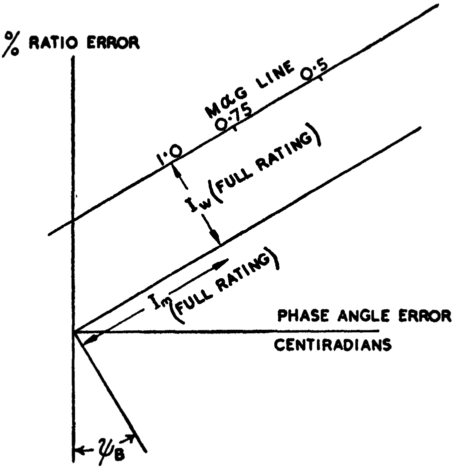

FIG. 124

The Essential Parts of Fig. 123 in the Form of an M & G Diagram for a Current Transformer at Various Fractions of Full-load.

The diagram as so far drawn enables the performance to be deduced for various currents but with a burden fixed in magnitude and power-factor. If the magnitude remains constant, the burden power-factor is dealt with by rotating the "M and G" line through the appropriate angle. If, however, the magnitude of the burden changes, the value of I_w is different and the "M and G" line is moved parallel to itself by the appropriate amount.

The Möllinger and Gewecke diagram can be used to predict the errors in a current transformer provided that the resistance and reactance of the secondary windings and burden are known, together with the turns ratio and the results of an "open-circuit" test. The assumption that the leakage reactance is shared equally between secondary and primary is not, however, usually valid in current transformers, where the secondary leakage is usually much smaller than that of the primary. This is particularly true of the "bar-type" transformers (see later). The method is accordingly more useful for checking a proposed design than for use with a completed transformer.

Reducing Errors

The Möllinger and Gewecke diagram for a current transformer shows clearly the magnitude and causes of the ratio and phase errors. These are shown to be due to the existence of current components in the primary circuit supplying magnetizing ampere-turns and core losses. If these current components could be eliminated, there need be no errors. It is worth noting that unlike the voltage transformer, the resistance and inductance of the windings do not directly introduce errors; they only affect the accuracy through increasing the exciting current. To give the lowest errors the magnetizing and loss currents must be kept as small as possible by suitable design of the core and selection of the material.

The magnetizing current is kept low by making the iron path as short as possible and of a large cross-section. An important feature is the elimination of all air-gaps. The laminations used in a current transformer are not usually of the split type which are assembled round a wound core such as the "E" and "I" shapes used in power transformers. The usual type of lamination is a continuous circular ring, upon a stack of which the windings are wound in toroidal form.

A more modern method is to wind into the completed coil system a continuous ribbon of core material. This enables the improved properties of the silicon-steel in the direction of rolling to be put to good use. In either case, air-gaps across the direction of the flux are avoided, so increasing the inductance and reducing the magnetizing current. The loss component of the exciting current is reduced by the use of high-grade laminations usually rather thinner than those used in power transformers. Care is taken not to introduce eddy-current paths through faulty assembly or in the clamping arrangements.

Even when all precautions have been taken to reduce I_m and I_w, the errors in silicon-steel cored current transformers are usually rather higher than is generally desirable. In particular the phase-angle error is often excessive for wattmeter measurements and until recently various expedients were used to lower the errors further. For example, since the actual ratio is higher than the turns ratio for all values of burden power-factor which are likely to be experienced, it is possible to compensate the ratio error at an average burden by reducing the secondary turns slightly. Again, by using resistive or capacitative shunts across primary or secondary, partial correction of the phase errors can be obtained. The flux-density used in the core of a current transformer is quite low (under 2 000 lines per cm^2) in order to keep the iron loss down, but at this value of B, the permeability is below its maximum, which is reached at a value of B which is several times greater. An auxiliary excitation was, therefore, sometimes provided so that the iron could be worked at its highest permeability. Most of these devices, with the exception of the simple one of adjusting the turns ratio slightly, have now been rendered obsolete, however, except perhaps for special laboratory current transformers, by the introduction of the new nickel alloys which enable the exciting current to be reduced at least ten times for the same maximum output.

Although the high-permeability nickel alloys were developed for the purpose of adding inductive loading to submarine cables, their excellent magnetic properties quickly proved extremely valuable in a number of branches of transformer engineering. The high cost prevents their use in power transformers, except for very special purposes and in the smallest sizes, and indeed the advantages are not very marked at normal power frequencies. This is because the nickel alloys are essentially low flux-density materials; at the values of B used in most 50-cycle transformers a good silicon steel is better.

The alloys in general use in England for current transformer cores are known as "Radiometal," "Mu-metal," and "Permalloy," and in the U.S.A. "Hipernik" is also used. "Hipernik" and "Radiometal" are 50 per cent nickel 50 per cent iron alloys, whilst "Mu-metal" and "Permalloy C" are 76–79 per cent nickel with small percentages of copper and molybdenum or chromium. The properties of these materials are described in rather more detail in Chapter XXII; it is sufficient here to note that at the flux-densities normally employed in current transformers the total loss per unit weight at 50 cycles is about one-tenth of that of 4 per cent silicon steel under similar conditions, whilst the magnetizing volt-amperes per unit weight are also reduced in a similar ratio.

Bar-type Current Transformers

It has hitherto been tacitly assumed that the primary winding of a current transformer consists of a number of turns of wire or strip, and this is the case for currents up to a few hundred amperes. Since the secondary rated current is always 5 amp, it is evident that transformers for primary currents exceeding 5 amp will necessarily be step-up in ratio. The secondary turns for a given class of transformer (i.e. for a stated burden and degree of accuracy) are independent of primary current, and it therefore follows that the primary turns are reduced in inverse proportion to the primary current. A limiting case will, therefore, be reached at high currents, in which the primary "winding" consists simply of a straight-through bar or conductor. From the point of view of simplicity and ease of construction the "bar-type" current transformer is very good. In high-voltage circuits, it can also be much more satisfactorily insulated by a simple porcelain bushing than would be possible if the winding consisted of a number of complete turns. In power distribution applications, there is another most important advantage, viz. the robustness of the straight-through conductor under conditions of heavy overload, when a short-circuit imposes very severe mechanical stresses on the windings. Alternatively, it is possible to dispense entirely with a primary and any special insulation, simply by passing the cable carrying the current to be measured through the centre of the core and winding. The insulation of the cable has, of course, to be capable of withstanding any strain likely to be thrown upon it, so that no further insulation is necessary in the transformer itself. As a precaution, however, a porcelain tube is often fitted; the transformer can then also be used for bare bus-bar conductors. For ease of assembly around a cable which is already installed, the core is usually rectangular in form. Two or sometimes four of the sides carry sections of the secondary and these are connected in series after assembly. The laminations forming the four sides of the core are "imbricated," i.e. arranged in small packets overlapping first one end and then the other, so as to reduce the reluctance of the iron path. In spite of this, the performance is necessarily inferior to that

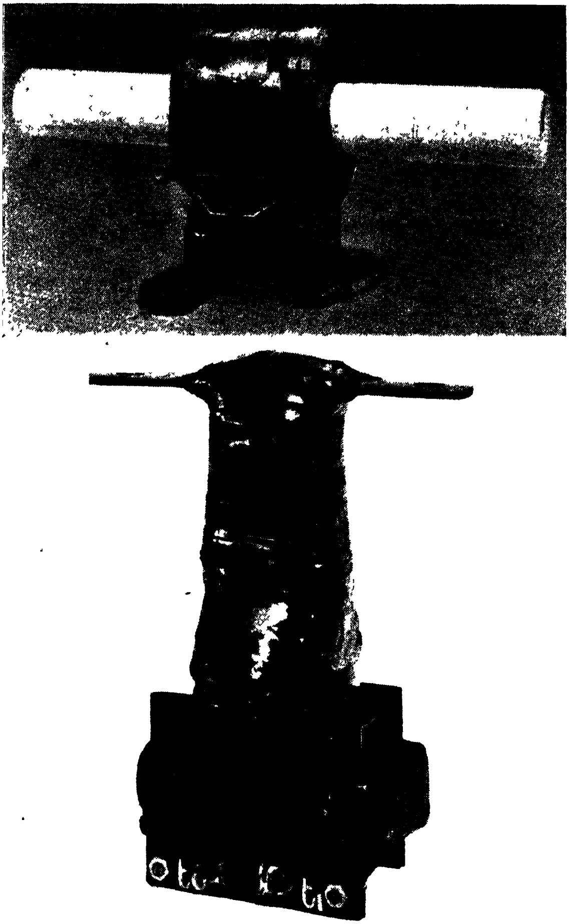

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Fig. 125

Types of Current Transformers for Different Primary-voltage Ranges.

obtainable from an unbroken ring core of similar dimensions, but the advantage of easy assembly is evident.

An extreme example of the split-core type is the "clip-on" current transformer used for temporarily fitting around cables or bus-bars. The round core is then in two halves carried on a pincer-like fitting which springs them together except when opened by the handles.

The advantages of the bar primary are so apparent that this method of construction is used whenever the current is sufficiently large. Unfortunately it is not possible even using the best nickel-alloy cores to secure Class 1 accuracy below a few hundred amperes without making the core impracticably large, so that the light electrical engineer is unlikely to have much to do with bar-type transformers. He may, however, have occasion to use the clip-on arrangement during the installation or checking of equipment. By passing the conductor several times through the core, the minimum current for satisfactory reading is, of course, reduced in proportion to the number of turns.

Construction of Current Transformers

The method of construction depends principally on the voltage which has to be withstood between primary and secondary. When this is only small, simple taped-up primary and secondary coils can be used; at voltages exceeding about 5 kV, porcelain or other bobbins are used to separate the primary from the rest of the transformer. At the highest voltages oil-immersion is necessary or, alternatively, a cascade-connected group of lower voltage types can be used. Typical examples are shown in Fig. 125.

3. INSTRUMENT TRANSFORMERS FOR RECTIFIER INSTRUMENTS

An important application of voltage and current transformers is in connexion with copper-oxide rectifiers. The measurement of alternating voltage and current has been revolutionized in the past fifteen years by the introduction of the instrument rectifier which enables moving-coil meters, with all their well-known advantages, to be used instead of thermal and moving-iron types.

The voltage-current characteristics of a copper-oxide rectifier are such that, irrespective of size, a pressure of about 0·5 volt per couple is required even in the forward direction before an appreciable current flows. (In the reverse direction it is about 6 volts.) By using a bridge-connected group of four rectifier couples and the usual series resistance, a very satisfactory a.c. voltmeter can be constructed for voltage ranges of 100 volts or higher, without the use of a transformer, but unless the lowest voltage to be read exceeds 0·5 volt by a considerable factor, inaccuracies become serious. It is, of course, possible to calibrate the scale so that the 0·5-volt reading is practically at zero deflexion and many early meters were made in this way. Apart from the possible inconvenience of omitting the lower part of the scale, the readings even on the upper portions of the range were considerably dependent on the forward characteristics of the rectifier, and these vary with temperature. To attain reasonable accuracy in low reading a.c. voltmeters, e.g. 5 volts full-scale, it is necessary to use a step-up transformer and a series resistance sufficiently large to swamp the variations in resistance of the rectifier. In multi-range instruments it also avoids the need for a different scale for each voltage range, as would be necessary if the first 0·5 volt had to be omitted; this would represent a different proportion of each scale. By using a tapped-primary transformer the rectifier and meter are working under identical conditions on each range.

To keep the power transformed as low as possible and so enable a smaller instrument transformer to be used for a given degree of accuracy it is preferable to put the series resistance in the primary circuit rather than the secondary. The resistance is then still effective in swamping the rectifier forward resistance but the transformer output voltage is much lower. Multi-range voltmeters thus have a suitably tapped series resistance feeding a fixed-ratio transformer which behaves in effect as a current transformer measuring the milliamperes passing through the resistance. If the same instrument has also to measure current, it is only necessary to tap the primary at suitable numbers of turns and pass the current to be measured directly through these turns. The product of turns and full-scale current is, of course, a constant, e.g. 1 turn for 10 amp, 10 turns for 1 amp, 100 turns for 100 milliamp, etc.

The secondary winding of the transformer, which is not changed whatever the input, is arranged to suit the full-scale current of the meter movement and the rectifier. The usual currents are 1, 2, or 5 milliamps. When the meter is arranged so that it can be switched away from the rectifier and transformer so as to measure D.C., it is obviously desirable to adjust the meter movement to give full-scale deflexion with an integral number of milliamperes. Since a factor of 1·11 is introduced when the rectifier is brought into operation in a.c. measurements, allowance for this must be made in selecting the transformer ratios. This factor arises from the fact that the mean value of the rectified current is $2/\pi$ (= 0·635) times the a.c. peak value, whereas the meter is calibrated to read r.m.s. values which are $1/\sqrt{2}$ (= 0·707) of the peak.

Although the importance of phase errors has been stressed in connexion with instrument transformers likely to be used for power measurements, it is unnecessary to consider them in this particular application; a transformer for a rectifier meter need only have a low ratio error. As already noted, a rectifier voltmeter can most conveniently be built as a rectifier milliameter with a series resistance, so in case of either a voltmeter or an ammeter, the transformer can be designed as a current transformer. The voltage which must be applied to the rectifier to produce full-scale current, e.g. 2 milliamps, is of the order of 0·9 volt. The standard meter drop is 75 millivolts, so the secondary voltage from the transformer must be approximately 1 volt. Since the input to the rectifier is 1·11 times the mean output current, the output VA from the transformer will be 2·22 millivolt-amperes. The core section and turns must now be chosen so that the exciting current ampere-turns are a sufficiently small fraction of the burden current ampere-turns. For example, if a ratio error of 1 per cent is permitted at full-scale, the exciting current must not exceed 1 per cent of the primary current. Strictly speaking, account should be taken of the large phase difference between the exciting current and the load component, so the allowance of 1 per cent is well on the safe side. At half-scale deflexion, the error is likely to be somewhat greater because, as we have already seen, the magnetizing component does not fall off quite in proportion to the applied voltage, although this is practically true of the loss component. To attain these small ratio errors in a reasonable sized transformer, it is necessary to use one of the high-grade nickel alloys such as Mumetal or Permalloy C. Suitable flux-densities are of the order of 100 lines per cm^2, or even less.

Taking a square stack of number 68 Mumetal laminations (Ref. γ, Table XXIV, p. 483), having a weight of 0·50 lb, the total excitation must not exceed 1 per cent of the output, i.e. 0·0222 millivolt-amperes. This amounts to 0·044 millivolt-ampere per lb and taking published figures for continuous strip cores of 0·015 in. Mumetal, this result is obtained at slightly over 100 lines per cm.^{2} As laminations will not give quite such good results as spiral strip cores, it would be advisable to take a somewhat lower figure, say 80. The secondary turns can now be calculated for 1-volt output at 50 cycles, in the usual way.

$$T_s = \frac{3\,490}{0{\cdot}08 \times 0{\cdot}335 \times 50} = 2\,600, \text{ approximately.}$$

The exact figure will have to be decided after allowances have been made for winding resistances, because it will not be practicable to make any corrections to the primary winding if it consists of perhaps only one turn as would be the case when measuring currents of the order of 10 amp. The primary should, therefore, be decided next, taking an integral number of turns for the largest current to be measured and multiplying up in inverse proportion for smaller currents. The actual turns for the secondary can then be fixed using not less than the 2 600 already calculated as giving 1 per cent error at full-scale. As this error would increase somewhat at lower readings, it would be desirable to over-compensate somewhat at full-scale, thus bringing the error to zero at half-scale and under-correcting at the bottom of the scale.

The particular core described should be quite large enough for a multi-range voltmeter-ammeter with a range of primary coils. A smaller one would be satisfactory for a single-range instrument or for lower accuracy.

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(2) J. MÖLLINGER AND H. GEWECKE: "Zum Diagramm des Spannungwandlers." *Elekt. Zeits* 32 (1911), p. 922.

(3) J. Möllinger and H. Gewecke: "Zum Diagramm des Stromwandlers." *Elekt. Zeits.* 33 (1912), pp. 270–271.

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CHAPTER XX

Audio-frequency Types

In its passage through a radio receiver, the signal assumes several different forms, requiring amplification stages suitably arranged to deal with the different ranges of frequency involved. In all forms of communication equipment, however, the signal is finally handled in the form of alternating voltages of the same frequencies as those in the sound waves received by the microphone. These voltages are known as audio-frequency (A.F.) voltages and the frequencies involved range from about 20 cycles to 15 000 cycles. A much smaller range than this is capable of reproducing understandable speech or music, but the complete range is necessary for an accurate reconstruction of the original sounds. The audio-frequency part of an amplifying system often contains small transformers connected in the signal chain and these transformers are expected to reproduce as accurately as possible all the complicated wave-forms produced by musical instruments or speech. The wave-forms which have to be copied faithfully are not necessarily those applied to the primary, but are those of the e.m.f. generated in the source, as for example in a moving-coil microphone.

1. REQUIREMENTS FOR TRUE REPRODUCTION

Although the wave-forms of the sounds to be reproduced are very complicated, they can, by Fourier's Analysis, be shown to be equivalent to a series of sinusoidal wave-forms of different frequencies (p. 197). If the audio-frequency circuit can reproduce each of the components correctly without distorting their wave-forms or changing their relative amplitudes or phases, it can reproduce accurately the complex wave-form which is equivalent to these separate components. The behaviour of a transformer in the audio-frequency chain is accordingly examined from two distinct aspects which are discussed in detail later—

(1) Freedom from amplitude or harmonic distortion, i.e. a

sine-wave input at any one frequency in the range must be reproduced as a sine wave at the output.

(2) Freedom from frequency and phase errors, i.e. the ratio of output to input must remain unchanged (vectorially) over the whole range of frequencies.

Failure to satisfy either of these requirements causes incorrect reproduction of a complex wave-form to a greater or lesser extent. As regards (1) it should not be forgotten that the source from which the transformer is supplied nearly always has an impedance comparable with that of the primary at the lower end of the frequency scale, so that harmonic distortion would become serious unless suitable precautions were adopted (Chapter X). Regarding (2) it should be noted that not only does a constant ratio have to be maintained over a wide range of *frequencies*, but the same ratio must be maintained over considerable changes of input *voltage* as well. To complicate matters the capacitances of the circuit and winding begin to take effect at frequencies of several kilocycles, so that quite a number of new factors have to be borne in mind when designing audio-frequency transformers.

The transformers in common use can be divided into two groups which differ considerably in their design and construction—

(*a*) *Input and Intervalve Transformers.* The input type is used to couple telephone lines, gramophone pick-ups, etc., to the grid circuit of the first valve in an amplifier. Intervalve transformers couple the second and succeeding valves to the anode circuits of the preceding valves. In most cases, this group of transformers supplies no power output, since the valve grids are usually biased negatively so that no current flows even with the strongest signals. Both input and intervalve transformers usually have step-up ratios, so raising the amplification of the stage with which they are associated.

(*b*) *Output Transformers.* These are used to couple the last valve of an amplifier to the load such as a loudspeaker, telephones, morse-recorder, etc. These transformers differ from those used in earlier stages, because they are called upon to deliver power to the load. Their function is one of impedance matching of the load to the source. They are usually step-down in ratio.

The task of the speech transformer will clearly be much harder than that of a power transformer for the following reasons—

(*a*) The amount of distortion which can be tolerated is smaller.

(*b*) The primary circuit, since it includes a valve anode, will be of quite considerable impedance.

(*c*) The input voltage will vary over a large range, instead of remaining constant.

(*d*) The same output to input ratio must be maintained over a wide range of frequencies.

(*e*) In many cases, a large d.c. component flows through one or more of the windings.

(*f*) Some types of audio-transformers (e.g. intervalve couplings) may have always to work unloaded.

(*g*) Output transformers must be of high efficiency to avoid wasting valuable audio-watts.

Points (*a*), (*b*), and (*c*) involve considerations of the magnetic behaviour of the iron core. In order that a low degree of distortion shall be achieved (to suit (*a*) above), it is necessary to work at a low flux-density in the iron. Instead of the figure of some 12 000 lines per cm^2 suggested for 50-cycle power transformers, it is necessary to keep down to about 4 500 lines per cm^2, at the lowest frequency and highest voltage with which the transformer is supplied, and the value of B will, of course, decrease in proportion at higher frequencies. High-quality transformers may require even lower flux-densities, particularly if there is a d.c. component flowing through the primary.

Working at a low value of B necessitates an increase in the number of primary turns for a given core size, frequency, and input voltage. The primary inductance is accordingly increased, and the magnetizing current reduced, so that decreasing B keeps harmonic distortion down in two ways—

(1) By reducing the percentage distortion in the magnetizing current.

(2) By making the magnetizing current a small proportion of the total primary current.

Point (*d*), in our preceding list of difficulties, involves a factor with which we have not previously had to contend, namely, the fact that the input may vary greatly in frequency.

As already noted, the magnetic circuit must be designed on the basis of the lowest frequency that is supplied to the primary; then the higher frequencies can be satisfactorily dealt with as far as that particular aspect of the problem goes. When the frequency becomes considerable (i.e. several thousand cycles per sec.), we have to take into account two other factors, the effects of which have been considered negligible at 50 cycles: first, the capacity of the windings to ground, and to each other; second, the magnetic leakage between the windings.

We shall see later that these effects are made greater on account of the necessity for correct working at low frequencies as well. Little trouble would be experienced in making a transformer to work at 2 000 cycles only; in fact, it would be smaller and more efficient than one of similar output intended for 50 cycles; but a wide range of frequencies makes matters difficult.

Section (*e*) involves rather a long story, and a later chapter will deal with this problem of the calculation of the inductance of iron-cored coils carrying D.C. At this stage it is sufficient to note that the inductance of a transformer will be reduced, often very considerably, by the passage of D.C. through one of its windings.

As regards (*f*), the presence of a load (such as a loudspeaker or an artificially-fitted resistance) tends to swamp the effects of any stray capacities across the windings; it does, however, make the effects of magnetic leakage more apparent. Item (*g*) requires no comment.

2. HARMONIC DISTORTION

Although the use of ferro-magnetic materials for the cores of transformers enables the efficiency and other characteristics to be improved very greatly over those of air-cored types, the fact that the permeability varies considerably with the magnetizing force introduces special problems. Although the effects of this variation of permeability become important to the power engineer in a few instances, as in certain three-phase transformers, it is generally true to say that they cause him little trouble. The reason for this is evident from Chapter X, in which it is shown that if the impedance of the source of alternating e.m.f. is negligible compared with that of the primary, the marked distortion of the exciting current wave is not transferred to the output-voltage wave-form. This is, of course, the state of affairs in most power-transformer circuits. In a.f. circuits, on the other hand, the source (usually a valve anode) has an impedance at least comparable with that of the transformer, and often much higher. If the transformer is working unloaded, i.e. with the secondary on open-circuit, it is very difficult to reduce the distortion to reasonably small proportions even by working at low flux-densities. In some cases it is necessary in order to minimize distortion to apply artificial loads to intervalve transformers which would otherwise be working into circuits of very high impedance. In the case of loaded transformers, such as output types, sufficiently small values of distortion can be achieved by good design.

It will be recalled that the distortion arises basically from the shape of the B-H curve. If the source impedance is high, so that the exciting current is maintained in a sinusoidal form, the flux wave is not sinusoidal since B rises less rapidly than H (which is proportional to I). The induced e.m.f., which is proportional to the rate of change of B, is also distorted, actually to a greater extent, because the relative magnitudes of the harmonics in the flux are multiplied by their order number when they appear in the e.m.f. In the absence of d.c. polarization of the core, the B-H curve is symmetrical so that only odd harmonics appear in the flux wave. If the flux at any instant is represented by

$$\phi = \Phi_{1_{max}} \sin \omega t + \Phi_{3_{max}} \sin 3\omega t + \Phi_{5_{max}} \sin 5\omega t \ . \ . \ . \quad (97)$$

then the induced voltage e is given by

$$e = -T\frac{d\phi}{dt} = -T(\omega\Phi_{1_{max}} \cos \omega t + 3\omega\Phi_{3_{max}} \cos 3\omega t + 5\omega\Phi_{5_{max}} \cos 5\omega t \ . \ . \ .) \quad (98)$$

An accurate calculation of the amount of distortion produced by a transformer under actual working conditions is very difficult and would usually be rather pointless owing to the variations between different specimens of a given grade of iron. Individual transformers made to the same design would vary appreciably because of differences in the particular packets of laminations fitted to each. A sufficiently good assessment can, however, usually be made as follows—

(1) Determine the harmonic percentages in the output voltage wave from the transformer working without load and supplied from a very high impedance source so that the exciting current is sinusoidal. The determination is made for the particular core material to be used when it is working at the intended peak flux-density. The distortion found under these conditions may be called the "inherent" distortion.

(2) Divide the percentages obtained in step 1 for an unloaded transformer, by a factor determined from the relative magnitudes of primary impedance, source impedance and load impedance expected under the working conditions.

The calculation of step 1 is made under the conditions assumed in Section 3 of Chapter X, i.e. with the sinusoidal magnetizing current which results from a source impedance which is high relative to that of the primary of the transformer. The method of obtaining the flux *wave-form* under these conditions has been described in Chapter X, but for the present purposes what is finally required is the percentage of each harmonic in the induced e.m.f. Although this could be obtained by analysing the wave-form diagram given by differentiating the flux wave traced out as described, it is better, when possible, to obtain the harmonic percentages for the particular type of core material at the flux-density to be used, by measurement with a harmonic analyser. This instrument enables the percentage of each harmonic to be measured in turn as a percentage of the fundamental voltage. The distortion measured under these conditions is characteristic of the core material itself, so that once a curve has been obtained for the particular material at various flux-densities, it can be used for any other transformers of comparable dimensions. To carry out the measurement, a representative core is selected and a known number of turns of reasonably large section wire are wound upon it, together with a secondary which can be of quite small gauge as it is only used to feed the high impedance input of the harmonic analyser. The primary is then connected in series with a high resistance to an a.c. supply of 50 or 60 cycles. The value of the resistance should be sufficiently high to ensure that the *current* wave form is sinusoidal as required by the conditions of Section 3 of Chapter X. The resistance can conveniently be adjustable so as to enable the voltage across the primary to be set at the figure required to produce the desired flux-density in the core. If the primary voltage were sinusoidal, the well-known relationship—

$$B_{max} = \frac{3\,490 \times V_p}{T_p \times a \times f}$$

where B_{max} = flux-density in kilo lines per cm^2
a = cross-section in in.^{2}
f = frequency
V_p = r.m.s. primary voltage,

could be used, but to be accurate it would in this case require modification to allow for the non-sinusoidal wave-form of the applied voltage. It will be remembered that in the derivation of this expression (Chapter III) it is the *mean* voltage which is related to the peak flux-density. For convenience the mean value is then converted to r.m.s. by multiplying by the "form factor" which is 1·11 in the particular case of a sine wave. It is better, however, in cases of appreciable departure from the sinusoidal shape, to retain the mean value for the voltage and make measurements by means of a rectifier instrument which although calibrated to read r.m.s. values on a sine wave, actually reads the mean value multiplied by 1·11 on any other wave-form. Hence V_{mean} = meter reading/1·11. If this value is now substituted in place of V_{mean} it will be seen that the original constant 3 490 may be used if the voltage is measured by a rectifier-type instrument calibrated to read r.m.s.

When the correct input conditions have been obtained, the output voltage from the secondary is examined by the analyser and the percentage of each harmonic is measured. The input voltage is then altered to give a different value of B_{max} and further harmonic measurements taken. It is, of course, important to check that the percentage harmonic in the supply voltage is sufficiently small to be neglected.

If facilities allow, it is advisable to make measurements of this kind on the particular make and grade of core material to be used, although the variations between the different silicon grades are not so great in this respect as in loss characteristics. Representative figures for a 4 per cent silicon content steel are shown in Fig. 126. The laminations used were the $\frac{3}{4}$ in. No-waste type, 0·014 in. thick.

Another method which may have to be employed when a harmonic analyser is not available is to work from the static magnetization curve for the particular peak flux-density in use, as described in Chapter X. The flux wave-form is first deduced for the application of a sinusoidal magnetizing current and the slope of the flux wave-form is measured at intervals to give the e.m.f. wave. Graphical methods can then be used to obtain the harmonic percentages.(1, 2) If the distortion is assumed to be third harmonic only, the percentage is very easily obtained, but a complete analysis is laborious. In this method, the use of a static B-H curve assumes, of course, that eddy currents in the iron are negligible. At the lowest audio-frequencies where harmonic distortion is highest, this assumption is reasonably valid.

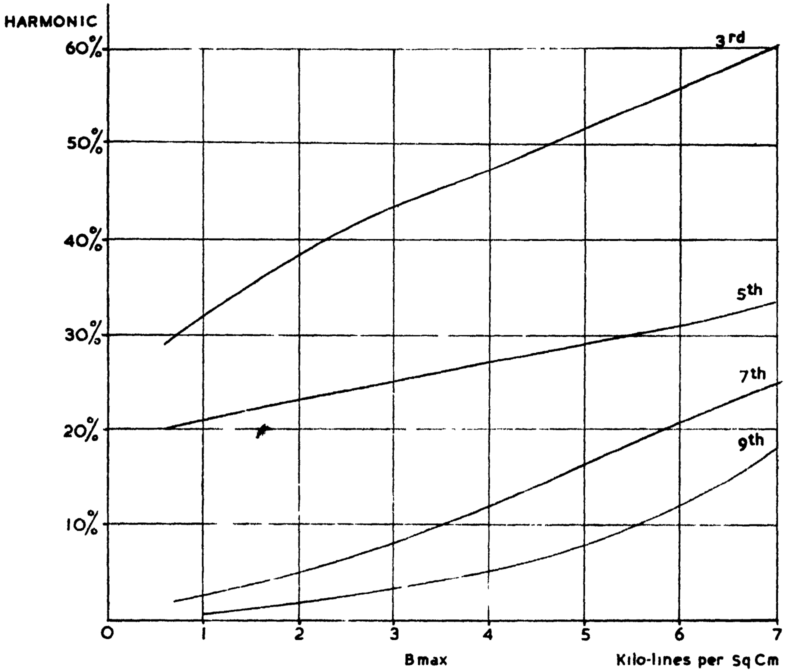

FIG. 126

Inherent Harmonic Distortion Percentages for a Typical Sample of 4 per cent Silicon Steel.

The harmonic percentages obtained in step 1 are the greatest that the transformer could produce at that particular input and frequency, however unfavourable the impedance conditions of the circuit might be. These conditions would be approached if the transformer were fed by a high-impedance pentode valve and no load were applied to the secondary. If, on the other hand, a triode with a low anode impedance is used to feed the transformer and a comparatively low resistance load is applied to the secondary, the distortion is considerably reduced.

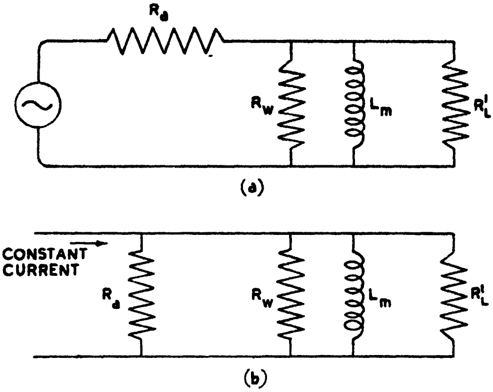

FIG. 127

Alternative Methods of Representing a Thermionic Valve feeding into a Transformer. (*a*) Constant-e.m.f. Generator. (*b*) Constant-current Generator.

The most frequently used representation of the anode circuit of a thermionic valve is a source of e.m.f. (of magnitude equal to the input voltage applied between the valve grid and cathode, multiplied by the amplification factor of the valve) in series with a resistance equal to the differential impedance of the anode circuit. An alternative but equivalent representation, which is more convenient in this application, is to consider the valve as being a source of constant current of magnitude equal to the grid voltage multiplied by the mutual conductance g of the valve, in parallel with a resistance equal to its anode impedance R_a. The transformer circuit is thus seen to be as shown in Fig. 127, the transformer primary impedance (to the fundamental frequency) being Z and the load R_L, referred if necessary to the primary. In addition, the resistance R_w represents the eddy-current and hysteresis losses. If the operation of the circuit is now examined it will be seen that any variation of Z over the cycle in such a way as to cause a peaky current wave, must result in a corresponding flattening of the current through the other resistances R_a, R_L, and R_w because the generator supplies a sinusoidal current. The peaky nature of the magnetizing current is due to odd harmonics (chiefly third) so that a harmonic current of equal magnitude, but in the opposite phase, must flow through the load. The harmonic current can thus be regarded as circulating around the closed loop formed by the resistances and the primary reactance under the influence of a harmonic e.m.f. situated in the reactance branch. The fraction of this e.m.f. applied to the load as a harmonic potential difference, depends on the relative magnitudes of the reactance (at the harmonic frequency concerned) and the load resistance. The problem is analogous to that of determining the terminal potential difference of an alternator supplying a resistive load, given the e.m.f. and internal reactance. Owing to the increase in internal reactance at higher frequencies, the effect of loading is greater on the upper harmonics than the lower. In the case of triode valves, the load is usually chosen to be several times greater than the impedance of the valve, and the primary reactance at the lowest audio-frequency to be considered is again made greater than the load. The resultant parallel load R is thus often only about one-tenth of the primary reactance at third-harmonic frequency and even less at higher harmonic frequencies. The third-harmonic percentage in the output is thus reduced to one-tenth of the "inherent" distortion of the transformer.

A word of warning is perhaps desirable here because the calculation of the reduction factor in step 2 is not quite as straightforward as might be suggested by the previous paragraph. This is because the inductance of an iron-cored coil varies considerably with the voltage applied. The value of the inductance is, of course, proportional to the permeability of the core material and this is low at very small values of the magnetizing force, rising fairly rapidly to a maximum value and declining less rapidly at high values of magnetization. The permeability figures applicable to a.c. magnetization are those known variously as incremental, differential or reversible permeability. The values are lower than those for steady magnetization and in the presence of d.c. polarization the values fall lower still. This aspect is considered in detail in the next chapter; for the present it is sufficient to recognize that the incremental permeability falls considerably at low values of H (and therefore of B). This fall in permeability (and therefore of primary impedance) offsets, and in many cases nullifies, the reduction in harmonic percentage which results from the use of very low flux-densities. If the transformer is loaded or working from a source of moderate impedance, the effect of a reduction in primary impedance is to reduce the factor by which the "inherent distortion" is divided in step 2 and if the impedance drops at the same rate as the inherent distortion, the distortion under working conditions remains constant. Actually the impedance drops more rapidly than the distortion below about 3 000 lines per cm^2, so that the distortion under working conditions rises, reaching a maximum at a few hundred lines per cm.^{2} Since the harmonic percentages and incremental permeability are both properties of the particular grade of core material, a factor proportional to the ratio of these quantities is also characteristic of the material. Curves of this factor against flux-density enable the best working conditions to be selected. In the absence of d.c. polarization, the shape of the curve is slightly concave upwards below 5 000 lines per cm^2 and there is accordingly no point in reducing the flux-density (for maximum input signal at the lowest frequency) below that figure. When D.C. is present, the distortion factor is always higher and the shape of the curve is also changed, being no longer concave, so that the distortion decreases continuously as the flux-density is reduced. For reasonably low distortion it is accordingly necessary to work well below 5 000 lines per cm^2 when D.C. is flowing in the circuit.

The problem of the calculation of harmonic distortion in output transformers has been examined in considerable detail by N. Partridge.(3) It may be said that his work has advanced the subject to a very considerable degree. Although Partridge's work is concerned with output transformers, the principles are applicable to other audio-frequency types, providing that they are loaded. Details are given in the next chapter, which is devoted to output types.

The Use of Air-gaps in the Core

The "inherent" distortion in a transformer can be reduced considerably by the use of an air-gap in the core, but in the case of a loaded transformer there is no improvement under operating conditions because the reduction factor is correspondingly smaller. If the same input voltage is applied to a gapped transformer as an ungapped one, the flux-density in the iron is unaltered (apart from leakage effects), so that the current required to magnetize the iron is also unchanged. In addition, however, current is required to produce a similar flux-density in the air-gap. The magnitude of this current can be calculated from the usual relationship.

$$B = H = \frac{4\pi IT}{10l_a}$$

where l_a = length of air-gap in cm,
I = magnetizing current in amperes
T = number of turns.

The wave-form of the air-gap magnetizing current is sinusoidal since the permeability of the air is constant. The fundamental frequency in the total magnetizing current is thus increased by the introduction of the gap, whilst the harmonics remain unchanged in magnitude. The *percentage* harmonic is accordingly reduced. Since the reactance of the primary is, however, reduced by this arrangement, the harmonic reduction resulting from a low source impedance or output load is correspondingly less and little or no improvement results. The method may, however, be useful in unloaded transformers working from high-impedance valves.

These remarks are only applicable, of course, if the winding is left unchanged when the gap is introduced.

3. FREQUENCY DISTORTION

It has already been stated that one of the requirements of an audio-frequency transformer is that its ratio of output to input voltage should remain constant over the whole band of frequencies to be covered. Strictly speaking, it is in many cases the ratio of output from the transformer to input to the grid of the preceding valve which has to be constant, or more generally to vary with frequency in a predetermined manner. For example, it is often advantageous to have a transformer characteristic which rises somewhat at one or both extremities of the pass band, so compensating for unavoidable losses elsewhere in the circuit. Alternatively the characteristic required may have a sharp cut-off at the upper frequency end, so reducing interference from heterodyne whistles, etc. In this section it will be assumed, however, that the intention is to produce a level or "flat" response over as wide a range of frequencies as possible. Departure from the level response, i.e. a change in the ratio of output volts to input grid volts, is known as *frequency distortion*. The causes of frequency distortion are most clearly seen by reference to the equivalent circuit of a transformer.

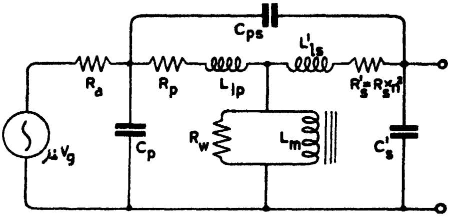

Fig. 128

Equivalent Circuit of an Audio-frequency Transformer and Preceding Amplifying Valve.

The effects of harmonic distortion have been seen to be most evident at the lowest frequencies, because the flux-density for a given input voltage increases inversely as the frequency. At the low-frequency end of the audio-frequency range it was possible to omit the leakage reactance and capacitance elements of the equivalent circuit, but when considering *frequency* distortion which may also occur at the higher frequencies, it is not permissible to omit these elements from the circuit altogether. At some frequencies, however, simplifications can be made.

The circuit elements which normally have to be taken into account in audio-frequency transformer designs are shown in Fig. 128. It is evident that circuit calculations for this network are relatively complex and that a general solution would be unwieldy. It is better, therefore, to consider particular cases in which some elements can be omitted without prejudicing the result, and so solve the problem much more easily. Output and intervalve coupling transformers call for different treatment for that reason, and each can with advantage be treated differently in the various sections of the audio-frequency band. At the low-frequency end, for instance, the effects of winding and interwinding capacitances can be neglected, whilst at the highest frequencies the shunt inductance can be omitted.

Output transformers are dealt with in Chapter XXI and Intervalve and Input transformers in Chapter XXII.

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CHAPTER XXI

Output Transformers

A GENERAL survey of audio-frequency transformer design has been made in Chapter XX. The difficulties are quite considerable and in the interests of simplicity it is desirable to concentrate on one type at a time. The most straightforward is the output transformer and again the simplest variety is that intended to operate from a push-pull output stage, so this is treated first.

The function of an output transformer is to match the loudspeaker (or other device such as a recorder) to the anode circuit of the output valves. For reasons of robustness, freedom from shock risks, and such-like, the loudspeaker is usually wound to a resistance of only a few ohms, whereas the optimum load for the output valves is measured in thousands of ohms. The transformer is, therefore, required to act as an impedance matching device, and is step-down in ratio. It has been shown that a load R_L across the secondary of a transformer of ratio n, looks like $R_L \times n^2$ ($= R_L'$) when seen from the primary terminals. Conversely, if the winding resistances and iron losses are neglected, the ratio required to match a low-resistance speaker to an output stage is given by the square root of the impedance ratio. For instance, if a 3-ohm loudspeaker is to be matched to an output stage having an optimum load of 3 000 ohms, the ratio would be

$$n = \sqrt{\frac{3\ 000}{3}} = 32$$

i.e. a step-down ratio of 32 to 1. In practice, however, the winding resistances particularly, and to some extent the iron losses also, appreciably affect the apparent load on the valves, so that the ratio has to be modified to make the transformer and load together appear as the required value.

The performance of an output transformer must meet certain requirements, of which the principal ones are—

(*a*) The frequency response must lie within specified limits.

(*b*) The degree of harmonic distortion must not exceed a given percentage.

(*c*) The efficiency must reach a specified figure at full input.

Push-pull types are, as already mentioned, the simplest to design, because the effects on the core of steady magnetization due to the passage of D.C. through the windings is avoided by cancellation. They are first considered in detail and examples of design are then worked out.

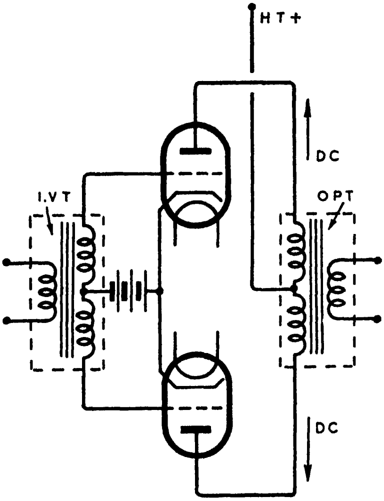

FIG. 129

Push-pull Circuit Showing Cancellation of Steady Magnetizing Effects of the Two Anode Currents.

1. PUSH-PULL OUTPUT TRANSFORMERS

The primary circuit of a push-pull output transformer consists of two similar windings in opposite directions, with one end of each connected together and to h.t. positive. In this way the steady anode currents of the two valves circulate round the core in opposite directions and, if the valves are accurately balanced, there is no resultant magnetization of the iron. Since the grid inputs to the two valves are in opposite phase, the a.c. components in the two portions of the primary are in the correct sense to add up across the ends of the winding (Fig. 129). We can accordingly regard the output transformer as a power transformer, supplied with varying frequencies, and feeding into a load which is assumed to be resistive.

The three important factors requiring attention in the design are first examined separately and the reaction of one on the other is then discussed.

Frequency Response

As with most audio-frequency transformers the response is specified in terms of a "level" input to the grids of the output valves, that is to say, a constant voltage at all frequencies within the audio range is applied to the grids and the output across the secondary load is required not to vary by more than a given percentage between specified extreme frequencies which lie towards the ends of the range. The permissible variation in level is usually stated in *decibels* (abbreviated to db) by the communications engineer, since it gives a better indication of the audible effect than would a percentage figure. The decibel scale is a logarithmic one, and since the response of the ear is logarithmic, an increase of 5 decibels at any sound level appears as an equal increase in loudness. The change of output power in decibels is calculated as ten times $\log_{10}$ of the power ratio or twenty times $\log_{10}$ of the ratio of voltages. A two-to-one drop in output voltage at a particular frequency relative to that at a standard frequency (usually 400 or 1 000 cycles) is thus said to be 6 decibels down ($= 20 \times 0{\cdot}3010$). For the purposes of quoting response, a figure of 3 decibels is usually regarded as a permissible variation in level from that at the standard frequency—a choice which is doubtless due to the ease with which this particular change can be calculated from relative values of load and shunt inductance or capacitance. If the shunt reactance at any frequency is equal to the resistance load, the resultant impedance is $1/\sqrt{2}$ of that of either component. In a high-impedance circuit the voltage across such a combination would also be $1/\sqrt{2}$ of that across the unshunted resistor and this is, of course, 3 decibels down.

The normal shape of an output transformer response curve plotted against frequency is a level central portion falling off at both high and low frequencies. It is convenient to divide the range into three sections—

(1) up to 100 cycles,
(2) 100–2 000 cycles,
(3) above 2 000 cycles.

Within each of these bands a different simplification of the equivalent circuit of the output transformer can be made. The general circuit is shown in Fig. 130; all elements are referred to the primary. It is very seldom necessary to take into account the capacitances, since their impedance even at the highest frequencies is usually large compared with the load resistance. In the central band (100–2 000 cycles) all reactances can be neglected and the shunt resistance R_w representing iron losses is also very high. The circuit thus becomes a straightforward series connexion of valve anode impedance R_a, winding resistances R_p and R_s', and load R_L'. The output voltage V_{sm}' is then readily calculated in terms of the circuit e.m.f. μV_g

$$V_{sm}' = \mu V_g \left(\frac{R_L'}{R_a + R_p + R_s' + R_L'}\right)$$

$$= \mu V_g \left(\frac{R_L'}{R_{ser}}\right) \quad . \quad . \quad . \quad . \quad . \quad (99)$$

where $R_{ser.} = R_a + R_p + R_s' + R_L'$.

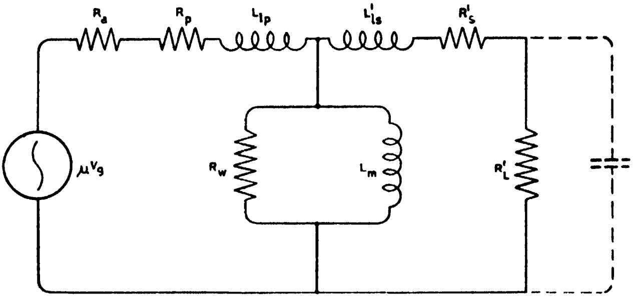

FIG. 130

Equivalent Circuit of an Output Transformer Referred to the Primary. The Load Capacitance (Shown Dotted) can Usually be Neglected.

Since this expression contains only resistance values it is independent of frequency, so the response is level over this range.

In the low-frequency range (below 100 cycles), the shunting effect of the primary inductance L_m becomes appreciable, whilst the leakage inductances remain, of course, negligible. The output voltage in the low-frequency range V_{sl}' is then

$$\mu V_g \left\{\frac{R_L' j\omega L_m}{(R_a + R_p)(j\omega L_m + R_L' + R_s') + j\omega L_m(R_L' + R_s')}\right\} \quad (100)$$

The ratio of the output in this range relative to that in the medium frequency range is

$$\frac{R_L' j\omega L_m}{(R_a + R_p)(j\omega L_m + R_L' + R_s') + j\omega L_m(R_L' + R_s')} \times \frac{R_a + R_p + R_s' + R_L'}{R_L'}$$

$$= \frac{j\omega L_m(R_a + R_p + R_s' + R_L')}{j\omega L_m(R_a + R_p + R_s' + R_L') + (R_a + R_p)(R_L' + R_s')}$$

$$= \frac{1}{1 + \dfrac{(R_a + R_p)(R_L' + R_s')}{(R_a + R_p + R_L' + R_s')} \Big/ j\omega L_m}$$

$$= \frac{1}{1 + \dfrac{R_{par.}}{jX_m}}$$

where $R_{par.}$ is the resistance of $(R_a + R_p)$ and $(R_s' + R_L')$ in parallel and $X_m = j\omega L_m$.

$$= \frac{1}{\sqrt{1 + \left(\dfrac{R_{par.}}{X_m}\right)^2}} \quad . \quad . \qquad (101)$$

Expressed in db the drop in output relative to the middle frequencies is $20 \log_{10} \sqrt{1 + \left(\dfrac{R_{par.}}{X_m}\right)^2}$

$$= 10 \log_{10} \left\{1 + \left(\frac{R_{par.}}{X_m}\right)^2\right\} \qquad (102)$$

When $R_{par.} = X_m$ the drop is 3 db.

In the high-frequency section of the pass-band (above 2 000 cycles), the primary shunt reactance is too high to affect the response, but the leakage reactances (in series with the load) can no longer be neglected. The capacitances of the windings may also have some effect, although it is unlikely to be appreciable. The effect of the leakage reactances is to cause a drop in output voltage. If V_{sh}' is the output voltage in the high-frequency section,

$$V_{sh}' = \mu V_g \left\{\frac{R_L'}{R_a + R_p + R_s' + R_L' + j\omega L_{Lp}}\right\} \quad . \qquad (103)$$

where L_{Lp} is the transformer leakage inductance referred to the primary.

Relative to that in the middle frequencies, the output is

$$\frac{R_L'}{R_{ser.} + j\omega L_{Lp}} \times \frac{R_{ser.}}{R_L'}$$

$$= \frac{R_{ser.}}{R_{ser.} + j\omega L_{Lp}} = \frac{1}{1 + \dfrac{j\omega L_{Lp}}{R_{ser.}}} = \frac{1}{\sqrt{1 + \left(\dfrac{X_{Lp}}{R_{ser.}}\right)^2}} \qquad (104)$$

The high-note loss in db is thus equal to

$$20 \log_{10} \sqrt{1 + \left(\frac{X_{Lp}}{R_{ser.}}\right)^2} = 10 \log_{10} \left\{1 + \left(\frac{X_{Lp}}{R_{ser.}}\right)^2\right\} \qquad (105)$$

This loss becomes equal to 3 db when $R_{ser.} = X_{Lp}$.

The division of the pass-band into three separate ranges is apt to give the impression that the quantities involved, i.e. primary inductance and leakage inductances, are independent quantities. Actually this is not the case, for unless alterations are made in the arrangement of the windings, as for example by further subdivision and interleaving of primary and secondary, an increase in primary inductance, made with the intention of improving the bass response, causes a proportionate increase in the leakage inductances and a corresponding loss in high-note response. The "pass-band" of an audio-frequency device is usually regarded as that portion of the audio spectrum over which the response does not differ by more than 3 db from that at the standard frequency. At the bass end, it has been shown that this condition is reached at a frequency f_l such that $R_{par.} = \omega L_m$, where

$$R_{par.} = \frac{(R_a + R_p)(R_s' + R_L')}{R_a + R_p + R_s' + R_L'}$$

and in the treble at frequency f_h when

$$R_{ser.} = \omega L_{Lp}, \text{ where } R_{ser.} - R_a + R_p + R_s' + R_L'$$

The ratio of the highest to the lowest frequency in the pass-band is thus—

$$\frac{f_h}{f_l} = \frac{R_a + R_p + R_s' + R_L'}{2\pi L_{Lp}} \times \frac{2\pi L_m(R_a + R_p)(R_s' + R_L')}{R_a + R_p + R_s' + R_L'}$$

$$= \frac{L_m}{L_{Lp}} (R_a + R_p)(R_s' + R_L') \quad . \quad . \quad . \quad . \quad (106)$$

As the resistance terms are substantially determined by R_a and R_L', which are fixed, the ratio of highest to lowest frequencies can be increased only by improving the coupling between the primary and secondary, thus reducing the leakage reactance. An increase in L_m without such an improvement in coupling merely shifts the whole response bodily downwards; it will still cover the same number of octaves.

Calculation of the leakage inductance in a particular instance is difficult, and the winding lay-out is usually decided by experience. The normal shape of stamping with a window length (i.e. along the axis of the winding) several times greater than its width, gives lower leakage inductances when the two windings are put on in layer formation, each winding running the full length of the window. The alternative method of construction, dividing the window space into two nearly square section coils, results, as one might expect, in much higher leakage inductances. Output transformers are, therefore, practically always wound in layer fashion. Even so, the leakage inductance is usually too large for high-fidelity work, and the windings have to be "sectionalized" and interleaved. Thus the secondary may be divided into three sections, and the primary into two.

For the best response it may be necessary to go farther and to use four primary sections, and either three or five secondaries. Nevertheless, unnecessary subdivision should be avoided, as it is wasteful of space on account of the extra interwinding insulation which is required. In any case, a push-pull output transformer will need to have an even number of primary sections, and these should be connected up in such a way as to provide as nearly as possible an accurate balance between the two halves.

Harmonic Distortion

The salient points of harmonic distortion calculations have been given in the previous chapter which deals generally with the matter, and the dependence of the final result on both the inherent distortion figure of the transformer and on the modifying factor determined by the external circuit conditions has been stressed.

Reference has also been made to Partridge's work on harmonic distortion in output transformers.(1, 2) He adopts a somewhat different method of approach from that described in the previous chapter in that he relates the distortion of the output-voltage wave-form to that of the magnetizing current when operating from a source of *negligible* impedance. It will be remembered that although the magnetizing current and induced voltage wave-forms are very difficult to deduce when the source impedance and primary impedance are comparable, this is no longer true in two particular cases, namely when the source impedance is either very small or very large. If small, the induced voltage is sinusoidal and the current wave-form can be plotted using the B-H curve as described in Chapter X, Section 2. Partridge's method thus goes to the other extreme to that used in the first method.

Starting from the general case of an unloaded transformer of impedance Z_f at the fundamental frequency, working from a source of sinusoidal e.m.f. of impedance Z_a, he shows that since there is no harmonic e.m.f. in the source, the harmonic voltage V_h across the primary must be equal and opposite to that across Z_a, i.e. $V_h = -I_h Z_a$. We also know that $V_f = I_f Z_f$, so that

$$\frac{V_h}{V_f} = -\frac{I_h}{I_f} \cdot \frac{Z_a}{Z_f} \quad . \quad . \quad . \quad (107)$$

The proportion of any particular harmonic in the output voltage is thus related to the proportion of that harmonic in the magnetizing current which is flowing *under these particular impedance conditions* (not under $Z_a = 0$ conditions). The behaviour of a *loaded* transformer is then shown to be the same as that of the unloaded one if the value of the source impedance is taken to be that of Z_a and Z_L in parallel. This has already been noted in the first method outlined in Chapter XX. Calling the resultant impedance Z,

$$\frac{V_h}{V_f} = -\frac{I_h}{I_f} \cdot \frac{Z}{Z_f} \quad . \quad . \quad . \quad (108)$$

In order to proceed farther, Partridge makes a number of assumptions which accord well with normal practice in audio-transformers. These are that Z is resistive (i.e. $= R$), that the flux-density does not exceed 10 000 lines per cm^2, and that R is never greater than Z_f, a condition usually imposed in any case by frequency-distortion limitations. From measurements made under these conditions, Partridge arrived at an empirical relationship between I_h for the third harmonic and I_H the

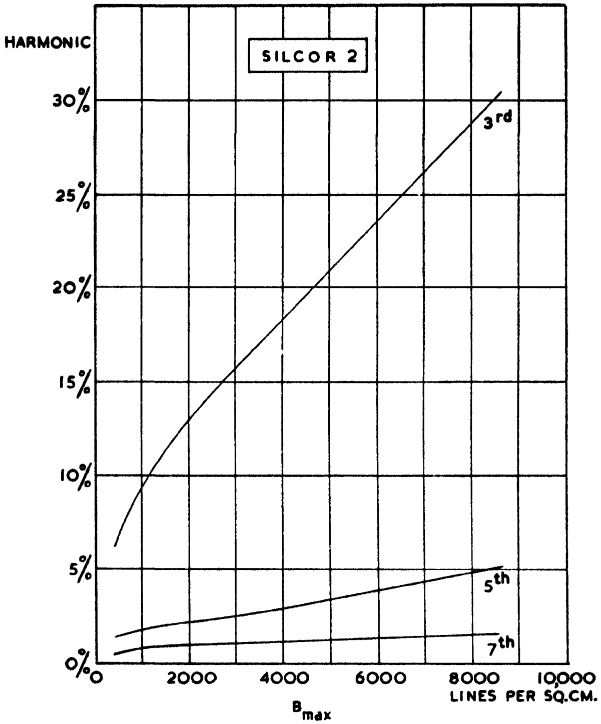

Fig. 131

Partridge's Harmonic Curves for Silcor 2 at Various Flux-densities.

corresponding third harmonic current in the particular case when R is negligible compared with Z_f. This relationship is

$$I_h = I_H\left(1 - \frac{R}{4Z_f}\right)$$

and it is claimed to give a maximum error of 5 per cent for all silicon steels in general use at all normal flux-densities. The particular virtue of I_H is that it is both easily measured with a harmonic analyser and is a property of the core material at the flux-density, frequency, and lamination thickness in question. The two latter factors have little effect at low audio-frequencies.

Equation (108) now becomes

$$\frac{V_h}{V_f} = \frac{I_H}{I_f} \cdot \frac{R}{Z_f}\left(1 - \frac{R}{4Z_f}\right) \quad . \quad . \quad . \quad (109)$$

neglecting the negative sign of equation 107 for numerical purposes. This equation is only applicable to the third harmonic (except as modified later for d.c. polarization), but since the third is much larger than the others and the total harmonic is the r.m.s. value of the individual harmonics, no great error is made in working with the third only.

Partridge gives curves(3) (reproduced in Fig. 131) of the percentage harmonic currents against flux-density for a number of grades of silicon steel, and by making use of these data the harmonic voltages under operating conditions can be calculated from equation 109, provided that particulars of the core dimensions, turns, etc., are known. These curves show that the magnetizing *current* distortion is continuously reduced as the flux-density is made lower and lower and it might appear that it would be advantageous to work at very low values of B in order to keep down output-voltage harmonic percentages. This is not correct, however, for a loaded transformer, because the primary impedance also falls rapidly at low flux-densities so that the shunting effect of the load is less. This becomes so marked at low values of B that the distortion actually rises again at very low values to a figure equal to that at 5 000 lines per cm.^{2} (This applies only to the case in which there is no d.c. polarization.) The magnitude of the primary impedance varies with the incremental permeability of the material, so that Partridge deduces a "distortion coefficient" for a core material which contains the current harmonic in the numerator and a quantity approximating to the incremental permeability in the denominator.

If, as is justifiable in view of the small departure of the flux wave-form from the sinusoidal shape, the well-known expression $\frac{4{\cdot}44 B_{max} T_p a f}{10^8}$ is substituted for $I_f \times Z_f = V_f$ in equation 109 we get—

$$\frac{V_h}{V_f} = \frac{I_H R 10^8}{4{\cdot}44 B_{max} T_p a f}\left(1 - \frac{R}{4Z_f}\right)$$

$$= \frac{4\pi\sqrt{2}\,I_H T_p}{10\,B_{max} l} \times \frac{10^9}{8\pi^2} \times \frac{lR}{T_p^2 af}\left(1 - \frac{R}{4Z_f}\right)$$

$$= \frac{I_H T_p}{0{\cdot}56 B_{max} l} \times \frac{10^9}{8\pi^2} \times \frac{lR}{T_p^2 af}\left(1 - \frac{R}{4Z_f}\right) \quad . \quad (110)$$

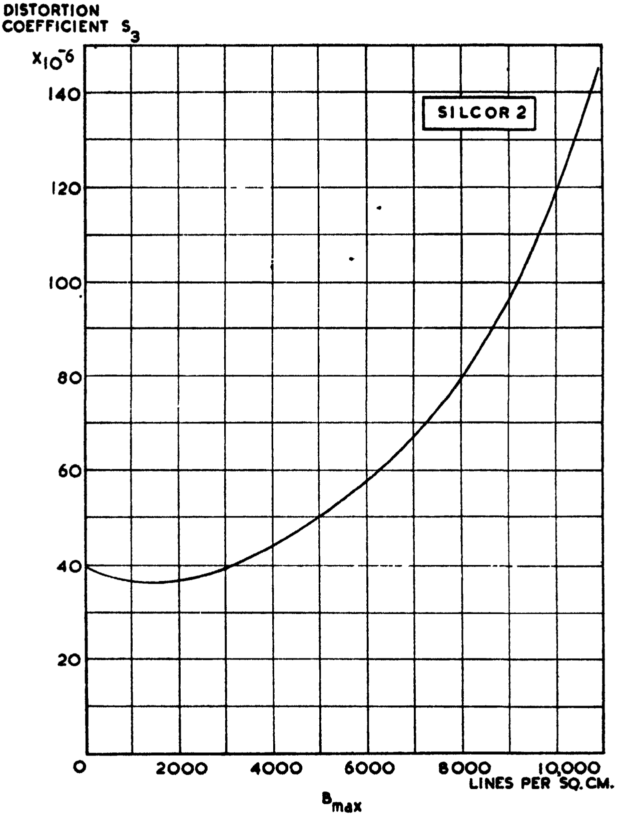

FIG. 132

Partridge's "Distortion Coefficient" for Silcor 2. Note that the Distortion Does not Change Appreciably Below $B_{max} = 4\ 000$ lines per cm.^{2}

Partridge gives the name "distortion coefficient" to the quantity $\frac{I_H T_p}{0{\cdot}56 B_{max} l}$, and in this form it is a property of the core material at a given flux-density (and d.c. polarization, if any). It is independent of frequency, lamination thickness, and shape, etc., being in fact the ratio of the harmonic component of the actual magnetizing current to the r.m.s. value of the hypothetical magnetizing current of a similar transformer with an air core. In an air-cored coil $B = H \div 1$ and the magnetizing current is sinusoidal, so that

$$B_{max} = \frac{\sqrt{2} \times 4\pi T_p I_m'}{10l}$$

where I_m' = r.m.s. current of air-cored coil. Hence

$$I_m' = \frac{10 B_{max} l}{4\sqrt{2}\pi T_p} = \frac{0{\cdot}56 B_{max} l}{T_p} \quad . \quad . \quad (111)$$

The distortion coefficient is thus I_H/I_m'. This is, of course, equal to $\frac{I_H}{I_f} \times \frac{I_f}{I_m'}$ and, if core losses are neglected, it becomes $\frac{I_H}{I_f} \times \frac{1}{\mu_t}$ where μ_t is the value of the permeability of the core applicable to the condition of a.c. magnetization, possibly in the presence of a superimposed steady magnetization.

Using this approximation, it will be seen that Partridge's expression, given in equation 110 in the form

$$\frac{V_h}{V_f} = S_H \times \frac{10^9}{8\pi^2} \times \frac{l}{T_p^2 a f} \times R\left(1 - \frac{R}{4Z_f}\right)$$

can be written as-

$$\frac{V_h}{V_f} = \frac{I_H}{I_f} \times \frac{10^9 l}{4\pi T_p^2 a \mu_t} \times \frac{R}{2\pi f}\left(1 - \frac{R}{4Z_f}\right)$$

$$= \frac{I_H}{I_f}\left(1 - \frac{R}{4Z_f}\right) \times \frac{R}{Z_f}$$

It will be seen that this expression is also of the form "Inherent Distortion divided by the ratio of primary impedance to load impedance." The inherent distortion in this case is of the magnetizing current when operated from a source of zero impedance, whereas that in the previous chapter (p. 370) was of the induced voltage when working from a high-impedance source. This difference is an important one in principle and it is worth while pursuing the matter a little farther. Referring back to the two special cases of source impedance discussed in Chapter X, it will be recalled that the magnetizing current wave-form can be deduced directly from a *B-H* curve when the source is of negligible impedance, and the flux wave-form when the source is of very high impedance. The degree of distortion in the two wave-forms is similar if the same *B-H* curve is used, although the harmonics are in the opposite phase. But the flux wave-form is not of direct interest; the induced e.m.f. wave-form is what is required *and this is proportional to the rate of change of flux with time.* Harmonic percentages in the flux are, as pointed out in Chapter X, multiplied by the order number of the harmonic when they appear in the induced voltage. It is, therefore, to be expected that the "Inherent Distortion" percentages in the induced voltage figures will be decidedly greater than the percentages of corresponding harmonics in the magnetizing current figures. This factor (the harmonic order number) actually cancels, however, in the method of Chapter XX with the ratio of primary reactance at harmonic frequency to that at fundamental, since the former is used in that method and the latter in Partridge's method.

The expression for the fractional distortion given in equation 110, namely

$$\frac{V_h}{V_f} = S_H \times \frac{10^9}{8\pi^2} \times \frac{l}{T_p{}^2af} \times R\left(1 - \frac{R}{4Z_f}\right)$$

enables the distortion to be calculated from curves of S_H, the core dimensions and number of turns together with a knowledge of the circuit impedance. The value of Z in the final factor need not be known to a high degree of accuracy; it can be calculated from the usual expression for L_m

$$L_m = \frac{4\pi T_p{}^2 a\mu_r}{10^9 l}.$$

Partridge gives figures of S_3 (i.e. distortion coefficient for third harmonic) for a number of representative grades of silicon steel, and there should be no difficulty in estimating figures for intermediate grades if required. The particular ones chosen are Silcor I, II, III, IV, and Vicor, all manufactured by Magnetic & Electric Alloys, Ltd. The values are stated to be typical and not necessarily average.

The particular investigation with which Partridge was concerned was in connexion with push-pull output transformers, so that d.c. polarization was not important. Even in push-pull types, however, some polarization is likely to occur owing to lack of balance between the valves, and some information is

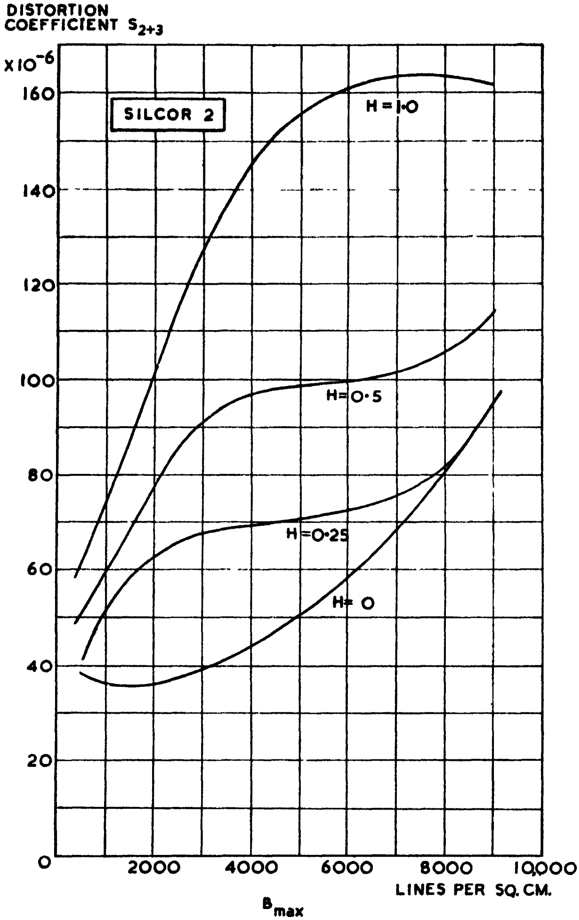

FIG. 133

"Distortion Coefficient" Values when a Small Resultant D.C. Magnetization is Superimposed (Partridge).

given enabling calculations to be made when there is a small d.c. component. If D.C. passes through the winding, the *B-H* curve is no longer symmetrical so that even order harmonics are also present. Partridge found that the quantity $\sqrt{I_2^2 + I_3^2}$

could then be substituted for I_3 as a useful index of harmonic distortion, I_2 and I_3 being the second and third harmonic currents respectively. Although neither the second nor the third harmonics under conditions of appreciable source impedance was found to be related to the magnetizing current harmonics by equation 109, when D.C. was present, the quantity $\sqrt{I_2^2 + I_3^2}$ was found to obey this empirical relationship. Curves are given for S_{2+3} for different values of $H_{D.C.}$ up to $H = 1{\cdot}0$, and some of these are reproduced in Fig. 133.

A number of useful principles come out of Partridge's work—

(1) The use of a high-grade (i.e. high silicon content) steel does not give an appreciably lower inherent distortion, but the distortion under working conditions is reduced by the higher impedance given.

(2) If a particular bass cut is specified (i.e. primary inductance is fixed) distortion in push-pull transformers can be reduced only by the use of an impracticably low flux-density of the order of 100 lines per cm^2.

(3) A proportionate reduction in distortion can be produced if the inductance is allowed to rise, but this is permissible only if further interleaving of primary and secondary is used to keep the leakage inductance from becoming excessive.

(4) The distortion of an output transformer is not reduced under working conditions by the introduction of an air-gap in the core.

Efficiency

The efficiency of an output transformer is more important than that of most small power transformers because the audio-frequency power has become quite expensive when it has reached the output of the amplifier. To waste watts in the transformer involves the use of larger output valves, larger power supplies, larger mains transformers, and more power from the supply. The highest efficiency would, of course, be obtained by making copper and iron loss equal at maximum output, but the harmonic distortion would then be quite excessive and in practice the copper losses are much greater than the iron losses. It should be noted in passing that maximum output from an output transformer does not mean quite the same thing as from a power transformer. The latter works from a constant input voltage and the variation of load, which is discussed in relation to the efficiency, is brought about through a change in load resistance and therefore of currents through the windings. The copper losses accordingly increase as the square of the output, whilst the iron losses remain unchanged. In the case of the output transformer, however, the variable quantity (at any particular frequency) is the *input voltage*; the load resistance remains constant. Under these conditions the iron losses vary as well as the copper losses, so that the efficiency does not fall quite so rapidly at low outputs as it would in a power transformer. As already noted, however, the iron losses are always much smaller than the copper losses, so this effect is not of great importance.

The copper losses can be reduced only by using larger gauge wires or reducing the numbers of turns. If the same performance is to be given in respect of frequency and harmonic distortion, this can be arranged only by using a large core size. In consequence, the output transformer of a high-fidelity amplifier is often as large as the power transformer, although it may only handle some 15 watts compared with perhaps 200 watts delivered by the power transformer.

Heating considerations seldom play any part in the design, not only because the losses have been kept as low as possible in the interests of efficiency, but also because of the intermittent nature of the signal, which reaches peak levels only for very short periods at relatively long intervals. The only point which requires watching is that the heating effect of the d.c. anode current in the primary windings does not become excessive. This is likely to happen only in quite small transformers where freedom from distortion has had to be sacrificed for reasons of cost.

Correlating the Various Factors

Until recently, production of a level frequency response was regarded as the primary requirement for an output transformer, but it is now becoming realized that harmonic distortion is much more noticeable and disturbing in its effects than the loss of a few decibels response at the extreme frequencies. The harmonic distortion is serious not so much on account of its direct effect on a single frequency, although this can be quite unpleasant, but chiefly because it is an index to the amplitude of intermodulation products formed in a non-linear circuit when two or more frequencies are present. When a single note is played on almost any musical instrument the sound contains not only the fundamental frequency but many harmonics which give the characteristic timbre to a particular instrument. A slight addition to the harmonic content by the reproducing system is not therefore easily detected, but if two notes are sounded together a distorting system produces sum, difference, and other combination tones which bear no musical relationship to the two original notes. These foreign frequencies are very easily detected as they give a harshness to the reproduction, and the effect is also quite apparent as a direct modulation of high notes by bass instruments when no intermediate frequencies are present. The tendency in high-quality output transformer design is accordingly to reduce the harmonic distortion to much lower figures than were previously considered satisfactory. A figure well below 1 per cent is desirable when working at full output at the lowest frequency it is wished to reproduce. If lower frequencies are present in the transmission, it is important that they should be cut out before the transformer, because they will still cause inter-modulation distortion even if they are not themselves reproduced.

The starting point in the design is thus seen to be the selection of a flux-density in the core which under the particular working conditions of load, valve impedance, etc., will give a sufficiently low degree of harmonic distortion. When a primary winding has been evolved which will satisfactorily meet the requirements, a check is made to see that the bass response is good enough, but it will nearly always be found to be amply good. The degree of subdivision of the primary and secondary windings has then to be decided so as to make the leakage reactance small enough to satisfy the high-note response specification. The secondary turns are next calculated on the basis of the nominal ratio n where

$$n = \frac{T_p}{T_s} = \sqrt{\frac{\text{optimum valve load}}{\text{loudspeaker impedance}}}$$

The primary and secondary turns are now known and suitable wire gauges are selected to enable the windings to be fitted into the space available after making suitable provision for insulation. The winding resistances are then calculated from the estimated lengths of the mean turns. The winding resistances particularly, and to some extent also the iron losses, will appreciably affect the apparent load on the valves, so that the ratio has then to be so modified that the transformer and load together appear as the required value. This can best be done (as is usual in transformer designing) by the method of an approximate solution followed by a correction, leading to a sufficiently accurate solution at the second attempt. The method is shown most clearly by working through an example in fair detail.

The efficiency of an output transformer is related to its frequency response and harmonic distortion. Attempts to improve these two requirements lead to a lowering of the efficiency as judged at the middle frequencies. This is because the need for a high primary inductance and low working flux-density results in a large number of turns on each winding. A high efficiency can be obtained in a high-quality transformer only by increasing its size.

The first example will be an output transformer for feeding a 3-ohm speaker from two Pen 44 valves in push-pull. A receiver employing such an output stage would probably be of a fairly high fidelity type, so that the specification for this transformer has been made on the tight side, although not sufficiently stringent to represent a really first-class performance.

To cater for those requiring a still better response, suggestions are made as to how to correct the faults which appear in the design produced.

EXAMPLE 1

An output transformer is required to match a pair of Pen 44 valves in push-pull to a 3-ohm loudspeaker. Response (measured across a 3-ohm resistance) to vary not more than 3 db between 50 and 7 000 cycles.

The first step is to look up the valve particulars published by the manufacturers. We find the following figures for operation at 250 volts—

Anode current, 64 mA.
Anode to anode load, 5 000 ohms.
Anode watts per pair (for 5 per cent harmonic), 18.

A suitable core for the required output is shown in Fig. 134 and the completed transformer, together with samples of the principal components from which it is built up, is illustrated in Fig. 135.

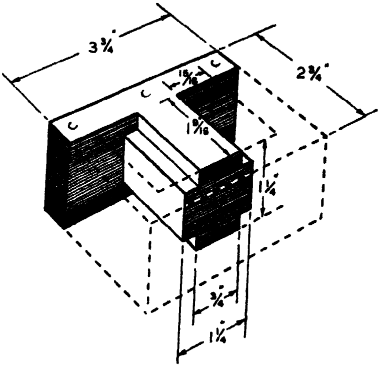

FIG. 134

The Stepped Core for the Transformer Designed in Example 1.

All calculations are based on the production of full power output at the lowest working frequency, this being the worst condition to be met. The maximum anode to anode voltage swing can easily be calculated from

$$W = \frac{V^2}{R_L'}, \text{ where } W \text{ is the anode output in watts.}$$

$$\text{Anode to anode volts} \quad \sqrt{WR_L'} = \sqrt{18 \times 5\,000} = 300 \text{ volts}$$

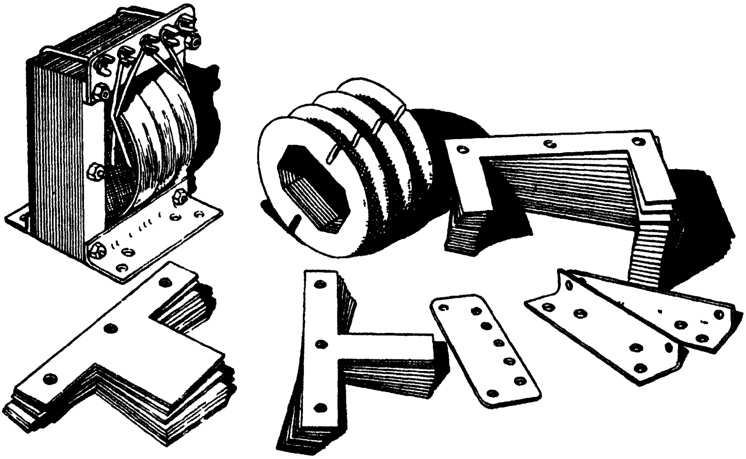

FIG. 135

The Completed Transformer of Example 1, with its Chief Component Parts.

Assuming a maximum flux-density of 5 000 lines per cm^2, the iron loss at 50 cycles will be found from makers' loss curves to be 0·19 watt per lb. Since the weight of the core is 2·3 lb, the total iron loss for the core is $0{\cdot}19 \times 2{\cdot}3 = 0{\cdot}44$ watt.

From the known anode to anode voltage (300 volts), and the iron loss 0·44 watt, we can calculate the value of the resistance which would dissipate this number of watts when the 300 volts are applied across it. The value of R_w representing this iron loss will be $\frac{300^2}{0{\cdot}44} = 205\ 000$ ohms. This figure will be required when we attempt to correct the turns ratio to allow for the transformer losses.

The total primary turns (i.e. from anode to anode) can now be calculated from the standard formula previously used for power transformers.

I.e.—

$$B_{max} = \frac{3\ 490 \times V_p}{T_p \times a \times f}$$

where B_{max} = peak flux-density in kilo lines per cm.^{2}
V_p = input voltage (r.m.s.).
T_p = primary turns.
a = effective cross-section in in.^{2}
f = frequency.

Substituting 5·0 kilo lines per cm^2 for B, 1·18 in.^{2} for a, and 50 cycles for f, we get—

$$T_p = \frac{3\ 490 \times 300}{5{\cdot}0 \times 1{\cdot}18 \times 50} = 3\ 550 \text{ turns.}$$

This winding must, of course, be centre-tapped for the h.t. feed.

We must next estimate the wire gauge. If it were not for the d.c. feed current in the primary, the most efficient arrangement would be produced if the winding space were divided equally between primary and secondary, but to minimize the d.c. voltage drop in the primary, it is desirable to readjust matters to allow the primary rather more room.

A factor which in one respect eases the design of this particular transformer is the fact that the valves are pentodes, and consequently they have a very high internal impedance. The tendency is for a constant current to be driven through the external circuit, more or less irrespective of its impedance. In consequence, the leakage reactance does not cause a drop in output volts relative to the input to the grids of the valves, because the volts on the primary rise due to the reactance.

At first sight, it would not appear to be wise to take advantage of this fact, since the rise in primary volts at high frequencies means, of course, that the load is higher and the valves will not be correctly matched. Fortunately, however, the depth of modulation of a signal is nearly always quite low at the high audio-frequencies, full output at 5 000 cycles being practically unheard-of in normal broadcasting.

The valve distortion resulting from mismatching at low outputs is seldom serious, so that we can in practice help ourselves by not worrying unduly (when using pentodes) about an amount of leakage reactance which would cause a fall in response at frequencies over 4 or 5 kc/s if triodes were used. In this transformer it will be found that two primary and one secondary sections will give sufficient "sandwiching" of the windings to keep the leakage reactance to reasonable proportions, and the space which would otherwise be taken up by insulation will be saved.

The window space available for windings, after allowing for the bobbin cheeks, etc., is 0·46 in.^{2} A suitable wire would seem to be 37 enamel and single silk- (or rayon-) covered, since this should, with careful winding, give 12 500 turns per in.^{2} of winding space. We shall thus occupy 0·28 in.^{2}—rather more than half the available space, as previously explained.

For the purpose of this first approximation we shall neglect the effect of losses on the ratio, so that this would work out to be—

$$\sqrt{\frac{5\ 000}{3}} = 40{\cdot}8$$

i.e. a step-down of 40·8 to 1. The secondary turns will then be

$$3\ 550/40{\cdot}8 = 87.$$

The space available (0·18 in.^{2}) would permit SWG 19 enamel and single silk-covered wire to be used.

The first attempt is, therefore, a primary of 3 550 turns of E. & S.S., split into two halves with the secondary 87 turns of 19 E. & S.S. in the middle. The resistances of the windings (from wire tables) are 390 ohms and 0·27 ohm respectively; referred to the primary, the secondary resistance will be 450 ohms. The resultant resistance between the input terminals can be calculated as 5 720 ohms.

The load for the valves will thus be greater than the recommended load of 5 000 ohms by a resistance representing the transformer losses.

In order to correct this mismatching, the ratio must be adjusted so as to make the load between the input terminals appear to be 5 000 ohms. This state of affairs will be brought about if we make the value of the output load (when transferred to the primary) look like 4 260 ohms; in conjunction with the transformer losses, this will give the required load of 5 000 ohms total. In order to transfer the load of 3 ohms on the secondary to a value of 4 260 ohms on the primary, the ratio will have to be

$$\sqrt{\frac{4\ 260}{3}}$$

i.e. 37·7 to step down. The secondary winding will accordingly be adjusted to 94 turns (having a resistance of 0·30 ohm).

The position is now that, having ascertained the valve output watts and optimum matching load, a likely-looking core has been selected and the windings calculated to match the given loudspeaker to the valves, assuming a flux-density that will give sufficiently small harmonic distortion at the lowest frequency, and highest signal level. The harmonic distortion will be less at higher frequencies and lower inputs, so that condition (*b*) (i.e. harmonic distortion limit) will be satisfied. There is, however, no indication that the other two conditions will be satisfied, so it is necessary to calculate the performance as regards frequency response and efficiency.

The drop in the bass response will be determined, as explained previously, by the relative magnitudes of L_m and the transferred load R_L' in parallel with the valve anode impedance R_a (this latter in the case of pentodes is too high to have any appreciable effect and will be neglected). We must accordingly calculate the primary inductance and see at what frequency the corresponding reactance value is equal to the transferred load resistance. At this frequency the response will be 3 db down, i.e. $\sqrt{2}$ to 1 in volts.

It will be remembered that a coil is said to have an inductance of 1 henry if an e.m.f. of 1 volt is set up across its ends when the current through it is changing at the rate of 1 amp per sec, and an e.m.f. of 1 volt represents a change of 10^8 linkages per sec between the coil and its magnetic lines of force.

Since L_m is equal to the voltage produced when the current falls from 1 amp to zero in a second, the expression for L_m is given by

$$L_m = \frac{4\pi\mu a T_p^2}{10^9 l} \text{ (with dimensions in cm)}$$

or

$$L_m = \frac{3{\cdot}19 \times 10^{-8} T_p^2 \mu a}{l} \text{ (with dimensions in in.)}$$

As explained in Chapter II the value of μ for iron varies considerably with the degree of magnetization, and it is this variation which is responsible for the harmonic distortion which, as we have seen, is liable to arise if the core is worked at a high flux-density. As the value of μ tends to decrease at flux-densities below 5 000 lines per cm², the bass cut will be greatest at low outputs. At a flux-density of 500, the corresponding value of μ is 2 000, and we shall assume this value in our calculation. The magnetic length l for this particular stamping is 7·25 in., and the cross-section 1·18 in.,² so that

$$L_m = \frac{3{\cdot}19 \times 10^{-8} \times 3\,550^2 \times 2\,000 \times 1{\cdot}18}{7{\cdot}25}$$

$$= 131 \text{ henrys.}$$

The frequency at which this inductance will have a reactance equal to 5 000 ohms is under 10 cycles, so that the condition of not more than 3 db loss cut at 50 cycles is very easily met. It should not be forgotten, though, that the inductance has been calculated on the assumption that the two output valve currents are exactly equal, so giving no resultant d.c. magnetization.

In practice this exact balance is unlikely to occur, and there will be some appreciable reduction of inductance owing to the out-of-balance d.c. field. If the valves are individually set up so as to draw the recommended current (as would be the case in P.A. gear, etc.), this reduction would not be serious. In mass-produced sets, where there is no special selection of valves, etc., it is possible that the difference in anode currents may be serious, so that it is usual in that case to assume that there may be a resultant current of, say, 20 mA through the winding, and to calculate the inductance by a method to be shown later. The core would then in all probability be "gapped."

The question of high-note response has already been discussed. We can safely assume that the pentode valves will maintain the response up to 9 kc/s. This point will be referred to again later when dealing with triodes.

At full-load the input is 18 watts at 300 volts. Neglecting the out-of-phase component through L_m, the primary current is 18/300 amp = 0·06 amp, and the secondary current is 2·2 amp. The various losses can now be calculated—

Iron loss 2·3 lb at 0·19 watt per lb = 0·44 watt
Primary copper loss $0{\cdot}06^2 \times 390$ = 1·40 watts
Secondary copper loss $2{\cdot}2^2 \times 0{\cdot}3$ = 1·45 watts

Total loss 3·29 watts

$$\begin{aligned}\text{Eff.} &= \frac{\text{output}}{\text{input}} \times 100 \text{ per cent} \\ &= \frac{18 - 3{\cdot}29}{18} \times 100 \text{ per cent} \\ &= 82 \text{ per cent}\end{aligned}$$

These calculations indicate that the transformer which has been designed should meet the requirements reasonably well in so far as the efficiency is satisfactory and the frequency response quite good.

The harmonic distortion is next calculated from equation (110), p. 389. Partridge's curve shown in Fig. 133, gives a figure 50×10^{-6} for S_3 at 5 000 lines per cm.^{2} Hence the fractional distortion at 50 cycles is

$$50 \times 10^{-6} \times \frac{10^9}{8\pi^2} \times \frac{7{\cdot}25}{3\,550^2 \times 1{\cdot}18 \times 50 \times 2{\cdot}54} \times 5\,000\left(1 - \frac{5\,000}{8\pi \times 50 \times 327}\right) = 0{\cdot}012 \text{ or } 1{\cdot}2 \text{ per cent.}$$

Note 1. In the denominator of the final factor, Z_f is derived from an inductance of 327 henrys, instead of 131 henrys as previously calculated when working out the bass loss. This difference arises from the different values of μ_i at the two flux-densities. A figure of 5 000 is applicable at $B_{max} = 5\,000$ lines per cm^2, instead of only 2 000 at $B_{max} = 500$.

Note 2. The distortion has been calculated for a closed magnetic circuit (i.e. without the air-gap recommended to deal with unbalanced direct currents in the two halves). The introduction of the air-gap will not, however, greatly affect the percentage of harmonic, although it will give a greater bass loss than calculated previously.

If the alternative method of distortion calculation is used, in which the harmonic percentage obtained from a loaded transformer is derived from the "inherent distortion" of an unloaded one, the figures are rather higher. The inductance (for small signals) has already been calculated to be 131 henrys (p. 400). At the 3rd harmonic frequency of 150 cycles, this represents a reactance of 123 000 ohms. The harmonic voltage developed across the 5 000-ohm load is then approximately one twenty-fifth of the inherent distortion which from Fig. 126 is 51 per cent 3rd harmonic at $B_{max} = 5\,000$ lines per cm.^{2} The 3rd harmonic distortion is thus 2 per cent. The reduction factors for the higher harmonics are greater owing to the increased primary reactance at those frequencies, so the figures are all less than 1 per cent.

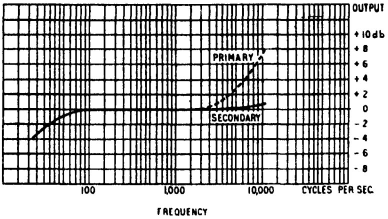

FIG. 136

Measured Response Curve for the Output Transformer of Example 1. Note the Effect of Leakage Reactance on the Primary Voltage.

Testing

Although an output transformer is intended to feed a loudspeaker, it is more satisfactory to carry out tests using a 3-ohm resistance as a load. A loudspeaker which is nominally 3 ohms will actually have an appreciable inductance in addition to a d.c. resistance, so that its impedance tends to rise quite considerably towards the higher audio-frequencies. If the output valves are pentodes, this rise in impedance has a considerable effect and it is the practice to shunt the loudspeaker (or more commonly the transformer primary) by a resistance and capacitor in series, of such values that the combination, with the loudspeaker, looks like a constant resistance of the nominal value at all frequencies. It is, therefore, more convenient and quite justifiable to use a plain resistance for testing purposes.

The first test made is for frequency response. The transformer was connected to a pair of Pen 44 valves and to a 3-ohm resistance, the grids of the valves being fed from a centre-tapped choke. The input volts to the grids were adjusted to be constant at all frequencies. The output across the 3-ohm resistance at frequencies between 30 and 10 000 cycles is shown in Fig. 136. The characteristic is seen to be well within the requirements laid down; i.e. within 3 db from 50 to 7 000 cycles.

Fig. 136 shows also the variation of volts across the primary winding, the rise at frequencies above 5 kc/s being quite considerable. This is due, as we have seen, to the leakage reactance, which would cause a corresponding drop in the output characteristics if the valves were of a low anode impedance, as would be the case with triodes.

Tests were next made to determine the harmonic distortion produced by the transformer, using a harmonic analyser. The test was made at

50 cycles. To avoid confusion with any harmonic produced in the output valves, the transformer primary was fed through a resistance, comparable in value to the anode impedance of the valves. This necessitated a much higher voltage from the source than the 300 volts actually developed across the transformer primary, and the production of this voltage in itself introduced a small amount of harmonic in addition to an appreciable amount of 5th harmonic present in the mains supply. Measurements made on the output from the transformer gave the following figures—

3rd Harmonic, 2·9 per cent
5th Harmonic, 0·7 per cent

As regards efficiency, tests showed that, at 50 cycles and full input, the figure obtained was 77 per cent.

It will be seen from these tests that the design gives a performance well up to the requirements laid down in the specification. If a higher efficiency had been asked for, or the output valves had been triodes, a rather larger stamping would have had to have been used. This would have permitted—

(*a*) Further sectionalizing and interleaving of primary and secondary windings to reduce the magnetic leakage. The high-note response could then have been made sufficiently good, even with a low impedance input.

(*b*) Larger section windings giving lower resistances. The losses, of which the larger part were copper losses, would be reduced by this means, and the efficiency correspondingly raised.

For comparison with the previous example, we shall next proceed to work out (in less detail) an output transformer for two large triodes in push-pull, giving an output similar to that from the pair of Pen 44's.

EXAMPLE 2

A push-pull transformer is required to match two PX25 valves to a 3-ohm speaker. The response from the grids of the PX25 valves to a 3-ohm resistance load must be within 1 db from 50 to 10 000 cycles.

The valve particulars for 400-volt working are—

Anode current (each valve)	63 mA
Anode to anode load	4 500 ohms
Anode watts (per pair)	13
Anode impedance (per pair)	2 530 ohms

A stamping which appears suitable is illustrated in Fig. 137. This is slightly larger than that employed in Example 1, and should permit the greater sectionalization necessary to keep the leakage reactance lower. The paper-interleaved layer-wound construction will be used.

From the valve data the primary volts will be $\sqrt{13 \times 4\,500} = 242$ volts. The primary turns will be—

$$T_p = \frac{3\,490 \times 242}{5 \times 1{\cdot}41 \times 50} = 2\,400 \text{ turns.}$$

The space available for the primaries is approximately 2 in. × 0·4 in.; SWG 31 enamel-covered wire which should wind 85 turns per inch interleaved with 0·002-in. paper appears suitable. This winding could conveniently be made up of four sections, each of 600 turns. The resistance would be 89 ohms. The uncorrected ratio would be—

$$\sqrt{\frac{4\,500}{3}}$$

= 38·7 to 1 step down, i.e. 62 turns. Three layers of doubled SWG 18 enamel look a reasonable arrangement for the secondary, giving a resistance of 0·06 ohm. Transferred to the primary this would represent 90 ohms. The iron loss at 50 cycles and a B_{max} of 5 kilo-lines per cm², will be (as before) 0·19 watt per lb. As the complete core weighs 3·85 lb the total iron loss will be 3·85 × 0·19 watt, i.e. 0·73 watt. This loss will be represented by a resistance R_w of 80 000 at the maximum input voltage of 242. The transformer with an uncorrected ratio would give an anode load which works out at 4 430 ohms. This is sufficiently close to the required value of 4 500 ohms to be not worth altering. The winding is, therefore, as follows—

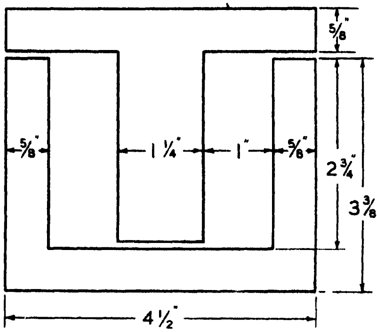

FIG. 137
The Core Suggested for Example 2 (Ref. 460A, p. 472).

Primary (in four sections): 2 400 turns (total) of SWG 31 enamel.

Secondary (in three sections): 62 turns (total) of two strands SWG 18 enamel in parallel.

Calculated Performance

First of all the efficiency has to be calculated.

Input Power: 13 watts.

Primary current—

$$\sqrt{\frac{13}{4\,500}} = 0{\cdot}054 \text{ amp}$$

Primary copper loss—

$$0{\cdot}054^2 \times 90 = 0{\cdot}26 \text{ watt}$$

Secondary current—

$$0{\cdot}054 \times \frac{2\,400}{62} = 2{\cdot}1 \text{ amp}$$

Secondary copper loss—

$$2{\cdot}1^2 \times 0{\cdot}06 = 0{\cdot}26 \text{ watt}$$
$$\text{Iron loss} = 0{\cdot}73 \text{ watt}$$

$$\text{Total loss (at full load)} = \underline{\underline{1{\cdot}25}} \text{ watts}$$

$$\text{Efficiency} = 100\left(\frac{13{\cdot}0 - 1{\cdot}25}{13{\cdot}0}\right) = 90{\cdot}5 \text{ per cent.}$$

To test the bass response, the primary inductance is worked out as before—

$$L_m = \frac{3{\cdot}19 T_p{}^2 \mu a}{l} \times 10^{-8} \text{ henrys}$$
$$(l = 10 \text{ in.})$$
$$(a = 1{\cdot}41 \text{ in.}^2)$$
$$= \frac{3{\cdot}19 \times 2\,400^2 \times 2\,000 \times 1{\cdot}41 \times 10^{-8}}{10}$$
$$= 52 \text{ henrys}$$

The effective input impedance is that of the source (2 530 ohms) in parallel with the load (4 500 ohms), i.e. approximately 1 600 ohms. The primary will have a reactance of 1 600 ohms at 5 cycles; the loss will then be 3 db. At 30 cycles the loss will be negligible.

The harmonic distortion, calculated as in the previous example, is 0·96 per cent third harmonic at 50 cycles and 2·7 per cent at 30 cycles.

2. Output Transformers Carrying D.C.

All the transformers which have so far been considered have been supplied only with A.C.; there has been no d.c. component as well. It is now necessary to examine the behaviour of a transformer in the anode circuit of a single-valve output stage, where a large d.c. component flows through the winding in addition to the A.C. The most evident effect of the D.C. is on the inductance of the windings, although it also increases the harmonic distortion. The effect on the primary side only of the transformer need be considered since the secondary windings can be calculated in the same way as before, once the primary is settled. It is sufficient, therefore, to regard the primary as a choke coil; in fact, the calculation for the inductance of iron-cored chokes carrying D.C. is identical with those for the primary inductance of a transformer carrying D.C. In the latter case the effect of distortion on the output wave-form must also be considered.

To refresh our memories on the subject of the magnetic properties of iron, Fig. 138 shows a magnetization (or *B*-*H*) curve similar to that described in detail in Chapter II. It will be remembered that *H* is the magnetizing force (proportional to the ampere-turns of the magnetizing coil), and *B* is the flux-density in the iron, resulting from the application of this magnetizing force. The *B*-*H* curve takes the form of a loop, because a given value of *H*, on account of hysteresis, produces a higher *B* when *H* is decreasing than when it is increasing. At any particular state of magnetization, represented by a point on the curve, the permeability μ of the iron is the ratio of B/H, and it will be remembered that the inductance is proportional to μ, other factors remaining fixed.

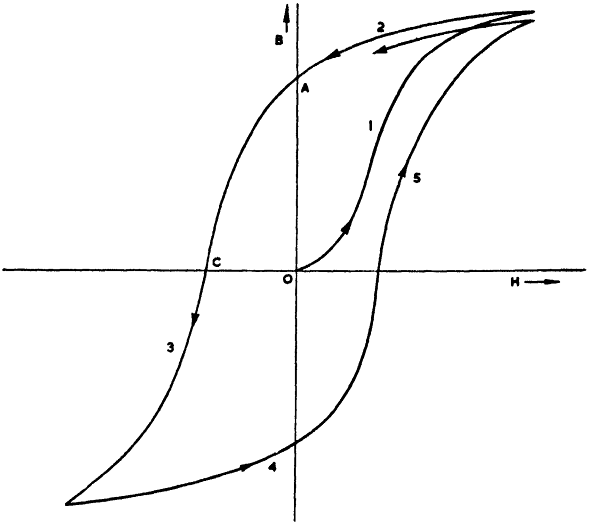

FIG. 138
A Typical *B*-*H* Curve for a Ferromagnetic Material.

To simplify matters, it will be assumed that the iron was initially demagnetized and that the effect of passing a certain steady current through the winding is to reach the state of magnetization represented by the point *P* on the central limb of the *B*-*H* curve.

The permeability μ is given by the ratio B/H, i.e. the tangent of the angle which *OP* makes with the *H* axis. This is the value of μ that should be used in d.c. magnet calculations.

If the primary current is not reduced to zero, but is caused to fluctuate by a relatively small percentage (as would be the case if a small a.c. component is superimposed on the D.C.), the operating point *P* does not move up and down the initial magnetization curve but proceeds to describe a small loop from the original position.(4, 5, 6)

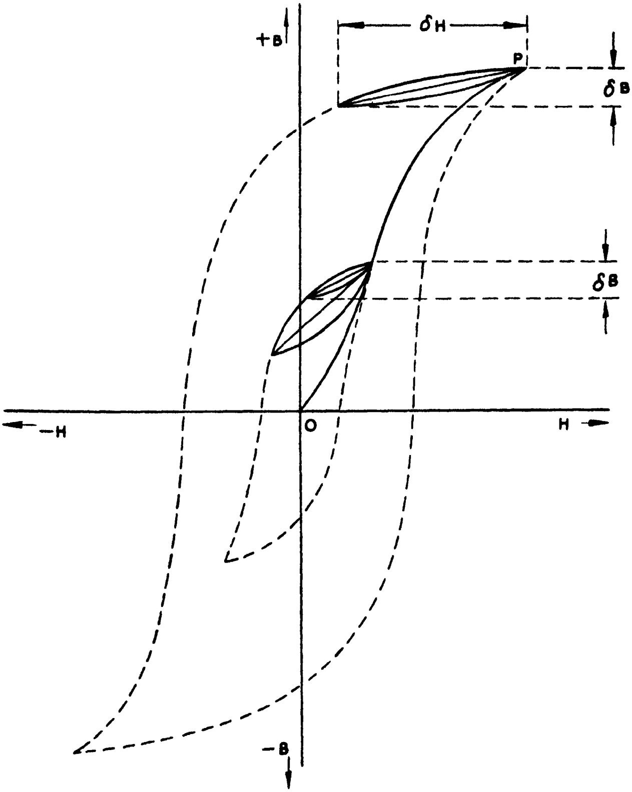

FIG. 139

The Small Loops which are Followed when a Small A.C. Ripple is Applied to a Sample of Iron Already Magnetized by a D.C. Component. Note that the Slope of Axis of the Ellipse Becomes Greater at *Small* D.C. Fields and *Large* A.C. Ripples.

If the small a.c. magnetizing force is superimposed on the d.c. force, and it is assumed that it is at its maximum positive value and about to decrease, the resultant magnetizing force will begin to fall. As in all materials exhibiting hysteresis effects, the operating point does not retrace its previous path but moves to the left in a more nearly horizontal direction along the return portion of the relevant *B-H* curve for the particular value of B_{max}. When the small a.c. magnetizing force has reached its most negative value and begins to decrease, the resultant *H* starts to increase again. Once more, however, it does not follow its descending path, but moves to the right below its previous value, eventually returning to the point *P*, having traced out a loop roughly elliptical in shape (Fig. 139). The slope of the axis of this small loop represents the *differential or incremental permeability* (i.e. the permeability figure to be used to calculate the inductance of the coil for a small a.c. ripple superimposed on the d.c. magnetization). The incremental permeability is defined as dB/dH and the symbol used is μ_i. This is the limiting value of the quantity $\Delta B/\Delta H$, the prefix in each case meaning "a small change in." Thus ΔB is a small change in B and ΔH a small change in H.

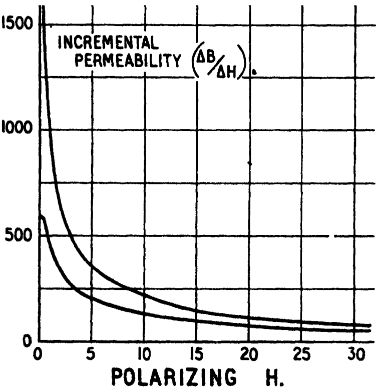

FIG. 140
Incremental Permeability Curves for 4 per cent Silicon Steel.

The slope of the axis of the loop tends to increase as the magnitude of the ripple increases; the incremental permeability, and hence the inductance, are greater for large ripple values. To give tables or curves for all the possible combinations of d.c. and a.c. magnetization would entail an enormous number of figures, so that values of incremental permeability are usually quoted against various d.c. magnetizing forces for two or three values only of a.c. magnetization. The values of a.c. magnetization chosen are: (1) for very small ripples; (2) for a value of $\Delta B = 400$. Fig. 140 shows representative curves for 4 per cent silicon steel.

It may seem strange that the a.c. ripple should be given in terms of ΔB (i.e. change in flux-density), whilst the d.c. magnetization is in terms of H (i.e. magnetizing force). This is, however, the most convenient method, because we almost always quote the ripple as so many *volts* applied to the coil, whilst the d.c. magnetization results from a specified *current* through the winding. Ripple volts can easily be converted to flux-density figures (i.e. ΔB), whilst the d.c. polarizing current can be translated in a given magnetic circuit to magnetizing force (i.e. H). It should not be forgotten that ΔB represents peak-to-peak value of the ripple. The value of B_{max} for the ripple is thus $\Delta B/2$.

Curves of incremental permeability must be used with caution, because they are *strictly* applicable only to the particular material from which the data were obtained, used in the particular laminations chosen and with the same methods of jointing. Nevertheless, the accuracy obtainable from typical curves of this kind, if intelligently used, is sufficiently good for normal design purposes, because variations between samples of a given grade of material are quite appreciable.

The particular form in which incremental permeability data are displayed in Fig. 140 is the one best suited to the needs of an audio-frequency transformer designer. For reasons connected with harmonic distortion, the a.c. flux-density is usually low relative to the d.c. polarization, so that the inductance value for small signals is usually required. For those rather infrequent cases in which there is a d.c. component in the windings of a power transformer, the data are better presented in the form of a series of curves of incremental permeability against B_{max} at various fixed values of $H_{D.C.}$. Transformers supplying half-wave rectifiers are probably the commonest examples of such cases; the values of B_{max} used there are similar to those normally employed, i.e. 12 000 to 14 000 lines per cm^2. The particular case in which $H_{D.C.} = 0$ is the usual condition in power transformers without D.C. in the windings. The incremental permeability then assumes the name *reversible permeability* and the *B-H* loop is symmetrical about the zero point. The slope of its axis is not necessarily exactly the same as that of the normal magnetization curve through the points of the loop, so that the reversible permeability is not quite the same as the d.c. permeability. Eddy currents also affect the shape of the a.c. loop to some extent.

If a particular core is selected, the inductance of a coil having a stated number of turns, and carrying a given value of D.C., can easily be worked out, provided curves or tables of incremental permeability are available. Start off by working out the polarizing (i.e. d.c.) magnetizing force H from the usual expression—

$$H = \frac{4\pi IT}{10l}$$

where T = no. of turns in coil,
I = current in amperes,
l = magnetic length of core in cm.

The makers' curves for the core material enable the value of incremental permeability corresponding to this value of d.c. polarization to be read off. There will (as mentioned previously) be two values of permeability: one for very small a.c. ripples, and the other for larger values. To decide which one is more nearly applicable to the particular conditions, the familiar formula is used to determine the value of ΔB reached when a stated value of a.c. ripple volts is applied across the coil.

$$\Delta B/2 = \frac{3\,490 \times V}{T \times a \times f}$$

where V = r.m.s. volts applied,
T = no. of turns,
a = cross-section of core in in.^{2}
f = frequency.

Having decided which value of ΔB is more suitable, the value of incremental permeability can be read off the curve. Substituting now in the formula which has been already derived for the inductance, the desired result is obtained—

$$L = \frac{4\pi\mu_i T^2 a}{10^9 l} \text{ henrys (dimensions in cm)}$$

or $$L = \frac{3{\cdot}19T^2\mu_i a}{l} \times 10^{-8} \text{ henrys (using inches)}$$

EXAMPLE 3

It is desired to calculate the primary inductance of a 4 *per cent silicon steel-cored choke coil when carrying* 70 *mA D.C. The cross-section of the core is* 1 *in.*^{2} *nominal* (0·92 *in. actual*) *and the magnetic length is* 5·25 *in.* (*Fig.* 141). *The coil consists of* 3 700 *turns of SWG* 36 *enamel-covered wire, and the ripple is* 1 *volt at* 50 *cycles.*

The polarizing H is—

$$\frac{4\pi \times 0{\cdot}07 \times 3\,700}{10 \times 5{\cdot}25 \times 2{\cdot}54} = 24{\cdot}5 \text{ oersteds}$$

$$\Delta B/2 = \frac{3\,490 \times 1}{3\,700 \times 0{\cdot}92 \times 50} = 0{\cdot}02 \text{ kilo-lines/cm}^2 = 20 \text{ lines/cm.}^2$$

This value of ΔB represents a very small ripple, so that the lower curve can be used. Working in this way the incremental permeability is found to be 75. Hence—

$$L = \frac{3{\cdot}19 \times 3\,700^2 \times 75 \times 0{\cdot}92 \times 10^{-8}}{5{\cdot}25}$$

$$= 5{\cdot}8 \text{ henrys.}$$

The inductance of 5·8 henrys which has been thus obtained, is a disappointingly low value. What is happening is that the D.C. is so nearly saturating the core that the changes in magnetization, brought about by the a.c. ripple, have such a small effect on the value of B that a large ripple current flows—in other words the inductance is low.

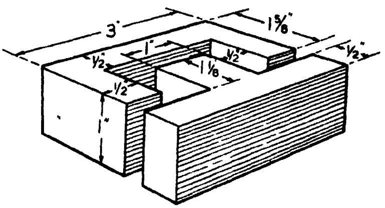

FIG. 141
Core for the Smoothing Choke Designed in Example 3.

The same core and coil can, however, be made by an artifice to yield a much higher inductance and, furthermore, one which varies much less with different inputs.

This artifice is the introduction of an air-gap into the magnetic circuit(7). By the choice of the correct gap length, the extent of the d.c. polarization can be so reduced that the operating point moves down the curve to a point where the inclination of the axis of the small hysteresis loop has increased so much that it more than offsets the effect of the air gap on the a.c. field (Fig. 139).

The most convenient way of calculating the optimum air-gap under any specified conditions is that evolved by C. R. Hanna.(8, 9, 10) From the known values of B, H, incremental permeability, etc., a curve can be produced showing the values of the quantity LI^2/V against the quantity IT/l (which is proportional to the d.c. polarization) for the best fractional air-gap. In these expressions—

L = inductance in henrys
I = D.C. in amp
T = no. of turns in coil
l = magnetic length of core in cm
V = volume of core in cm^3

The exact shape of the curve varies somewhat according to the grade of stamping used, but a typical curve for 4 per cent silicon-steel material is shown in Fig. 142.

The method of using the curves is best shown by again working out the example already given, but this time putting in an air-gap. The relevant figures for the choke are—

Volume of core . . .	4·73 in.^{3}
Magnetic length . . .	5·25 in.
No. of turns	3 700
D.C. flowing	70 mA

The d.c. magnetization IT/l is first calculated as $\dfrac{0{\cdot}07 \times 3\,700}{5{\cdot}25 \times 2{\cdot}54}$

= 19·5 ampere turns per cm.

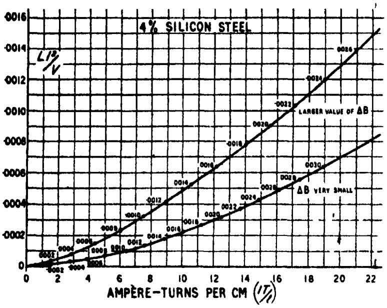

FIG. 142

LI^2/V Curves for 4 per cent Silicon Steel. Appropriate "Fractional air-gap" Figures Appear Alongside the Curve.

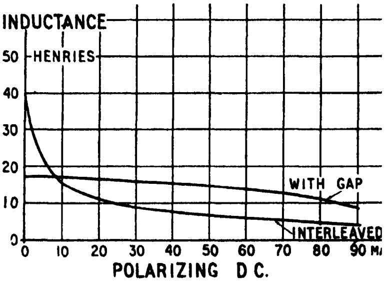

FIG. 143

Measured Inductance Values for the Choke Designed in Example 3, with and without Air-gap.

From the curve (Fig. 142) the corresponding value of LI^2/V comes out to be 0·00067 on the curve for ΔB very small, and the nearest value of fractional air-gap shown along the curve is 0·003. The inductance of the choke is quickly found to be—

$$0{\cdot}00067 \times \frac{V}{I^2}$$

i.e. $$\frac{0{\cdot}00067 \times 4{\cdot}73 \times 2{\cdot}54^3}{0{\cdot}07^2} = 10{\cdot}6 \text{ henrys}$$

To obtain this inductance, the length of air-gap has to be correct, although the figure is not particularly critical. The total magnetic path length of the core is 5·25 in., and since the fraction is 0·003 the actual length of air path is 5·25 × 0·003 in. = 0·015 in. It must not be forgotten that the gap-piece will be half the length calculated (i.e. 0·007 in. approx.) because there are two gaps in series.

Test Results

A choke, wound to the data given, was measured both with the core stampings interleaved, and with the air-gap pieces fitted. The inductance obtained at various values of D.C. is shown in Fig. 143. Without

D.C., or with only quite small currents, the inductance is considerably higher without an air-gap, but a useful improvement is obtained at higher values of D.C. The air-gapped type will be seen to be very little affected by the passage of D.C., and this constancy of inductance is in itself often of importance.

Alternative Methods of Calculation

Although the method of Hanna described above is the quickest once curves of LI^2/V are available, the preparation of these curves is laborious. If it is desired to make a few isolated calculations on a material for which there are no LI^2/V curves, other methods may be more convenient. One such method is first to find the state of d.c. magnetization in the iron by a graphical method, then to look up the incremental permeability corresponding to this condition, and hence to calculate the inductance.(11) Consider a d.c. electromagnet having a uniform magnetic circuit entirely of iron except for a short air-gap perpendicular to the flux, and of uniform length. The m.m.f. required to establish a flux Φ round the circuit, will be the sum of the m.m.f.s required for the iron section and for the air-gap. If a curve of Φ against m.m.f. is drawn for the iron portion of the circuit and a similar curve is drawn

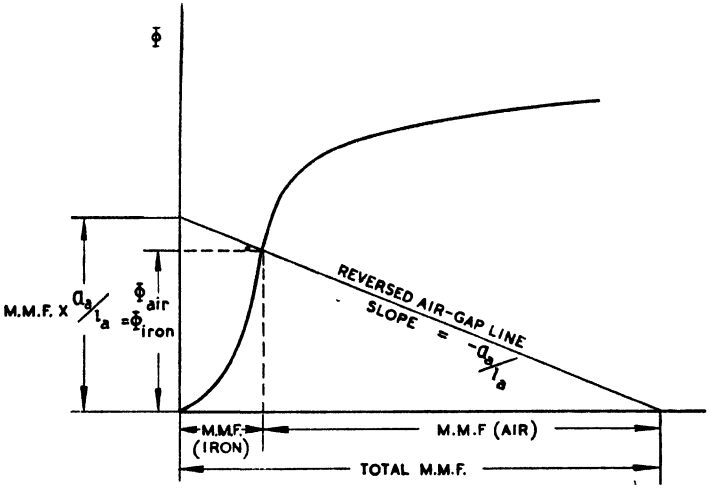

FIG. 144

The Application of the Air-gap Line to a Steady Magnetization Problem.

reversed left to right for the air-gap starting from the total m.m.f. available, the intersection will give the value of Φ for the circuit (Fig. 144). The value of m.m.f. to the left of the ordinate erected at the intersection is that required for the iron, whilst that to the right of the intersection is that required for the gap. The shape of the iron curve is, of course, that of the normal magnetization curve to a suitably chosen scale, whilst that for the gap is a straight line having a slope $-a_a/l_a$. In order to be able to use an actual B-H curve for the material, it is only necessary to divide the vertical scale by a (cross-sectional area), so obtaining B in the iron, and the horizontal scale by l, obtaining H in the iron. The figure obtained from the diagram for H in the air now requires conversion, but this is not usually a great disadvantage.

The use of a reversed air-gap line corresponds to the calculation of thermionic valve gain by means of a "load line."

Having first found the value of $H_{D.C.}$ in the iron by means of the air-gap line method, the next step is to look up the appropriate value of incremental permeability from curves for the material. The inductance can then be readily determined. In many practical cases, however, the air-gap is not stated, but has to be chosen for maximum inductance. As the method gives no guidance on the choice of the optimum gap, it is not very useful in that form. This difficulty does not apply to a method proposed by R. T. Beatty.(12)

Beatty's method involves drawing two curves for the material (Fig. 145). One of these is the normal magnetization (B-H) curve. This is drawn well to the right of the sheet, whilst on the left (i.e. on the negative side of the origin) is drawn a curve of $B/\mu_i - H$ against B using the same vertical scale for B. A point P is taken so that OP represents the average m.m.f. per cm over the whole magnetic circuit (air and iron). If the maximum inductance is desired (as is usually the case) a line is drawn from P as a tangent to the $B/\mu_i - H$ curve. Then the value of LI^2/V for these conditions is equal to twice the area of the shaded triangle in Fig. 145. To find the optimum gap ratio, draw RS horizontally to cut the B-H curve at S and join S to P. Then the tangent of the angle that SP makes with the vertical is the ratio of air-gap to magnetic length of the iron path.

Reference should be made to the original paper for proof of these statements.

Once the primary winding of a transformer has been decided upon, the rest of the design follows the lines of the previous examples in which there was no d.c. complication.

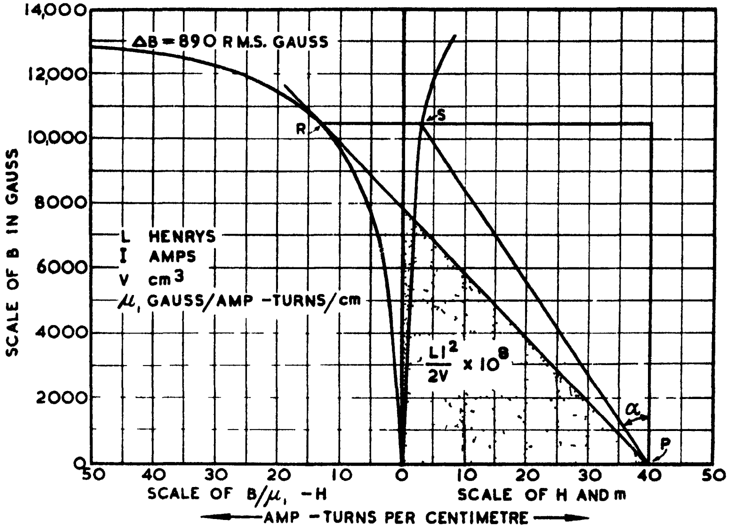

FIG. 145

Beatty's Method of Designing Iron-cored Inductances Intended to Carry Superimposed D C.

In the interests of low harmonic distortion, the maximum flux-density in a transformer with superimposed d.c. magnetization should be kept below that which would be satisfactory in a push-pull type. It will be remembered that in the latter case a reduction in flux-density below about 4 500 lines per cm^2 does not reduce the harmonic percentages, but this is not true of polarized cores. A progressive reduction in distortion takes place in those cases as the flux-density is reduced.

Inductance Measurements

An almost essential piece of equipment for those dealing with audio-frequency transformers is a means of measuring their inductance under working conditions. This often involves the passage of D.C. through the windings and also the application of different and known ripple voltages. Although simple impedance measurements can be made with sufficient accuracy for quantity production testing, the only completely satisfactory arrangement for laboratory purposes is a suitably selected bridge.

A form of the Hay bridge (Fig. 146) is a very satisfactory

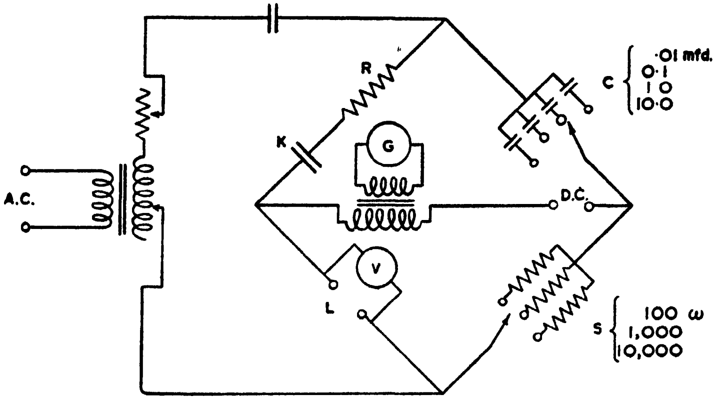

FIG. 146

An A.C. Bridge suitable for the Measurement of Iron-cored Inductances carrying D.C.

one to use, as the D.C. passed through the coil under test is not altered during the balancing operation. The d.c. path is from the supply unit through the galvanometer transformer (which should be suitably gapped), through the coil under test, and back via *S*. As balancing is carried out on *C* and *R*, the direct current is unaffected. The capacitance *K* is used only for power-factor adjustment, and as its value does not enter into the calculation of the inductance it need not be accurately calibrated. The procedure is to set the standard capacitance *C* and resistance *S* to values to suit the expected inductance figure and then to make alternate adjustments of *K* and *R* to obtain a minimum output in the detecting arm. The choice of detecting device is of considerable importance and a frequency-selective type is almost essential owing to the production of harmonics in the coil being measured. A vibration galvanometer tuned to the frequency of the supply to the bridge is very satisfactory and is preferable to telephones with tuned filters, because it shows up small changes in the large deflexion produced during the earlier stages of balancing.

The voltage supplied to the bridge may have to be quite high in order to obtain a ripple voltage across the coil similar to that found during operation. This may be some 100 volts or more, so that it is desirable to provide a tapped transformer supplying up to 500 volts to the bridge. The provision of appreciable power in this way almost dictates that the supply should be taken from the mains at 50 or 60 cycles, but this is not a disadvantage because high values of B can be obtained in the transformer under test without the application of excessive ripple voltages. At these frequencies a visual detector, such as the vibration galvanometer already suggested, is almost essential. The relatively high a.c. voltages which are applied, together with the considerable direct current which may be passed through some of the arms in addition, will generate appreciable heat in the coils. The resistance boxes should therefore be of the heavy-duty type which are rated to carry the currents expected. Since the incremental inductance is in any case a somewhat variable quantity depending on the previous magnetization history of the core the resistances need not be of the highest grade.

The ripple voltage across the coil under test can best be measured by means of a thermionic voltmeter, since this has a sufficiently high impedance to avoid modification of the voltage distribution around the bridge elements.

When an approximate balance has been obtained by successive adjustments of K and R, the ripple voltage across the coil under test should be checked and readjusted by variation of the input to the bridge, because alterations in the impedances of the arms affect the voltage applied to the coil. The voltmeter should not, however, be left permanently across the coil during the later stages of balancing, since it may introduce spurious e.m.f.s into the circuit. When a sufficiently close balance has been obtained, the inductance is calculated from the expression

$$L = RSC$$

the result being in henrys if R, S, and C are in ohms and farads respectively.

For further details *A.C. Bridge Measurements*, by B. Hague,(13) should be consulted.

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CHAPTER XXII

Intervalve Transformers

As the position of this chapter indicates, the subject of intervalve coupling transformers is one of the more complex problems of transformer designing. This difficulty arises from the fact that an intervalve transformer is nearly always operated with the secondary winding practically unloaded, although in some applications it may be necessary to apply an artificial load in order to obtain a sufficiently flat response. An intervalve transformer normally feeds the grid of a succeeding amplifying valve, and at audio-frequencies the input impedance of the valve is very high and almost completely capacitative. In parallel with the valve input, there may be grid leak resistances, and they would then form the principal load. Generally speaking, however, the transformer can be considered to be working substantially on an open circuit, or into a small capacity only. Open-circuit secondary conditions make the design difficult because capacity effects of the windings are no longer made negligible by the shunting effect of a resistive load. It is thus no longer possible to neglect any of the elements of the general equivalent diagram of a transformer. The self-capacitances and inter-winding capacitances, as well as leakage reactances, have all to be considered and the input frequency and voltage both vary within wide limits. Again, the source impedance of the valve feeding the transformer cannot be neglected, and so harmonic distortion must be watched closely. On the other hand, efficiency is of no account and there is usually no risk of over-heating.

1. GENERAL CONSIDERATIONS

An intervalve coupling transformer has its primary circuit connected in the anode circuit of the valve which supplies it, whilst the secondary is connected to the input terminals of the next valve in the chain. The purpose of the transformer is to supply to the second valve an accurate copy of the voltage wave-forms which were applied to the grid of the first valve magnified on as large a scale as possible. In order that the complex wave-forms should be reproduced accurately it is, of course, important that frequency, phase, and harmonic distortions should all be small over the pass-band of the transformer. These requirements can be summarized as follows—

(*a*) The volts applied to the grid of the valve following the I.V.T. must be proportional to those on the grid of the valve feeding the I.V.T., at all voltages and frequencies within the working range. This is what is usually meant by the brief statement, "a level response."

(*b*) The harmonic distortion must be as low as possible.

(*c*) The step-up ratio must be as high as is consistent with (*a*) and (*b*).

(*d*) In push-pull types a high degree of balance between half-windings is required.

Some of these requirements correspond to those of output transformers, so that considerations of flux-density in the core, primary inductance, and leakage reactance are dealt with in the same way as before. Self-capacities and inter-winding capacities of the primary and secondary are, however, new factors which have not previously been present. Their effects are sufficiently important to have a major influence on the design of intervalve transformers.

Since there is no resistance load on the output, the leakage reactance has little effect on the response at the middle frequencies; but at the higher frequencies it operates in conjunction with the capacity to cause a serious drop in response. It is, in fact, to maintain the performance at the higher frequencies that the design of this type of transformer differs from that of the other types we have so far considered.

The equivalent circuit of an intervalve transformer is shown in Fig. 147. (This should be compared with the corresponding equivalent circuit of an output transformer illustrated in Fig. 130 (p. 382).

Frequency Response

It will be readily understood that the analysis of a complex circuit of this type would be rather involved, and would in fact be unnecessary for most purposes, because the magnitudes of the reactive components (i.e. inductances and capacitances) are such that none of them is significant over the whole range of the audio-frequencies involved. We can as before divide this frequency band into low, middle, and high frequencies. In figures, these are roughly below 100 cycles, 100 to 2 000 cycles; and above 2 000 cycles, respectively.

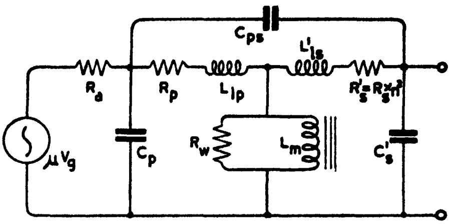


Fig. 147

The Complete Equivalent Circuit for an A.F. Transformer Contains too many Components to Handle with Reasonable Ease.

In the low-frequency section of the band the leakage inductances and the capacitances can be neglected completely, so that the circuit can be simplified to that shown in Fig. 148. The transformer falls short of the ideal because the primary inductance L_m (and to a small extent the iron loss resistance R_w) shunt the output terminals.

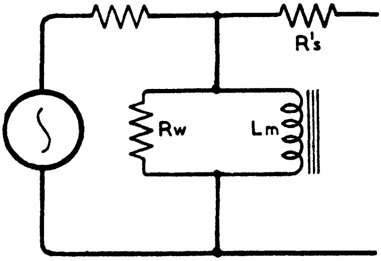


Fig. 148

The Equivalent Circuit at Frequencies Below 100 Cycles.

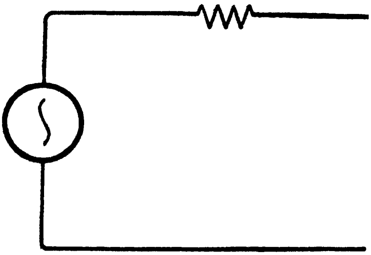


Fig. 149

The Transformer Becomes a Pure Resistance in the Middle Frequency Range, 100–2 000 Cycles.

The step-up ratio is thus less than the turns ratio by an amount which decreases as the frequency rises towards 100 cycles.

At a frequency of 100 the reactance of L_m will have become very high and the value of R_w (which depends on the flux-density in the core, and therefore at a fixed voltage, increases with frequency) has also become high, so that they can be left out of the picture. The transformation ratio is then equal to the turns ratio and the transformer becomes for practical purposes equivalent to an "ideal transformer" (Fig. 149).

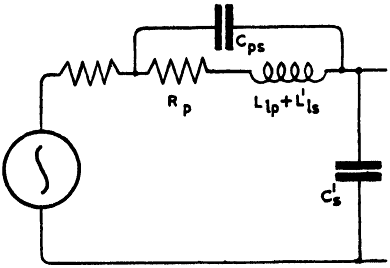


Fig. 150

At Higher Frequencies above 2 000 Cycles Winding Capacitances can no Longer be Neglected.

Above 2 000 cycles or so the effects of leakage inductance and winding capacitances can no longer be neglected. The equivalent circuit then takes the form shown in Fig. 150. If the transformer has an interwinding screen, capacitances between the windings (represented in the diagrams by C_{ps}) can be neglected and the simplified circuit of Fig. 151 remains. Even if a screen is not specially fitted between the two windings, much the same effect is produced by arranging that the finishing end of the inner winding (usually the primary) and the adjacent starting end of the outer winding are the ones connected to the "earthy" points of H.T. positive and grid bias respectively. The circuit of Fig. 151 will at once be recognized as a series tuned circuit connected across the hypothetical generator, and at the frequency at which the circuit is resonant, a step-up in voltage will take place across the output terminals, relative to the generator voltage. This step-up is, of course, over and above the normal step-up ratio of the transformer. The height which this response peak reaches in the neighbourhood of the resonant frequency depends on the circuit magnification (or Q), and this is, of course, dependent on the relative values of C_s', L_{Lp} and R. Properly controlled, this peak helps to maintain the response level up to the cut-off frequency (Fig. 152).

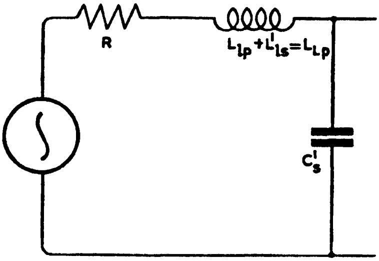


Fig. 151

When an Interwinding Screen is Fitted, the Primary-secondary Capacitance can usually be Neglected. The Transformer then Appears as a Series Tuned Circuit.

In order to damp down the peak, it has sometimes been the practice to wind the secondary of the transformer with resistance wire instead of copper wire, so putting up the value of R and reducing the Q of the tuned circuit. From the equivalent circuit it might have been expected that this result could have been more conveniently achieved by connecting an external resistance in series with a secondary winding of copper wire, but in practice this is not possible because the capacity C_s' shown connected across the output is actually distributed over the whole length of the winding and not concentrated in the way we have so conveniently assumed. It is, however, practicable to connect an external resistance as a load across the output, and this has the desired effect in damping down the peak. There is a disadvantage, though, in that it reduces the gain throughout the frequency range by an appreciable amount. This disadvantage does not apply to a resistance wire secondary winding.

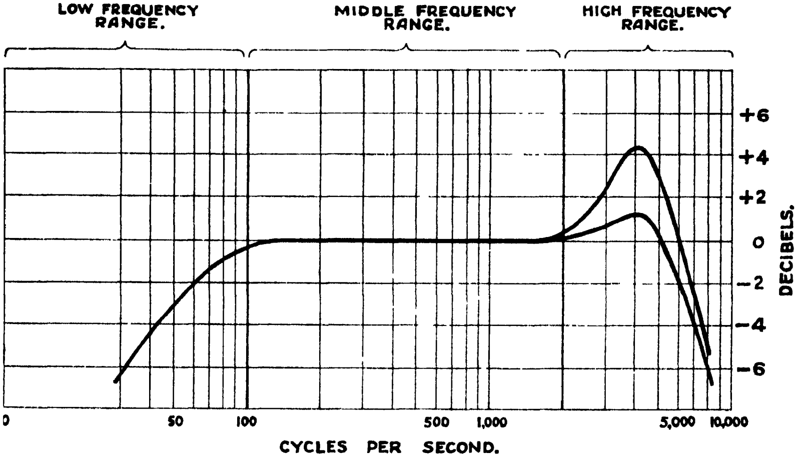


Fig. 152

A Representative Response Curve for an Intervalve Transformer with Two Different Values of Damping Resistance.

The primary must be kept to as low a resistance as possible because of the effect on the low-frequency end of the response. It should not be forgotten that R of Fig. 151 is actually the sum of R_p and R_s', and that in Fig. 147 another arm is connected at their junction.

At still higher frequencies (i.e. above the resonant frequency of the leakage inductance with winding capacity), the load is capacitative and its reactance falls with frequency. The leakage reactance and the capacities thus conspire to cause a sharp drop in the response above the cut-off frequency.

Whilst it is convenient in this way to examine the behaviour of a transformer in three separate bands, it must not be forgotten that eventually our design must produce a transformer which will operate without change of the values derived from consideration of Figs. 148 and 151. Since the upper end of the frequency range is likely to be the more difficult, it is best to start there and later calculate the performance of the bass end.

When calculation indicates that the high note response of an I.V.T. is unlikely to be satisfactory the factor most easily adjusted is the step-up ratio. By reducing the ratio, the effects of secondary capacities are rendered less important, although, of course, the gain of the amplifier is reduced correspondingly.

A high primary inductance is necessary in I.V.T.s in order to avoid a poor response in the bass, but unless the construction is well arranged, this is likely to make the leakage inductance between primary and secondary higher than can be tolerated. Unfortunately we cannot in an I.V.T. follow very far the method of sectionalizing and interleaving of the primary and secondary windings in the way we did with an O.P.T. Having decided upon the primary inductance from bass response considerations, it was possible to subdivide the windings of an O.P.T. to any desired extent in order to keep the leakage inductance down to a value permitting the desired top response to be obtained. Whilst it is true that the leakage inductance of an I.V.T. can be decreased indefinitely in the same way, the increased capacity effects which would thus be introduced would more than offset the reduction in leakage inductance. These capacity increases also take place in an O.P.T., but they are of little significance because the capacity effects are in any case swamped by the load on the secondary.

Construction

The earliest transformers for input and intervalve coupling purposes followed the general lines of an output transformer or a small power transformer. A 4 per cent silicon-steel core having a cross-section of the order of $\frac{1}{2}$ in.^{2} was wound to give a suitable primary inductance when carrying the steady d.c. anode current of the valve to which it was connected, and a secondary of finer wire with a larger number of turns was added, so obtaining a step-up of between 2 and 5 to 1 depending on the bass response demanded. This method of construction is still used in some applications where the passage of the d.c. anode current through the primary is essential. An example of this would be in an amplifier where only a very limited h.t. voltage is available, and here the voltage drop in a feed resistance would be intolerable.

Silicon-steel cores are also sometimes necessary where high-power levels are reached before the final output stage. The cost of expensive nickel-alloy cores in the large sizes necessary for these requirements is often prohibitive. Conditions such as these are found in transformers coupling the penultimate stages of public address amplifiers to high-power output triodes which often need a grid input of some 90 volts.

When a given geometrical arrangement of windings is adopted the ratio of the highest to the lowest frequencies reproduced remains more or less constant. Any increase of turns in the windings, although lowering the bottom limit of reproduction, also reduces the upper limit. On the other hand, complicated "sandwiching" arrangements of primary and secondary result in high winding capacities and do not keep the high note response. In practice it is not usual to go beyond the stage of two secondary divisions and one primary division.

Improvements in response of such a transformer can only be made by the use of higher permeability materials and most of the modern intervalve and amplifier input transformers use cores of nickel alloys such as Mu-metal or Permalloy. These alloys do not take kindly to d.c. magnetization and it is necessary to parallel-feed the primary winding through a resistance-capacity circuit. This system has also a very important advantage that there is no longer a high d.c. potential difference between the two windings, or between winding and core. When a potential difference of this kind occurs, particularly under damp or acid conditions of the atmosphere, as in a battery-operated receiver, the wire which is positive with respect to the rest of the circuit tends to corrode rapidly owing to electrolytic action. The wire is eaten away at any pinholes in the enamel covering and becomes open-circuited, sometimes in a matter of months.

Nickel-alloy Core Types

The most striking change which is effected by the use of a Mumetal or Permalloy core is the great reduction in the size of a transformer for a given performance. The permeability of these alloys is so much greater than that of silicon steel, that the core cross-section can be reduced to something of the order of 0·05 in^2. instead of 0·5 in.^{2} The diameters of the coils can be made correspondingly small and a great reduction in winding capacities results. The high permeability of the core enables the primary inductance to be maintained in spite of the reduced diameter, whilst the leakage inductance, which is practically independent of the core material since it depends on flux having a large air path, is considerably reduced.

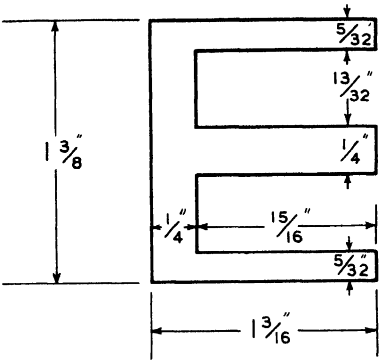


FIG. 153
A Nickel-alloy Lamination Frequently Used for Small Intervalve Transformers.

The small size of the laminations used for these applications allows clamping plates to be dispensed with, and so besides the conventional "E and I," and "T and U" stampings, there are others such as the wide yoke, "E" type (Fig. 153). To build up a core with these stampings, the "E's" are inserted from both sides in turn, without "I's." In order to keep up the full cross-section where there are air spaces making up half the depth of the core, the yokes are kept equal in width to the central limbs. The absence of "I" stampings is a help in rapid assembly.

It should be noted in passing that the magnetic properties of nickel-alloy materials are seriously impaired by mechanical distortion and it is most important to see that the stampings are not bent as they are put into the former. Even the distortion resulting from forcing a last stamping into the bobbin when there is not really room for it often results in the transformer actually having a lower inductance than if it had been left out. Needless to say, the remaining stampings must not be "kinked" to make a tight fit under these conditions; the stampings should be held by adhesive or wax.

The parallel-feed method of connexion is illustrated in Fig. 154. It will be seen that a blocking capacitor is connected in series with the transformer primary to isolate the winding from the h.t. line. The size of this capacitor is generally reduced to a value which resonates with the primary inductance at a frequency towards the bass cut-off point. In this way the bass response can be maintained level to a lower frequency than would be possible without the capacitor, although once the response has begun to fall it does so more rapidly.

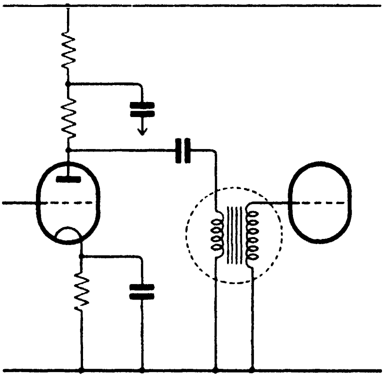


FIG. 154
The "Parallel-feed" Method of Connecting on I.V.T.

The parallel-feed method has another advantage in that it lends itself to the auto-transformer arrangement. As the primary is isolated from the h.t. line by the feed capacitor the low potential end of the secondary can be connected not to an earthy point (as would otherwise be necessary), but to the high potential (or anode) end of the primary. In this way the primary voltage is added to that induced in the secondary and the effective step-up ratio raised. An I.V.T. having a normal ratio of, say, 2·5 to 1, becomes 3·5 to 1 when auto-connected.

2. NICKEL ALLOYS

High-permeability alloys of nickel and iron were first used in submarine telegraph cables as a means of applying continuous "loading" to improve the propagation characteristics. Their value in measuring and audio-frequency transformers was quickly realized and they may be said to have revolutionized the design of these pieces of equipment.

In order to obtain useful results from nickel alloys, extreme purity of the constituents is essential. In addition careful heat treatment is required. It was for these reasons that although several eminent metallurgists examined the properties of nickel alloys during the nineteenth and early twentieth centuries, it was not until 1920 that the really remarkable properties of these materials were discovered.(1, 2, 3) Extremely pure iron and nickel, obtained either by electrolytic methods or by the decomposition of iron and nickel carbonyls, are fused *in vacuo* by high-frequency induction furnaces. The resulting alloy, after rolling, is annealed in an inert atmosphere. These processes are expensive and even now after continued development, the price of nickel alloys is too high to allow their use in transformers of any size.

The properties of the alloys vary in an irregular manner with the percentage of nickel, the useful range of which is between 30 and 80 per cent (Fig. 155). The 50 per cent nickel 50 per cent iron group (e.g. Permalloy B and Radiometal) have very high resistivity, giving low eddy-current losses, and quite a high saturation value, although lower than that of pure iron or silicon steel. The hysteresis loss of this material is the lowest of any of the binary Ni–Fe alloys. A higher initial permeability (μ_0) is obtainable from the 78 per cent nickel group, such as Permalloy, and this was important for the cable-loading application for which it was developed, but the saturation figure is lower, and in the case of the binary alloy the other properties are inferior. The addition of small quantities of molybdenum and manganese as in Permalloy C produces a marked improvement both magnetically and mechanically.

Another modification of the 78 per cent nickel alloy which produces excellent results is the addition of 5 per cent of copper again with small quantities of manganese. This alloy, known as Mumetal, and produced by the Telegraph Construction and Maintenance Co., Ltd.,(4) is widely used in Great Britain for transformers in which there is no d.c. polarization. Under a.c. magnetization conditions the permeability is extremely high, being similar to that of Permalloy C. When D.C. is present in the windings, Radiometal, prepared by the same firm, is more suitable. Although its permeability figures in the absence of D.C. are lower than those of Mumetal, they are less adversely affected by a moderate amount of D.C.

Other alloys which are useful for special purposes are Permendur, having permeability figures well maintained up to high values of *H*, and Perminvar having a nearly constant permeability. These alloys contain cobalt (Permendur, 48% Co, 49% Fe, 2% V; Perminvar, 45% Ni, 25% Co, 30% Fe).

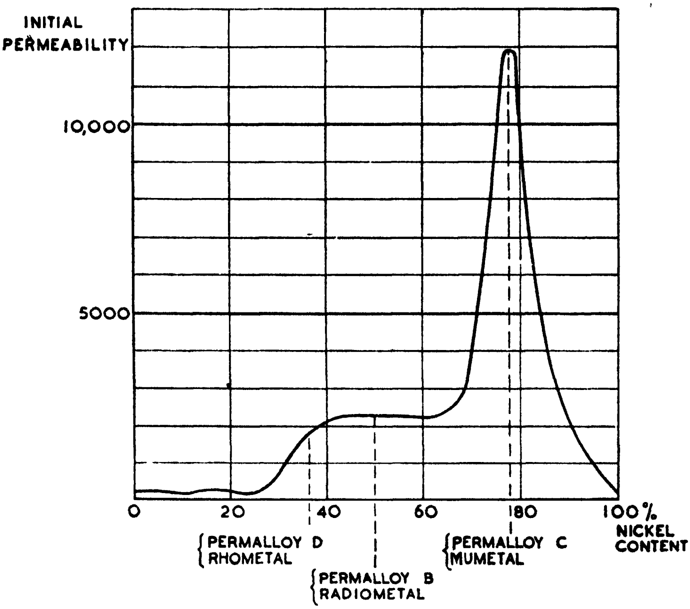


FIG. 155

The Variation of Initial Permeability of an Iron-nickel Alloy with the Percentage of Nickel.

A feature of many high-permeability alloys, which may cause trouble in certain applications, is the fall in initial permeability at the higher audio-frequencies. In a lamination thickness of 0·015 in. for instance, materials of the Mumetal, Permalloy C class drop to about one-quarter of their low-frequency value even at 1 000 cycles. If the thickness is reduced to 0·005 in., the frequency is raised to 5 000 cycles for a similar drop in initial permeability. The effects of this loss of permeability are not usually serious in most transformer applications, because even with the reduced values, the primary inductance is ample to make the reactance (at these frequencies) high compared with the circuit resistances. At the extreme upper end of the audio band, the leakage inductance is the important figure and since this is determined by flux having a large part of its path in air, it is hardly affected by the permeability of the core material. There are, however, applications in which a high permeability is required at the higher frequencies, and two materials, Rhometal and Permalloy D, have been developed for these purposes.

Table XV summarizes the properties of the more generally used alloys.

3. CALCULATION OF PERFORMANCE

Having made a general survey of the operating conditions and performance of intervalve and input transformers, we must now examine the matter quantitatively. The subject is considered here first of all from the standpoint of calculating the response of a transformer having known values of inductance, capacitance, etc., and then from the point of view of a designer wishing to produce a transformer having a specified performance.

The High-note Response

As already mentioned, the high-frequency end of the range is the more difficult to handle satisfactorily and so it is usually best to start there. At these frequencies the capacitance of the secondary circuit determines the impedance, and if the primary impedance is fixed by valve-loading limitations, the maximum step-up ratio is fixed. Since the secondary capacitance plays such an important part in the design, it is convenient to refer the various quantities to the secondary rather than to the primary. This has been done in Fig. 156, which shows the elements which are effective at the upper end of the audio-frequency band. If the e.m.f. of the generator representing the valve anode circuit is E_p, it will have a value E_p' when referred to the secondary. If the output voltage is V_s, it is evident that the ratio V_s/E_p' will indicate the relative response of the transformer at the particular frequency in question.

TABLE XV

PROPERTIES OF NICKEL IRON ALLOYS

MATERIAL	MAIN CONSTITUENTS	INITIAL PERMEABILITY (Closed magnetic circuit)	MAXIMUM PERMEABILITY	SATURATION (Gauss)	RESISTIVITY (Microhms per cm^3)	LOSSES (Watts lb at $B_{max} = 5\,000$ and $f = 50$ c p s for 0 015 in. sheet)
Permalloy B[1]	46% Ni, 54% Fe	1 800–2 400	10 000 20 000	16 000	55	0 11
Radiometal[2]	50% Ni, 50% Fe	2 500	25 000	16 000	55	0 045
Permalloy C[1]	77 4% Ni, 13 3% Fe, 5% Cu, 3 7% Mo	10 000–30 000	50 000–100 000	8 000	60	0 04
Mumetal[2] .	77% Ni, 13% Fe, 5% Cu, 4% Mo	15 000–30 000	100 000	7 600	60	0 08
Permalloy D[1]	36% Ni, 64% Fe	1 500–2 000	6 000–8 000	13 000	90	0 2
Rhometal[2]	36% Ni, 62% Fe, 2% Si	800–1 500	8 500	10 000	90	0 16
Permendur[1]	48 4% Co, 49 3% Fe, 2 3% V	700 1 000	3 000 5 000	24 000	26	0 3
Perminvar	45% Ni, 30% Fe 25% Co	365	1 800	15 500	19	
Silicon Iron	4% Si	300	6 000	18 000	51	0 19

[1] Standard Telephones and Cables, Ltd.

[2] Telegraph Construction & Maintenance Co , Ltd.

Now
$$\frac{V_s}{E_p'} = \frac{-j\dfrac{1}{\omega C_s}}{R' + j\left(\omega L_{Ls} - \dfrac{1}{\omega C_s}\right)}$$

where
$$R' = R_a' + R_p'.$$

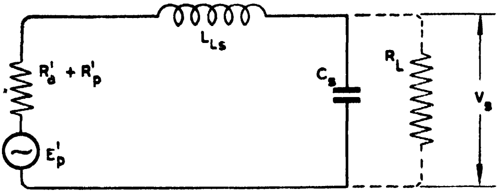


FIG. 156

The Equivalent Circuit of an I.V.T. at the Higher Audio-frequencies. All Quantities are Referred to the Secondary.

Rationalizing, this becomes

$$\frac{\dfrac{1}{\omega C_s}}{\sqrt{R'^2 + \left(\omega L_{Ls} - \dfrac{1}{\omega C_s}\right)^2}} \qquad (112)$$

The frequency at which the reactances of the capacitance and the leakage inductance become equal is known as the cut-off frequency and the corresponding value of ω is denoted by ω_0. The shape of the response depends on the relative magnitudes of the reactance $\frac{1}{\omega_0 C_s}$ and the resistance R'. which represents the valve and windings referred to the secondary. In the particular case in which this reactance is equal to the resistance, the ratio V_s/E_p' is 1 at ω_0 and slightly higher at frequencies just below the cut-off. J. G. Story⁽⁵⁾ gives generalized curves for the response of circuits having a range of ratios of reactance to resistance at ω_0. From these curves, some of which are reproduced to a decibel scale in Fig. 157, it is clear that the reactance/resistance ratio (= K) should be kept within the range 0·85 to 1·25 if a reasonably level response is required up to the cut-off frequency. If this is not possible in a particular case it is necessary to resort to loading the secondary with an external resistance R_L.

The introduction of the fourth element R_L into the circuit complicates considerably the calculation of the response(6) and it becomes impracticable to give curves of all the possible combinations of values. The range of variation actually occurring in practice is not, however, very great, and Story gives six curves in which certain relationships have been fixed between the capacitative reactance at the cut-off frequency (= leakage reactance) and the resistances. One set of curves, for instance, shows the relative response when R_L is fixed at a value equal to that of the reactance at cut-off, while the input resistance R' is varied. Under these conditions R' can vary from half to twice R_L without seriously affecting the response. Other sets of curves are given by Story for conditions in which the reactance value is two, three and four times the resultant of R' and R_L in parallel.

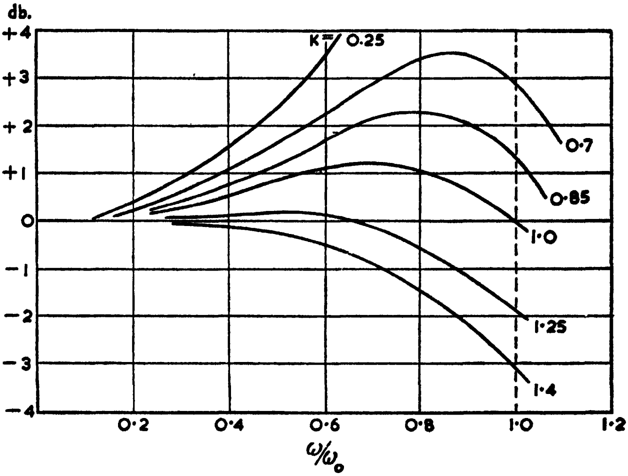


FIG. 157

Story's Design Curves for an Unloaded Transformer Redrawn to a db Scale.

The Bass Response

At the low-frequency end of the range, the response falls, owing to the shunting effect of the primary inductance. In this respect the performance of an intervalve transformer is calculated in the same way as that of an output transformer.

When the inductive reactance of the primary falls to that of the resultant resistance of the source and the load (if any) in parallel, the response is 3 db down (i.e. 0·707) on the response at the middle frequencies. The inductance in this case is, of course, the winding (or open-circuit) inductance L_m, and not the leakage inductance, which is the relevant inductance concerned in the calculation of the high-note response.

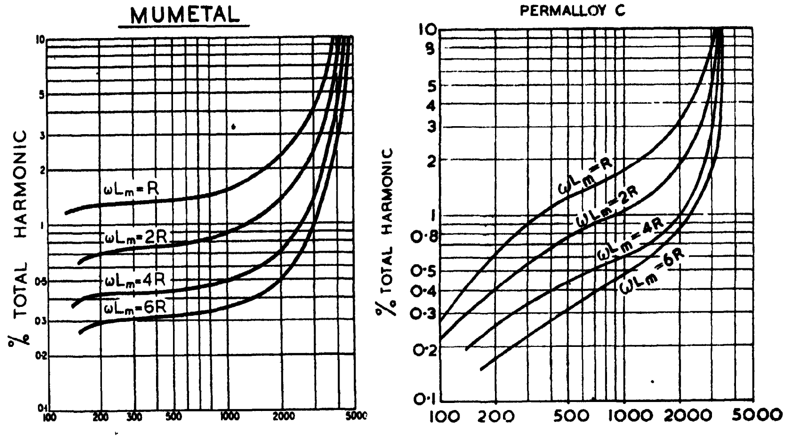
The determination of the frequency at which the bass cut is 3 db requires a knowledge of the inductance and it should be remembered that the effective value of the permeability of the core material to be used in the formula is the "initial permeability," i.e. the permeability for very small values of B. Unless this figure is used, the result will indicate a better bass response than will actually be obtained at low levels of audio signal. As the characteristics of the ear make the low notes relatively less prominent at small output intensities a further cut in the transformer is clearly most undesirable and an adequate inductance is essential.

Harmonic Distortion

As in any other audio-frequency transformer working from a source having an appreciable impedance, the output waveform from an intervalve transformer differs somewhat from that of the e.m.f. in the primary circuit. The extent of the difference depends on the characteristics of the material, the flux-density in the core, and the relative magnitudes of the source impedance and the primary reactance at the lowest frequency considered. Qualitatively, the calculation of harmonic distortion in a coupling transformer is thus seen to be similar to that in an output transformer, but when the magnitudes of the various quantities are examined it is found to be a less serious problem. At the earlier stages in an amplifier, the signal levels are quite low and it is also usually practicable to exclude D.C. from the windings. Both these factors have quite a considerable effect in reducing distortion. It will accordingly be found that if a good high-note response is required, involving as it usually does a fairly high leakage reactance, the primary inductance will be sufficiently high to ensure not only a good bass response, but also a low harmonic content.

Story gives curves of measured harmonic for Mumetal and

Permalloy C at various flux-densities and for primary reactances at 50 cycles of 1, 2, 4, and 6 times the effective source impedance (Fig. 158). This quantity is the resultant resistance of the actual source in parallel with the load resistances if any. These curves show that if the primary reactance can be made at least three times the effective source impedance, and a flux-density of 2 000 lines per cm^2 is not exceeded, then the total harmonic distortion should not be more than 1 to 1½ per cent. If the reactance is equal to the source impedance, the harmonic rises to 2½–3 per cent at $B_{max} = 2\ 000$.



Story

FIG. 158

Harmonic Distortion Curves for Mumetal and Permalloy C.

Calculating the Transformer Constants

In order to calculate the performance, it is essential to be able to estimate the values of the various quantities involved. The important quantities are primary inductance, leakage inductance, and winding capacitance. The calculation of primary inductance has frequently been made in previous applications and no further explanation is necessary. The leakage inductance has also been dealt with in Chapter VIII where an expression was derived for a transformer with concentric primary and secondary windings. In this derivation, it was assumed that the leakage flux is distributed around the whole core and not only in the portion between yokes. Story confirms that results obtained in practice agree well with this formula, although the theoretical basis suggests a more restricted distribution. In addition he gives a corresponding expression for the side-by-side method of winding the primary and secondary. The two formulae are—

(1) For Concentric Windings

$$L_{Ls} = \frac{4\pi T_s^2}{l}\left(m_0 b_0 + 0{\cdot}9\,\frac{m_1 b_1 + m_2 b_2}{3}\right) \times 10^{-9}\ \text{henrys} \quad (113)$$

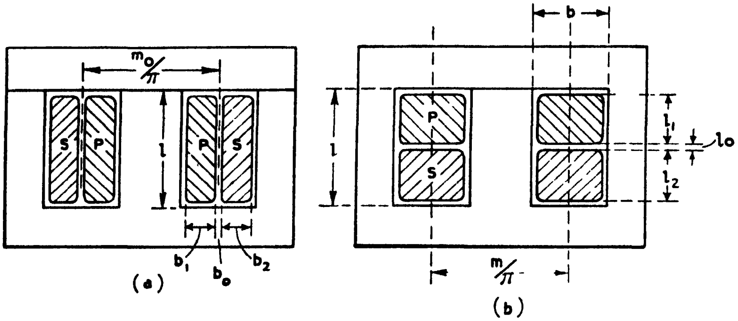


FIG. 159

(*a*) Concentric Coil Arrangement. (*b*) Side-by-side Arrangement.

where m_0, m_1, m_2 are the mean circumferences, and b_0, b_1, b_2 are the radial widths of the interwinding space, primary and secondary respectively; l is the axial length of each winding.

The factor 0·9 is introduced to allow for the fact that the coil space is not entirely composed of copper.

(2) For Side-by-side Windings

$$L_{Ls} = \frac{4\pi T_s^2 m}{b}\left(l_0 + \frac{l_1 + l_2}{3}\right) \times 10^{-9}\ \text{henrys} \quad . \quad (114)$$

where m is the mean circumference of each winding and l_0, l_1, l_2 are the lengths of the interwinding space, primary and secondary respectively; b is the radial width of each winding.

It will usually be found that the leakage inductance of the side-by-side type (Fig. 159*b*) is greater than that of the concentric type (Fig. 159*a*). This has already been noted in connexion with output transformer design. In that case it was desired to keep the leakage inductance as low as possible; here it may be necessary to raise the leakage somewhat in order to maintain the high-note response particularly in high-impedance circuits, so that the side-by-side method of winding may be preferred.

If in (1) above, a further approximation is made that the mean circumference of each winding is the same, i.e. m, and it is noted that $b_1 + b_2$ is approximately equal to b in (2) since b_0 is usually small, and that $l_1 + l_2$ in (2) is practically equal to l in (1), then the side-by-side arrangement will have a higher leakage inductance when $\left(\frac{l}{b}\right)^2$ exceeds 0·9. Since the length of the window in most stampings considerably exceeds the radial width, the side-by-side method of winding gives the higher leakage inductance.

The estimation of secondary capacitance requires a knowledge of the capacitance of the external circuit in addition to that of the transformer. The valve input capacitance and the wiring capacitance together with that of any switches must all be included. The capacitance of the winding is difficult to calculate from fundamental principles, but Story finds that the following expression gives a reasonably good result for the sizes in general use—

$$C_s = 20 + \frac{m_2 l}{11{\cdot}3 b_2}\text{ pf.} \qquad (115)$$

(dimensions being in cm)

The first term represents the capacitance of the terminal arrangements, lead-out wires, etc., whilst the second term is derived on the assumption that the self-capacitance of the coil is that between the innermost and outermost layers of wire. The full secondary voltage is, of course, operative between these layers, whereas the voltage between adjacent layers is low and they consequently contribute little to the total. If the inner and outer layers are considered to be the plates of a tubular capacitor and the separation is small compared with the mean diameter, the capacitance can be calculated to be equal to the second term in Story's formula $\left(\text{viz. } C = \frac{Ak}{4\pi d \times 900\,000}\text{ pf.}\right)$. The specific inductive capacity k is thus assumed to be unity, but Duerdoth(7) in a similar expression, retains k in the formula without, however, using a constant term.

When the constants have been estimated for the transformer under examination, the performance can be calculated for the upper and lower frequency regions in accordance with the methods outlined in Section 2. In these calculations it must not be forgotten that the core losses are represented by a resistance across the output, so that even if no external loading is apparent, it may be necessary to assume the presence of a resistance R_L of a value equivalent to the core losses.

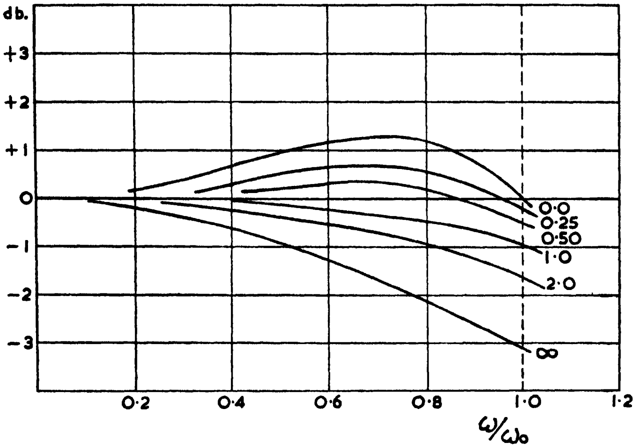
4. COUPLING TRANSFORMER DESIGN

Sufficient information has now been given to enable the performance of a transformer of the normal type to be calculated. The relevant constants are first worked out, and the response at high and low frequencies is then determined relative to that at the middle frequencies. The reverse process of designing a transformer to meet a given specification is not necessarily quite straightforward on account of the large number of variables, and the method recommended depends on the class of performance required. If the response has to be well maintained up to 10 000 cycles or higher, it is best to start with the high-frequency end, afterwards checking bass response and harmonic distortion, although these are likely to be found satisfactory. On the other hand, if only a moderate high-note response is required, it will usually be achieved without special attention to the upper end of the band and it is then preferable to start from the basis of securing a sufficiently high primary inductance to avoid a bass cut and harmonic distortion. The working then follows the lines laid down for output transformers and little further need be said, except as regards the choice of ratio which is made by the method to be outlined for the more ambitious class of transformer having a well-maintained high-note response.

The starting point for this latter class is clearly the estimation of the total capacitance of the whole secondary circuit. This determines the impedance into which the transformer has to work at the upper frequencies. A level response will be obtained only if this impedance is high compared with the effective value of the resistance, i.e. the secondary load resistance, if any, in parallel with the source resistance referred to the secondary. Since the capacitative reactance is largely decided by components outside the transformer and is, therefore, more or less fixed, the resistance arm has to be varied to meet the condition. Now the source impedance (i.e. that of the anode circuit of the previous valve) is also fixed, so that the highest step-up ratio allowable is that which causes the transferred resistance of the source to be so high that it is just beginning to be shunted by the capacitative reactance at the highest desired frequency. This can conveniently be assumed to occur when the transferred resistance is equal to the reactance of the capacity. The ratio is thus fixed.

At the frequency at which the capacitative reactance and resistance reach equality, the response would be 3 db down if there were no leakage inductance. The effect of leakage is to raise the response at frequencies near that at which series resonance occurs with the capacitance and to cause a sharp cut-off afterwards. The frequency of resonance is known as the cut-off frequency, but it must not be assumed that the response will necessarily be falling off at that point, because the resonance, if not damped very much, may raise the curve above the 1 000 cycle level. Generally speaking, however, the cut-off frequency does represent the upper limit of the useful frequency range. The next step is to determine the value of leakage inductance which will resonate with the secondary capacity at the desired cut-off frequency, so securing a rise rather below cut-off and a sharp drop above. The reactance of the leakage inductance is, of course, equal to that of the secondary capacitance at the cut-off frequency and, if that has been made equal to the total equivalent resistive load, we get the condition assumed in one of Story's curves reproduced to a db scale in Fig. 157. This is a good basis from which to start. Ratio of resistance to reactance can be modified later if necessary to obtain slightly differently shaped responses.

The curve obtained for the condition in which resistance and reactance terms are equal at cut-off, rises approximately 1·3 db at the highest point which is at a frequency a little below cut-off, and is level at cut-off. By raising the value of the transferred resistance, i.e. using a greater step-up ratio, a curve is obtained which droops all the way or, alternatively, by reducing the step-up, a greater rise can be obtained. A flatter response can be obtained only by loading the secondary and making a suitable reduction in step-up ratio. A practically level curve is then obtained if R_L is made equal to the capacitative reactance and the transferred input impedance is arranged to be one-third of R_L. The variations are then within the range $\pm \frac{1}{2}$ db up to cut-off frequency (Fig. 160).



(*Story, redrawn to db Scale*).

FIG. 160

Design Curves for Various Degrees of Secondary Loading
Figures refer to values of ratio R^1/R_L

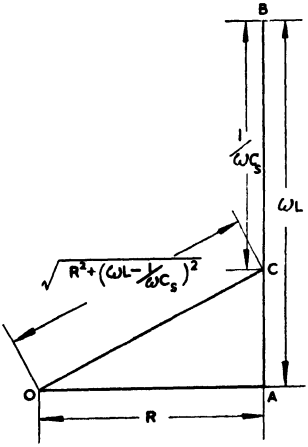
So far the calculations have been concerned with circuit constants rather than with the transformer itself. A theoretical response which is sufficiently close to requirements having been obtained, the next step is to calculate a transformer having a leakage inductance equal to the capacitative reactance of the whole secondary circuit at the selected cut-off frequency. This can be achieved in a variety of ways, but if a high standard of performance is desired a method of winding must be chosen which has the lowest self-capacity. The side-by-side arrangement may be desirable in some cases.

The step-up ratio has already been decided upon, so that the primary turns can be obtained from the secondary turns divided by the turns ratio.

Attention should now be turned to the low-frequency end, and the primary inductance calculated using the initial permeability figure for the core material. If this is Mumetal or Permalloy C, this step is likely to be in the nature of a check only, because the inductance will probably be quite high enough to maintain the response well into the bass. The harmonic distortion is also likely to be satisfactory; it should of course be calculated for the maximum input expected. In the rather unlikely event of either of these conditions being unsatisfactory, it will be necessary to increase the number of turns until the required primary inductance is obtained.

If the design is started off from the low-frequency end, the methods used by E. T. Wrathall(8) offer advantages in ease and speed. Suitable reactance values are selected by graphical methods, starting from the bass end. Wrathall employs the graphical method for the selection of the primary inductance, making allowance for the winding resistance of the primary when drawing the vector diagram. For most purposes, however, it is sufficiently close to calculate the primary reactance so as to be at least equal to the source impedance (and preferably several times greater) to keep harmonic distortion reasonably small. The number of turns required to reach the desired inductance on a likely core are calculated, using μ_0 for the particular material.

Calculations for the low-frequency end having been started from the primary side, it is desirable to continue at the high-frequency end using constants referred to the primary. The equivalent leakage reactance can be calculated in the usual way from the coil dimensions, or an estimate made by assuming that the leakage inductance referred to the primary is a given percentage of the primary inductance. Wrathall takes a figure of 1·8 per cent for the ordinary method of winding and 0·85 per cent if "sandwiched." A vector diagram is then drawn up, starting from the source resistance in the horizontal direction; this is shown as OA in Fig. 161. At A a line AB is drawn at right-angles to OA and of length proportional to the leakage reactance at the highest frequency it is desired to reproduce fully. A point C on AB is now found by trial to satisfy the condition that BC/OC is equal to the permitted ratio of deviation from the 1 000-cycle response. For example, if it is desired to make the response level to 1 db at 5 000 cycles, the ratio BC/OC is made 1·12/1 ($20 \log_{10} 1{\cdot}12 = 1$). The length of BC is proportional to the value of capacitative reactance referred to the primary at that frequency, and this is, of course, drawn vertically downwards, i.e. 180° out of phase with the leakage reactance (also at 5 000 cycles). The length OC is the resultant impedance of the source resistance and the capacitative and leakage reactances in series. As equation 112 shows, the ratio BC/OC is the ratio of output voltage to source e.m.f. for a 1/1 turns ratio transformer at the particular frequency. From the vector diagram the value of the capacitative reactance has been found when referred to the primary, i.e. C_s'. But the value of C_s, the actual secondary capacitance, can be determined from the capacitances of the external secondary circuit and the winding, so that the transformer ratio can be found from $\sqrt{C_s/C'_s}$. The secondary turns are now calculated at once from the step-up ratio multiplied by the number of primary turns. The wire gauges for the two windings can now be decided upon. The secondary gauge is made the smallest that can conveniently be handled, but even so it is likely to take up more than half the available space. The primary size is chosen to fill up the remaining space. The resistances of the windings are calculated, and that of the secondary referred to the primary. The total transformer resistance will probably be small compared with that of the source, but if necessary a correction can be made and the ratio re-calculated, using the source + transformer figure in place of that used previously for the source.



Wrathall

FIG. 161

Graphical Method of I.V.T. Design. A Point is Chosen by Trial so that $BC = 1{\cdot}12 \times OC$

So far, consideration of the cut-off frequency has been omitted from this method. It will be remembered that the cut-off frequency is that at which the leakage reactance is equal to the capacitative reactance. The frequency of this resonance should now be calculated; it will lie below that at which the permitted loss occurs (1 db at 5 000 cycles, in the case of the example used above). There will probably be a rise above the normal level at the cut-off frequency and the extent of this rise must now be checked. The transformer ratio at ω_0 is clearly—

$$\frac{1}{\omega_0 C'_s}/R \text{ since } \omega_0 L_{Lp} = \frac{1}{\omega_0 C'_s} \text{ at } \omega_0.$$

As already noted from Story's curves, the height of this peak depends on the relative values of resistance and reactance. If the rise is found to be too great in a particular case, an attempt can be made to reduce it without losing too much at the highest frequency by increasing the effective resistance. This is the condition covered by the use of one of Story's curves (Fig. 157), for which K is less than unity. There are limits, however, to what can be achieved in this direction and it may be necessary to load the secondary with an external resistance. Nevertheless this is hardly likely to arise if the starting point of the design has been carefully chosen, for if the high-note response was considered of the greatest importance, the previous method would have been adopted, making the high-frequency end the basis of the design.

The possibility of starting work from either the high or the low note end provides an interesting illustration of the technique of transformer design. It is not practicable to produce in a reasonable time a transformer which provides an absolutely optimum performance in all respects, and in any case this may be largely a matter of opinion. The user specifies certain limits to the more important forms of distortion and in an intervalve transformer asks for the highest ratio compatible with these figures. The designer tries to better these limits but within a reasonable time it is not possible to be sure that the transformer is as good as possible in every respect. In consequence some of the less important aspects may be left as "good enough" if within the specification. For that reason it is best to start off from the most difficult of the more important requirements, making the design as good as possible in these respects, and leave the easier ones to be "within limits."

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CHAPTER XXIII

Transformers for Television Receivers

1. SCANNING TRANSFORMERS

THE method of reception of television signals used almost universally at the present time employs a cathode-ray tube. A narrow beam of cathode rays is focused upon a fluorescent screen by an electrostatic or magnetic lens system and the resulting bright spot of light is caused to traverse the screen in a series of lines. The intensity of the spot is at the same time caused to vary in such a way as to build up the picture. The process of sweeping the spot over the screen in this systematic manner is known as "scanning." Two movements are involved in scanning: the "line scan," which causes the spot to move rapidly across the screen, usually in a horizontal direction, so as to appear as a line of varying intensity; and the "frame scan," which causes each line to appear slightly below the previous one until the bottom of the picture is reached.

The deflexion of the spot in the two directions can be carried out by either electrostatic or magnetic means. In the electrostatic system the deflecting voltages are applied to two perpendicular sets of plates arranged around the beam, whilst in the magnetic system a current of suitable wave-form is made to circulate around coils placed against the neck of the tube near the source of the electrons. The magnetic system has now practically ousted the electrostatic one, owing to the lower cost of tube replacement, although the equipment external to the tube is rather more complicated. The deflecting coils are coupled to their respective valves by transformers, so allowing a more robust construction by the use of thicker wire and also removing the d.c. component.

In order to produce an undistorted picture both scanning fields must increase linearly with time over the effective portion of their stroke and must then fall to zero as rapidly as possible before commencing the next stroke. The wave-form of the current required is accordingly a saw-tooth (Fig. 162), and this is generated either by a gas-filled triode valve or by a blocking oscillator and is subsequently amplified by a pentode or beam-tetrode power valve. Although the principle of the line and frame scan circuits is the same, the wide discrepancy in the repetition rates makes the design of the transformers very different.

The systems at present in operation use 400–800 lines to build up the picture (Great Britain, 405) and it is repeated twenty-five times per second (30 in U.S.A.). The number of frames shown per second is actually twice that number because an "interlaced" scan is used. By that arrangement only alternate lines are traced out by the spot during one frame, the intervening lines being covered in the next. The line scan frequency is thus of the order of 10 000 cycles per second, whereas the frame-scan frequency is only 50. The frequencies are in each case those of the fundamental; harmonics up to a fairly high order must also be correctly reproduced if the wave-form is to be preserved sufficiently well. It is usually considered necessary to retain up to the twentieth harmonic to obtain a correct reproduction of an angular wave-form of the type under discussion and in the case of the frame-scan transformer no particular difficulty might have been expected (from the previous discussions on Audio-frequency Transformers), since the frequency of the twentieth harmonic is only 10 000 cycles. Actually, however, imperfections are much more apparent here than in audio-frequency work so that a much higher primary inductance would be required to avoid

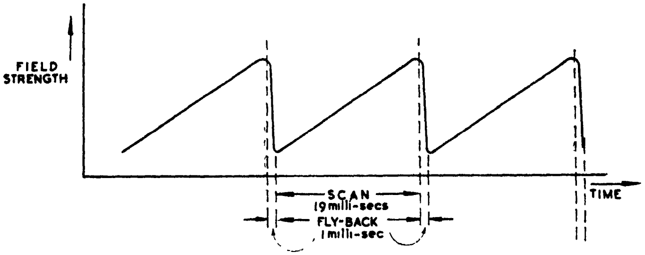

Fig. 162

A Typical Saw-tooth Wave-form. The Intervals are Marked in Accordance with the Frame Time-base Requirements.

distortion. Fortunately it is possible, as will be seen later, to balance some transformer distortion against valve curvature.

The twentieth harmonic of the line-scan frequency, on the other hand, is of the order of 200 000 cycles and the difficulties involved in working at this frequency are much more formidable. It is perhaps desirable at this point to refer to the statement made in connexion with audio-frequency transformers, that it is the *ratio* of the highest to the lowest frequency rather than the actual frequency which determines the difficulty of the design. In other words, the actual range of frequencies to be accepted can be shifted up or down somewhat, provided that the ratio of the extreme frequencies is unchanged. The particular portion of the frequency spectrum required in this case is, however, so markedly different from that used in audio-frequency work that in spite of the similarity in frequency ratio, it is not sufficient merely to change the magnitude of the inductances; alterations are also required in the basis of the design. The core material, for example, must be so chosen that there is not excessive heating due to eddy-current losses, and winding capacitances cannot be neglected. Again, the peak voltage produced across the primary reaches 2 or 3 kV, so that special precautions have to be taken to avoid insulation failures.

We have hitherto discussed distortion in transformers from the point of view of harmonic percentages, a method particularly well suited to audio applications, but one which is less useful in video work. The ear actually hears distortion as the introduction of spurious frequencies into the original sound; the percentage of harmonic is thus a direct index to the amount of distortion. Changes in the relative phases of the fundamental and the harmonics have no effect on the aural effect of *continuous* notes, since the ear operates through the response of a number of resonating systems to the different components of the sound. The wave-form of the resultant varies very markedly with the relative phases of fundamental and harmonic, e.g. third-harmonic in-phase produces a flat-topped wave; 180° out of phase it produces a peaky wave, but the ear cannot tell the difference. When scanning applications are considered, however, the wave-form is of primary importance and any departure from the ideal is at once evident, because it means that the spot is not at the right place on the screen at any particular instant and the picture is distorted. It accordingly makes all the difference if a given frequency component is shifted in phase, so actual wave-form distortion rather than harmonic distortion is the more satisfactory criterion to adopt. Lest any confusion should be caused by this distinction, the reader is reminded that to reproduce a complex wave-form correctly, it is necessary that every component frequency should be retained with the correct relative phase relationships as well as correct magnitudes. The ear is satisfied with the latter requirement only; the eye requires both to be correct.

If this question of the accuracy of reproduction at the transformer secondary of the saw-toothed wave-form applied to the grid of the amplifying valve is pressed too hard, however, there is some danger of an uneconomic design being reproduced. Although an accurately reproduced wave-form would be quite satisfactory, it is unnecessary to seek perfection at all parts of the cycle. Referring again to Fig. 162, the requirement is that the secondary current shall rise linearly with time over the scanning part of the stroke, but no restriction need be placed on the shape of the return stroke, although, of course, it must not be forgotten that it affects the peak voltage generated across the windings. Again, some distortion arises in the valve itself owing to curvature of the grid-volts anode-current characteristic and some compensating distortion in the transformer is not only permissible but necessary. Often, however, the valve curvature is insufficient to compensate fully for the limited inductance of the transformer in the case of the frame-scanning system, and the input to the valve grid has to be distorted deliberately from the saw-tooth form in order to compensate. The balancing-up of the various causes of distortion is quite a complicated business and it is rather outside the scope of transformer engineering as such. Some general principles will, however, be considered in the following sections. As the magnitudes of the quantities involved in the two types are quite different, frame- and line-scan transformers will be examined separately.

Frame-scan Transformers

The field required to scan an average magnetically-operated tube is of the order of 100 ampere-turns. The number of turns used can, of course, be varied over quite wide limits and the coil inductance will vary approximately as the square of the number of turns. For coils of given overall dimensions, the resistance will also vary in the same manner, so that the ratio of resistance to inductance, although dependent somewhat on the method of construction, will be independent of the turns decided upon. In practice the resistance is much larger than the reactance at the fundamental frequency of 50 cycles.

As already indicated, the conception of a fundamental and harmonics is not a very useful one in scanning problems and it is preferable to examine the response of the circuit to the saw-tooth wave. What is desired from the transformer secondary is a current which rises linearly with time during the period of scan, i.e. for 19 milliseconds, and then falls back to the initial value during the next millisecond allowed for the "fly-back." The exact form of the wave during this latter period is relatively unimportant provided that the fly-back is completed by the end of the 1 millisecond allowed for it in the transmission. Although iron yokes are often used to complete the magnetic circuit outside the tube, their effect on linearity is usually quite small owing to the very large air-gap. It is accordingly satisfactory to assume that the field is proportional to the current in the scanning coils, and to consider the current rather than the field in discussions of linearity. To simplify matters, let it be assumed to start off that no transformer is used for matching the coils to the amplifying valve, i.e. the coils are of suitably high resistance. The circuit is then equivalent to a straightforward series connexion of a generator having the desired saw-toothed e.m.f., the valve anode resistance, the coil resistance and inductance. The voltage distribution throughout the circuit is then given at any instant by the equation—

$$e = -L_L \frac{di}{dt} + i(R_a + R_L)$$

Now if the desired output current wave is obtained during the scanning period, that is to say the current rises linearly with time, $\frac{di}{dt}$ is constant whilst $i(R_a + R_L)$ rises linearly. A constant voltage drop accordingly occurs across the inductive component and rising ones across the resistive components. To discuss the relative magnitudes of these voltages it is necessary to examine the values of inductance and resistance obtained in typical scanning coils in general use. Representative figures for the low-resistance type intended for operation from a transformer are 1 millihenry inductance and 2 ohms resistance. Both inductance and resistance can be changed as convenient by altering the transformer ratio, but the proportions will remain the same. For use without a transformer, as we have postulated for simplicity, the values would probably be about 0·3 henry and 600 ohms respectively. The valve anode current rises from about 10 mA at the commencement of the scanning period to about 40 mA at the end (i.e. 19 milliseconds later) so that $\frac{di}{dt}$ is $(30 \times 10^{-3})/(19 \times 10^{-3}) = 1{\cdot}58$ amp per second. The inductive voltage is thus 0·48 whilst the resistive drop in the coil rises from zero to 12 volts. (It is assumed that the d.c. component of the anode current is diverted from the coil, so that only the fluctuations in current have to be considered.) It will thus be seen that the inductive back e.m.f., across the coils during scanning is negligible compared with their resistive drop. This will simplify the theory of the transformer considerably, since it can safely be assumed that the load is simply a resistive one. Actually, in the simplified circuit at present under consideration, the extremely high anode impedance of the pentode ensures that the current through the coils is, in any case, independent of their inductance. Since the anode current of the valve has been assumed to be proportional to the voltage applied to the grid, the linearity of scan is ensured.

Conditions during the fly-back are somewhat different, for at the moment the thyratron fires, the potential of the grid drops back practically instantaneously to that which it had at the commencement of scan. The anode current of the valve cannot drop so rapidly, however, owing to the inductance of the coils which, although practically negligible in effect during the relatively slow scan, cannot now be neglected. The time allowed during the transmission for the fly-back is 1 millisecond, so the anode current should return to zero during this period. Since the decay of current in an inductive circuit follows an exponential law, it theoretically takes an infinite time to reach zero, but this difficulty can be avoided by basing calculations on the assumption that it falls to some small fraction, say one per cent, of its maximum value. If i_1 is the current at the end of scan, then the current i at an instant t seconds later is given by—

$$i = i_1 e^{-\frac{R}{L_L}t}$$

where $$R = R_a + R_L.$$

If $i/i_1 = 0{\cdot}01$ and $t = 10^{-3}$ sec, then $\frac{R}{L_L}t = -\log_e 10^{-2} = 4{\cdot}6$. Since L_L has been taken to be 0·3 henry, R must be 1 380 ohms. This is the *minimum* resistance of the circuit allowable if the current is to fall to a value not greater than 1 per cent by the commencement of the next scanning stroke. Clearly in the simplified circuit at present being examined this condition will be amply fulfilled owing to the high anode impedance of the pentode valve, and in practice when a transformer is used no difficulty is experienced in meeting this requirement.

A point of some importance in connexion with the fly-back part of the cycle is the voltage developed across the coil. If it is assumed that the fly-back is linear and is completed during the period of 1 millisecond, the voltage will be $0{\cdot}3 \times 0{\cdot}03 \times 10^3 = 9$ volts. If the fly-back is completed more quickly or it is not linear as is normally the case, the maximum voltage will be higher. On the other hand, eddy currents in the cores and clamps reduce the value somewhat, so that in practice the voltages reached are not so high as to be difficult to handle by the normal technique adopted for small iron-cored components.

Turning now to the use of a transformer to couple low-resistance coils to the scanning amplifier, consider the conditions in Fig. 163 which shows at (*a*) the actual circuit, and at (*b*) the equivalent circuit for secondary values referred to the primary. Since we have already noted that the inductive back e.m.f. induced in the scanning coils is negligible compared with the resistive drop, L_L can be neglected during the scanning period. It will be seen that the only way in which the introduction of a transformer can influence the linearity of scan is by the effect of its finite primary inductance. This is in parallel with the output and some of the anode current passes that way

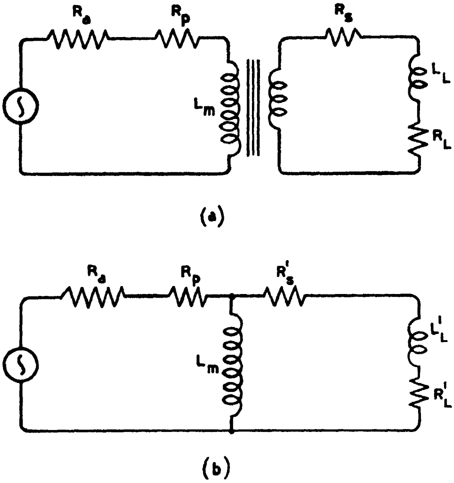

FIG. 163

(a) The Actual Circuit of a Frame-scan Transformer Showing the Components which are of any Significance. (b) Equivalent Circuit Referred to the Primary.

instead of flowing through the load. It is now necessary to examine quantitatively the magnitude of the primary inductance necessary to limit the distortion from that cause to a reasonable value. It is probably unnecessary to be as stringent here as in the discussion on the completion of fly-back because a small amount of compression spread over an appreciable width of picture is much less evident than an actual folding back due to failure to complete the return stroke in time. In order that there shall be no distortion, the current in the scanning coils (referred to the primary) must rise linearly to a value i_1 during the scanning stroke of t_1 secs, and at a time t, it should have a value $i'_L = i_1 \frac{t}{t_1}$. The output voltage v_s' would be $R'_L i'_L = \frac{i_1}{t_1} R'_L t$ and the current i_m through L_m would be

$$\int \frac{v'_s}{L_m}\, dt = \frac{i_1}{t_1} \frac{R'_L}{2L_m} t^2$$

the constant of integration being zero. If the valve delivers a linear current, the coil current will be lower than it should be by the amount passing through L_m; let us assume that at the end of the scanning period the current is 2½ per cent below what it would have been with a linear scan. This condition is satisfied if $\mathrm{i}_m = 0{\cdot}025\, i_L'$ at $t = t_1$, i.e. if

$$\frac{R'L}{2L_m} t_1 = 0{\cdot}025 \text{ or } L_m = \frac{R'_L t_1}{2 \times 0{\cdot}025}$$

Taking our previous figure for scanning coil resistance (now referred to the primary) $L_m = \dfrac{600 \times 19 \times 10^{-3}}{2 \times 0{\cdot}025} = 228$ henrys. Clearly this is an impracticably large value to obtain in any reasonable size of transformer when it is borne in mind that a direct-current component of some 25 mA is passing through the primary. We are therefore forced to accept a much larger degree of distortion in the transformer on account of its finite primary inductance and to "pre-distort" the input voltage applied to the grid of the amplifying valve. This is usually effected by means of a capacitance-resistance combination, the values of which are chosen or preset to suit the inductance obtained in the transformer. Another factor which must be taken into account is the non-linearity of the valve characteristic. Curvature at the lower end of the grid volts–anode current curve is concave upwards, so it is in the right direction to compensate for a low value of inductance, but in some valves an opposite curvature appears at the upper end and this accentuates the effect of the inductance. Negative feedback can be applied to the amplifying stage in order to reduce valve distortion and if the degree of feedback can be varied, the control also serves as a means of adjusting the picture height to the correct figure. In practice the various factors must be arranged to balance out the non-linearity so that the overall result is satisfactory. The final setting is made by trial to cover variations between individual samples of the components involved.

A typical frame-scan transformer for operation from a pentode of the type used to deliver some 5 watts of audio-output in a sound receiver, would be wound on a square-section core of ¾-in. no-waste laminations. The primary inductance of 25 henrys would be obtained with a winding of 3 000 turns, whilst the secondary would have approximately 100 turns to match the 2-ohm coils described earlier. As the induced voltages are low, no special insulation is required between the turns of the primary, but the interwinding insulation must be able to withstand the h.t. supply voltage of some 250.

Line-scan System

The current wave-form required for the line-scanning process is similar to that used for the frame-scanning, but the frequency is much higher. Once again perfection of wave-form is not essential provided that certain requirements are met. It has sometimes been suggested that in order to retain a sufficiently good approximation to the saw-tooth wave-form, harmonics up to the tenth or even the twentieth must be reproduced but, as Cocking shows,(1) the *scanning* period can be satisfactorily covered with a much lower degree of perfection. Linearity is sacrificed during the fly-back interval, but that does not matter.

The arrangement of the line-scan system differs basically from the frame-scan in that capacity effects cannot be neglected at the much higher frequencies involved. Again, the reactance of the scanning coils and the leakage reactance of the transformer cannot be neglected in comparison with the resistance. In consequence, all three elements, resistance, inductance, and capacitance are present and it is quite possible for the circuit to perform damped oscillations when excited by the saw-toothed input. Actually, as Cocking indicates, a degree of damping slightly below critical gives the best results. There is then a straightening up of the early part of the scanning wave-form as compared with the rounded shape obtained with critical damping.

Although frame-scan coils can be wound with a sufficiently large number of turns to be suitable for direct connexion in the valve anode circuit, a similar arrangement is hardly practicable in the case of the line-scan coils because a very high voltage is built up during the fly-back period and insulation difficulties are likely to be serious. Even the transformer needs to be thoroughly well insulated to withstand the 2 kV or more which may be set up across it.

If the secondary and inter-winding capacitances can be neglected, as is usually the case, the equivalent circuit of transformer and coils referred to the primary is that shown in Fig. 164. The secondary inductance and resistance are shown as well as those of the scanning coil and secondary winding, referred in each case to the primary. Even from this diagram it is clear that the problem is one of circuit design rather than simply that of designing a transformer alone, and even so no account has been taken of valve curvature, which still further complicates matters. It is not surprising, therefore, that values are usually finally selected by trial after some preliminary calculations have given some idea of the order of magnitude of the constants involved.

Since the important feature of this system of operation is the heavily-damped oscillation which is allowed to take place during the fly-back time, the design starts with the choice of the circuit inductance to bring the frequency of this oscillation to a suitable value. During the fly-back period of 14·8 microseconds, rather more than a half-cycle of the oscillation occurs. From the known (or estimated) capacitance of the primary circuit, the required circuit inductance can be calculated so that the desired frequency of some 50 kc/s is obtained.

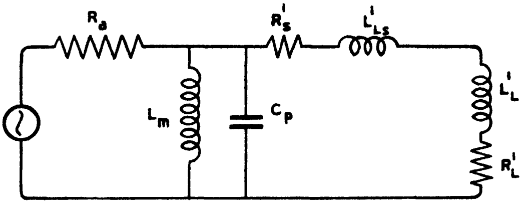

FIG. 164

The Equivalent Circuit of a Line-scan Transformer Referred to the Primary.

The method of construction generally adopted makes use of moderately small laminations in nickel-iron material. Silicon steel can be used, but although the losses are probably not excessive from the circuit point of view, since damping has to be applied in any case, some difficulty may be experienced owing to heating of the core.

As already noted, the voltage developed across the primary during the fly-back amounts to several thousand volts, so that the method of winding must be carefully chosen; the construction must in fact be carried out along the lines suggested in Chapter XIV, dealing with high-voltage types, although of course as this is a step-down transformer it is the primary which requires special treatment. Since the end of the primary that is connected to the h.t. supply remains at a constant potential of some 300 volts to earth, no particular precautions have to be taken in insulating that end, and so the "one end earthy" conditions apply. One of the best arrangements for the winding is the tapered sectionalized construction shown in Fig. 89 (Chapter XIV). This has the advantage over layer-winding of reducing the self-capacitance of the winding with consequent economy in scanning power. A high capacitance necessitates a lower value of circuit inductance, a lower damping resistance, and hence a higher primary current with a correspondingly high mean anode current through the valve. The rather complicated shape required for the coil former suggests a moulded construction. If phenol-formaldehyde is used, care must be taken to keep creepage distances as large as possible owing to the tendency to "track" over the surface between high-potential points and frame. The secondary is best arranged as a layer winding next to the core. If it is desired to reduce the leakage reactance, it can be divided into two sections, one inside and the other outside the primary. Dividing the secondary by putting it against the stampings at both extremes of the window space in this way does not entail any great loss of winding area, since it is at low potential and does not need much insulation.

A rather unusual feature of a line-scan transformer is the need for acoustical insulation. Transformers of any type emit a certain amount of sound, but at normal power frequencies the effect is not generally large nor is the nature of the noise particularly annoying. At 10 000 cycles, however, quite small surfaces become effective radiators of sound and the frequency is also a very disturbing one. Transformer noise may arise from various causes: loose turns vibrating in the field; laminations chattering together; or more fundamentally, magneto-striction effects in the core. A brief reference has already been made in Chapter II to the fact that a ferromagnetic material changes its shape slightly when magnetized, growing either longer or shorter along the axis of magnetization. It accordingly follows that a transformer core subjected to a continuously changing field will expand and contract very slightly every cycle, thus radiating sound from its surface. Loose parts, either windings or laminations, can be effectively silenced by proper care in winding or by impregnation, but this has no effect on the magneto-striction. The only way to deal with that is to encase the whole transformer in a jacket of sound-absorbing material such as sponge rubber.

Details of the construction and sound insulation of a typical transformer are shown in Fig. 165. A-$\frac{13}{16}$ in. stack of Permalloy C laminations $2\frac{7}{32}$ in. × $1\frac{27}{32}$ in. outside dimensions (ref. γ in Table XXIV) forms the core. The secondary is wound next to the central limb whilst the primary is divided into five sections decreasing in radial width towards the high-potential end. It consists of 1 400 turns of SWG 36, giving an inductance of 1·1 henrys. The step-down ratio is approximately 14 to 1 for the particular $1\frac{1}{2}$-ohm scanning coils for which it is intended. A sponge-rubber casing split into two halves for ease of fitting, surrounds the whole transformer, whilst a cylindrical aluminium screening can house the whole assembly.

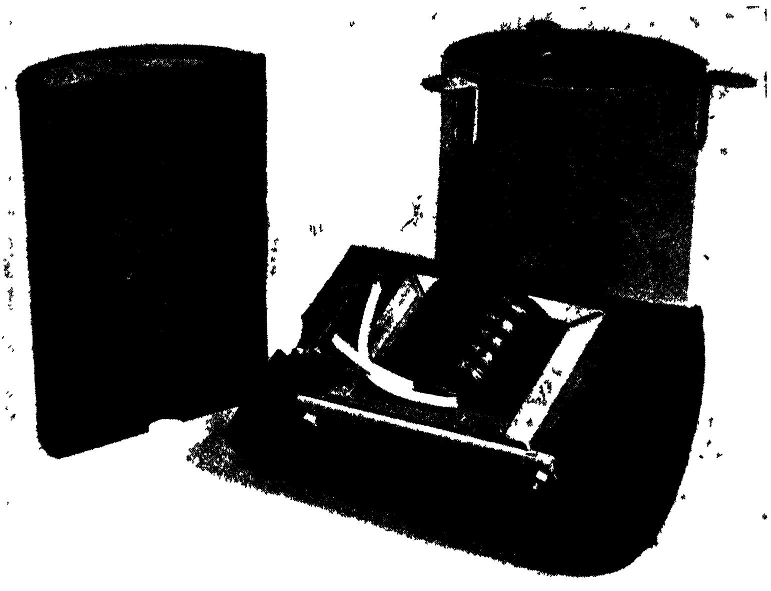

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FIG. 165

A Typical Line-scan Transformer with Primary Wound in Five Sections to Reduce Self-capacitance and Improve Insulation. The Sound-absorbing Material is Shown Ready for Fitting around the Transformer.

The desire to reduce the cost of television receivers has recently directed the attention of designers towards single-valve scanning circuits in which the output pentode is also an oscillator. In this arrangement the need for a separate thyratron or blocking oscillator valve is avoided. The pentode or tetrode used may be arranged to oscillate between control grid and anode or between screen and anode; wave-form correcting voltages may also be applied to the grids and altogether a large number of variations are possible. Naturally, the design of the transformers for these circuits is dependent on the requirements of the particular arrangement to an even greater extent than in the two-valve circuit already described and no rigid scheme can be formulated. Generally speaking, however, the anode circuit coil always needs to be of low self-capacitance and good insulation, characteristics which can best be secured by winding in four or five sections. These may be either random-wound into slots in a moulded former or wave-wound side by side on a plain resin-impregnated tube.

A development which is likely to become of major importance in many fields, including that of line-scan transformer design, is the introduction of a ceramic core material composed of "ferrites" of certain bi-valent metals.(2,3) The general formula of a ferrite is MFe_20_4, where M represents one of the bi-valent metals in question. The ferrites of copper, magnesium, manganese and nickel show marked ferromagnetic properties, whilst those of zinc and cadmium are non-magnetic. Mixtures of the magnetic and non-magnetic ferrites yield much higher permeabilities than those of the single ferrites. The action of the zinc compound is thought to be that of reducing the Curie point to a figure a little above room temperature. Near the Curie point the strain in the crystal lattice in the "unmagnetized" state due to the spontaneous magnetization of the domains is a minimum. The proportions of the different *magnetic* ferrites are adjusted to give zero magneto-striction, another cause of lattice distortions.

The ferrite mixtures yield materials which have a high permeability of some thousands and very low losses up to radio frequencies. The frequency limit depends on the composition of the mixture; "Ferroxcube," the name given by Philips Electrical Ltd. to their product, can be made in many varieties. The most generally useful, Ferroxcube III, is suitable for frequencies up to at least ½ Mc/s and has a permeability of approximately 1000. It will readily be understood that a material with these characteristics should prove invaluable in most iron-cored transformers and chokes intended for operation at the higher audio-frequencies where eddy-current losses in normal laminations are prohibitively high. Hitherto the designer has been compelled to use iron-dust cores for these applications, but owing to the very low effective permeability of this material, the coils have had the characteristics of air-cored coils rather than iron-cored ones. The new ferrite materials promise to change this state of affairs and allow good regulation, high K transformers at frequencies of 10 or 20 Kc/s or higher. Flux density limitations ($B_{max} = 2500$ lines per cm^2) at present render Ferroxcube unsuitable for use in power transformers for normal supply frequencies.

2. POWER SUPPLIES

The requirements of a television receiver in regard to valve-heater and anode-current supplies are similar to those of a radio receiver, except perhaps in magnitude. Since a television set normally uses some fifteen or more valves, the heater and anode currents are correspondingly greater than when only four or five valves are used. The principles of design are, however, the same as those employed in Chapter VII, which should be consulted for details.

In addition to these supplies, a television set requires a very small current at a relatively high voltage ranging from about four thousand volts (rectified D.C.) for a directly-viewed cathode-ray tube, to some thirty thousand for a high-intensity tube intended for viewing by optical projection. Until recently it has been the practice to derive these voltages from the 50-cycle supply by means of a step-up winding, high-voltage rectifier and capacitance-resistance smoothing circuits. Now, however, there is a growing tendency to utilize the line-scan system for the supply of e.h.t. It will be remembered that even in the normal use of such circuits, some 2 000 volts are developed across the primary winding of the line-scan transformer during the fly-back period and this can be stepped up to 7 000 volts or so by means of additional turns added after the anode connection.

Fly-back e.h.t. circuits can be based on either driven amplifier or self-oscillating line-scan circuits; in both cases the step-up windings have to be of the lowest possible self-capacitance and to be well insulated. The transformer is usually kept small to minimize the capacitance and leakage inductance and oil-immersion is advisable even if it involves some increase in self-capacitance of the windings. The economy of fly-back e.h.t. supplies is not due primarily to savings in the transformer but to the fact that very much smaller capacitors can be used in the smoothing circuit since the ripple frequency is some hundreds of times as high as in a 50-cycle arrangement.

A disadvantage of the fly-back method is its relatively poor regulation and this causes the picture size and focus to vary as the beam current of the cathode-ray tube changes with the white content of the picture. Although this effect may not be regarded as objectionable in lower priced receivers, it is sufficiently noticeable to make some designers retain the 50-cycle system in more ambitious equipment. In that case the high a.c. voltage can either be derived from a separate step-up transformer or from a winding on the main power transformer. When the voltage is twenty or thirty thousand, as it is for projection tubes, the requirement comes clearly into the scope of Chapter XIV, which deals with high-voltage types. One end of the winding may be earthy or, if arranged for use in a voltage doubler circuit, the whole winding may have to be insulated to withstand the full d.c. output voltage to case. Transformers delivering output voltages of this order are best arranged to be separate from the heater and anode-current supply windings, because it is often desirable to be able to adjust the secondary voltage by variation of the voltage applied to the primary. This would, of course, be inadmissible if other windings were placed on the same core. The design of a transformer of this type follows the lines suggested in Chapter XIV. Oil-immersion is very desirable to minimize the tendency to breakdown following corona discharges, and a hermetically sealed construction can be used with advantage.

At the other end of the scale, supplies of four to ten thousand volts can be obtained most economically from an additional winding on the same core as the low-voltage windings. A constant accelerating voltage is in these cases applied to the cathode-ray tube, control of brilliance being obtained by the use of auxiliary electrodes. The *power* output of a high-voltage winding of this kind is extremely small, so that its effect on the primary current of transformers supplying other secondaries can be neglected. The core size of such a transformer is only increased over one without an e.h.t. winding, by the space required to house and insulate the secondary itself. The wire size used for this winding is made as small as mechanical considerations allow, since the current is only some thirty micro-amperes. Another advantage in combining the e.h.t. winding with the low-voltage ones is that the number of turns per volt is likely to be considerably lower, owing to the larger core section used. If the e.h.t. transformer were a separate one, it would not be practicable to use such a large section of iron.

Yet another advantage of using a common core for high- and low-voltage coils is the effect of the heater windings in limiting voltage surges at the moment of switching off the primary. If a snap-action switch is used the rate of change of current through the primary is quite high when the circuit happens to be broken at certain parts of the cycles (see Transients, Chapter XI), and in the absence of other windings an e.m.f. several times the normal peak value may appear in the secondary. When other windings carrying loads are also coupled to the primary, however, currents circulate in them in such a direction as to tend to maintain the flux and its rate of change is accordingly much lower, and so also is the magnitude of the voltage surge.

If a stamping of normal proportions is used the e.h.t. secondary is best wound in the form of a short slab occupying most of the radial width of the window. The inner end is made earthy so that no special precautions are required to insulate it from the core. Being short axially, the number of turns applied per layer is not large even when small-diameter wires such as SWG 42 are used. The voltage between layers is then within the capabilities of interleaving paper of reasonable thickness and there is no need for a sectionalized construction. The potential difference to frame is built up steadily as the diameter of the coil increases, so that the width per layer can be reduced progressively as winding proceeds, so following the tapered winding technique. Alternatively, breakdown to the stampings and other windings can be prevented by thick discs of insulating material assembled on each side of the high-voltage winding.

Another winding which requires careful insulation is that supplying the heater of the e.h.t. rectifying valve. This winding is of course at the potential of the rectified d.c. output, so that although the voltage generated in the winding is only a few volts it all has to be carefully isolated. This result can be achieved quite satisfactorily by winding the few turns required into a thick-walled bobbin capable of withstanding the full d.c. voltage. An alternative method which takes up slightly more space but is much easier, is to use heavily insulated connecting wire for the winding (Fig. 166). No special bobbin is then

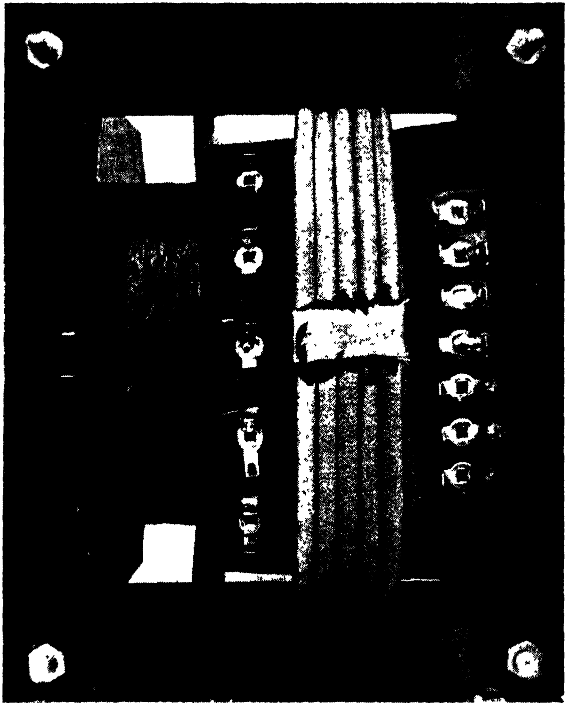

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FIG. 166

A Television Power Transformer with E.H.T. Secondary in a Short Section on the Left and Rectifier-heater Winding of Polythene-covered Wire on the Right.

required, for the wire provides its own insulation. Rubber covering is *quite unsuitable* for this purpose because it perishes very quickly in the presence of corona discharges, but thick P.V.C. or polythene coverings do well.

Astatic Windings

Since a cathode-ray tube is easily affected by stray magnetic fields from mains transformers, etc., and can only be screened at some appreciable expense, it is often useful to arrange the transformer windings so as to reduce the stray field as far as possible. This is best done by adopting the double-winding core-type construction rather than the more usual shell-type.(4) The primary and principal secondary windings are then divided into two sections, one on each limb and connected in series. At a distance which is large compared with the spacing of the limbs, the magnetic effects of the two sections cancel out.

It is necessary to split the secondaries as well as the primaries to keep the leakage reactance reasonably small, but in the case of the e.h.t. winding this is not important and it is satisfactory to place this on one limb only.

Although large core-type transformers are built on cores made up from rectangular strips of steel, smaller ones can be more easily assembled using L-shaped laminations. These are, of course, arranged in opposite ways in adjacent layers so as to minimize the effects of air-gaps at the joints.

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(1) W. T. COCKING: "Electromagnetic Deflection." *Wireless World* 52 (July, 1946), pp. 217–222.

(2) J. L. SNOEK: "Non-metallic Magnetic Material for High Frequencies." *Philips Tech. Rev.* 8 (Dec., 1946), pp. 353–360.

(3) E. J. W. VERWEY, P. W. HAAYMAN, E. L. HEILMAN: "On the Crystalline Structure of Ferrites and Analogous Metal Oxides." *Philips Tech. Rev.* 9 (No. 6. 1947), pp. 185–190.

(4) E. B. HARRISON: "High Quality Communication and Power Transformers." *Jour. Soc. Mot. Pict. Engineers* 43 (Sept., 1944), pp. 155–167.

CHAPTER XXIV

Design Data

NOTE: *The punching processes involved in the production of laminations necessarily cause some deterioration of the magnetic properties of the material near the edges. To obtain figures equal to those found on large sheets, strips must be at least two inches wide and be cleanly sheared off. In practically all small laminations the metal near the punched edges forms a large proportion of the whole and the effect on the performance is quite appreciable. Whenever they are available, curves taken on stampings should be used in preference to those given for sheet. Curves taken on stampings are indicated thus *. These remarks apply only to laminations not re-annealed after punching; re-annealed stampings give figures as good as, or better than, full sheets.*

THE following pages summarize in the form of tables and graphs, all the numerical data which are required to design the transformers described previously. That is to say, once the reader has become familiar with the principles and the general procedure all he requires for reference are the figures for the shapes, sizes, and other properties of the materials he is going to use. These are presented in this chapter, grouped as far as practicable according to the type of transformer being considered.

1. POWER TRANSFORMERS

Core Materials

$$T_p = \frac{3\,490 V_p}{B_{max}\, af}$$

where a = section area in in.^{2},

B_{max} = peak flux-density in kilo-lines per cm.^{2}

At 50 cycles, B_{max} can be 11 to 13, the larger value being possible for transformers over 1 kVA. Silicon content need not exceed 2½ per cent and thickness of 0·020 in. is suitable for most applications, since the limitation is usually magnetizing current rather than loss, particularly in small sizes.

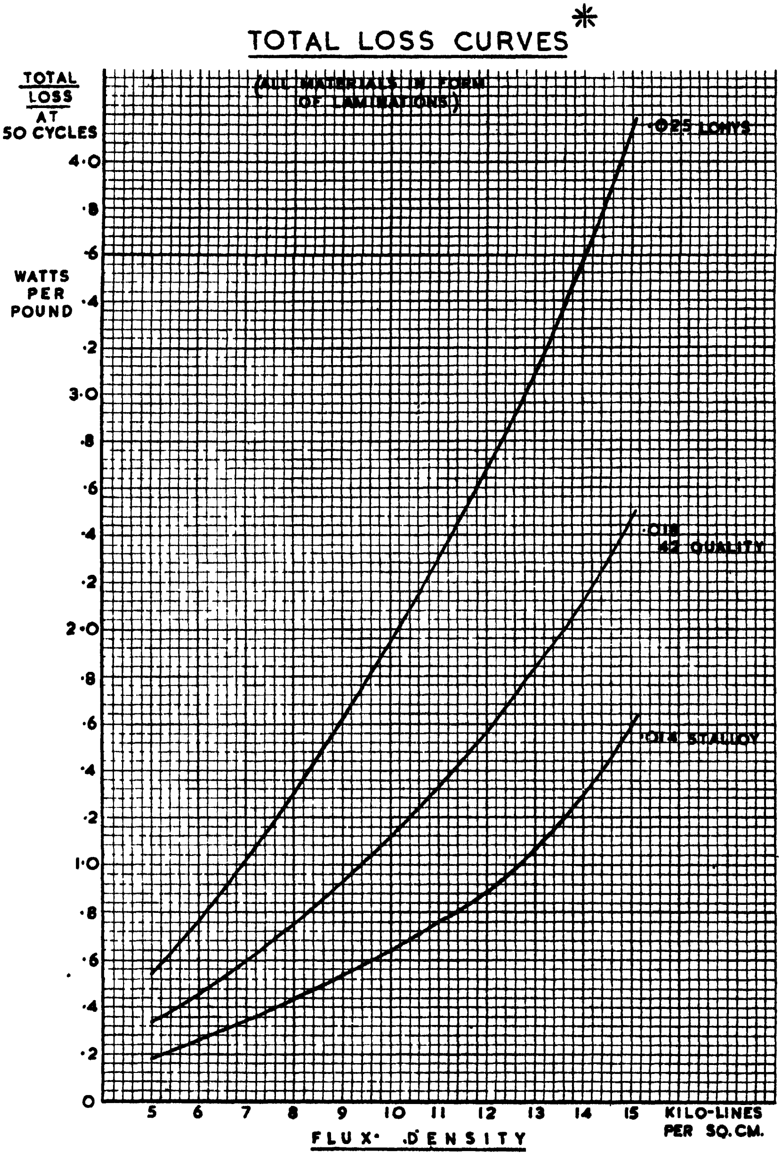

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FIG. 167

Total Loss at 50 Cycles in Different Grades of Silicon Steel in the Form of Small Laminations.

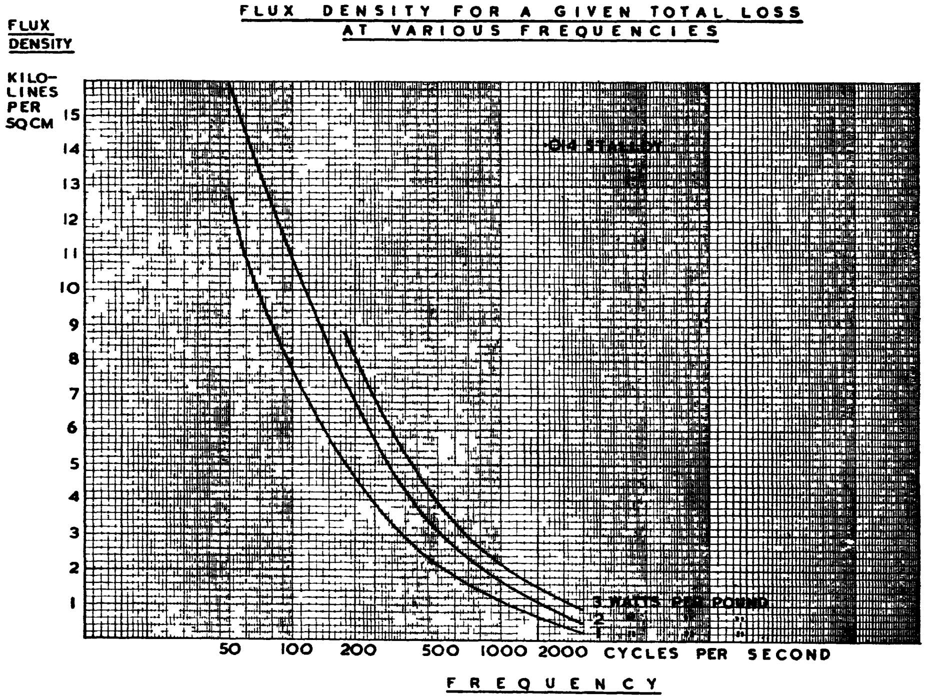

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FIG 168

The Flux-density at which 0 014-in. Stalloy Gives a Total Loss of 1, 2 or 3 Watts per lb at Different Frequencies

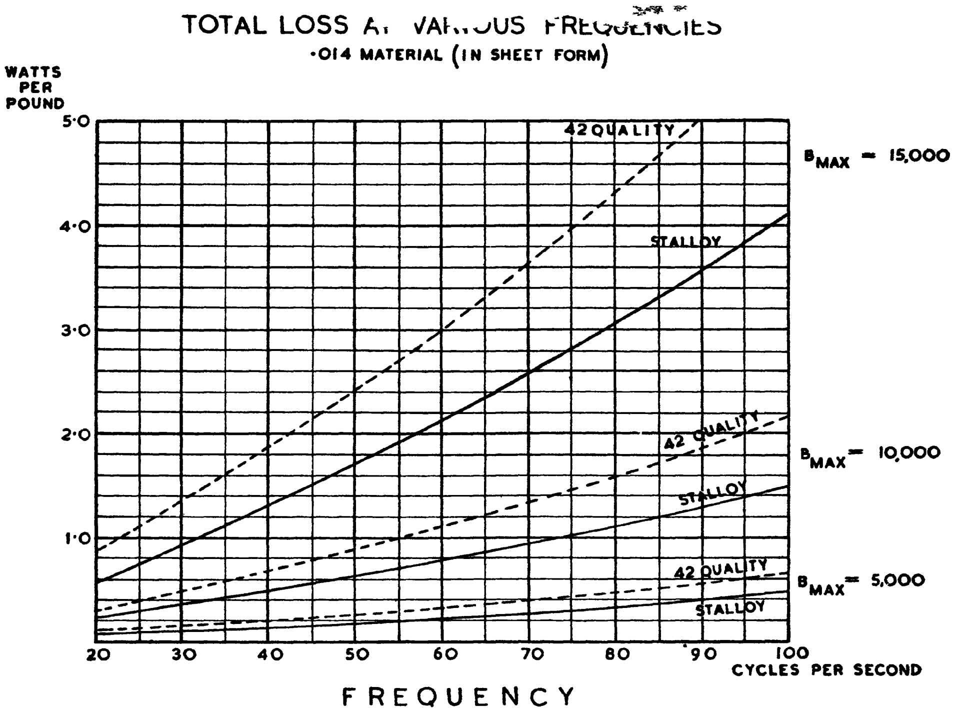

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FIG. 169

The Total Loss at Different Frequencies for Stalloy and 42 Quality at B_{max} = 5 000, 10 000, and 15 000.

Above 50 cycles, total iron loss tends to become the determining factor, so lower loss materials (4 per cent silicon, 0·014 in. thick) are advantageous.

Below 50 cycles, the magnetizing current becomes of major importance in determining the flux-density to be used. Quite

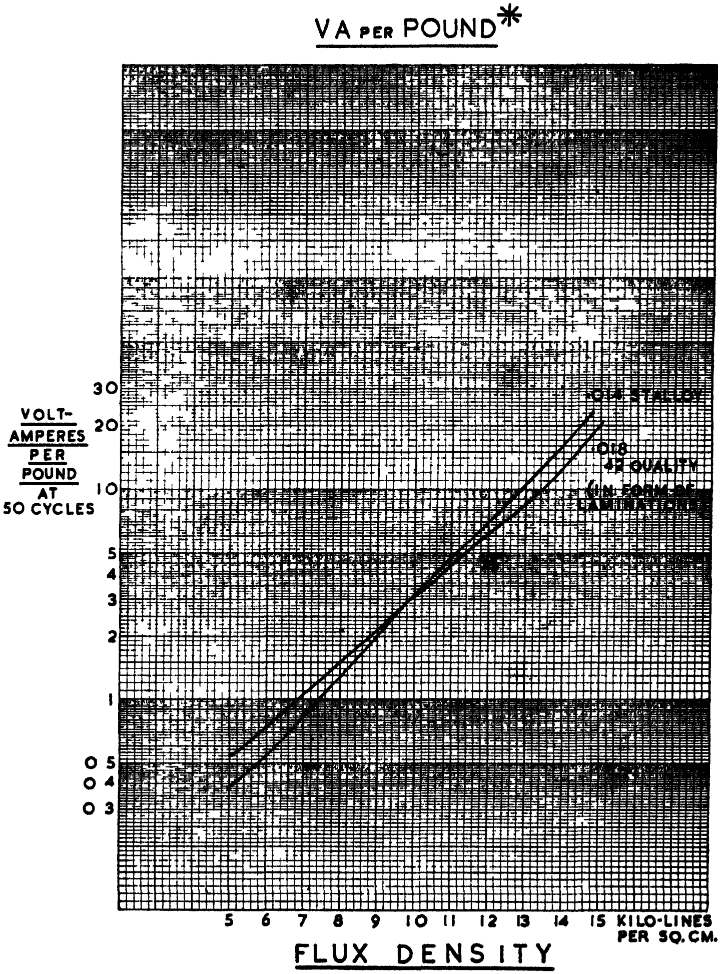

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Fig. 170

Volt amperes per lb for Stalloy and 42 Quality in the Form of Small Laminations.

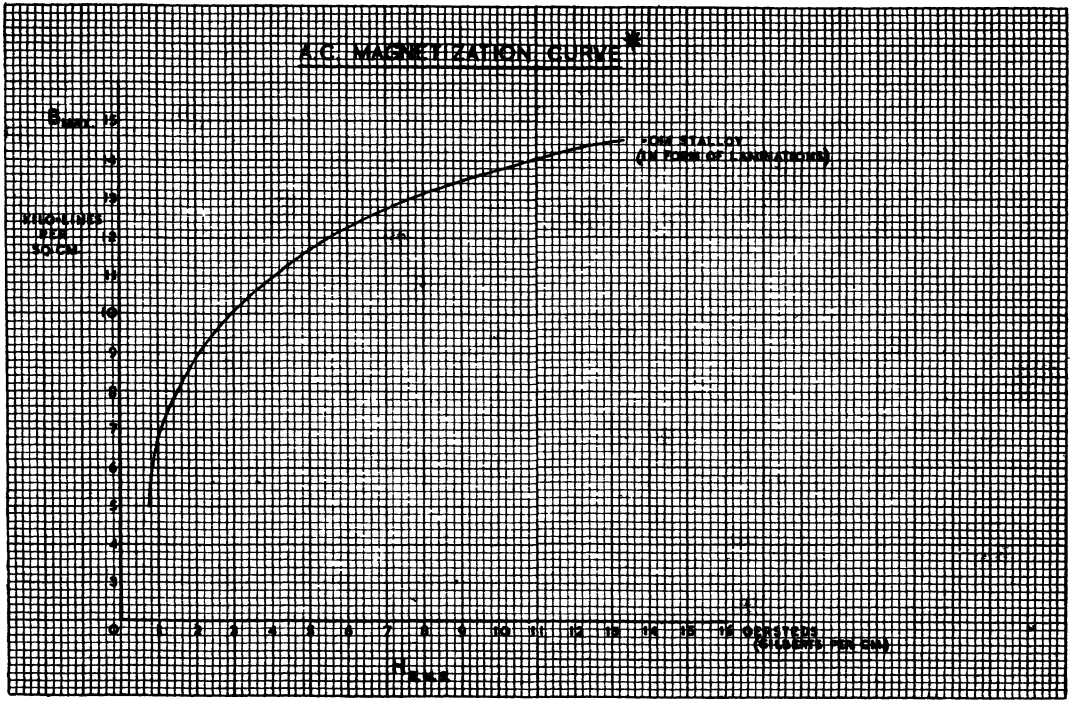

FIG. 171

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A.C. Magnetization Curve for Stalloy in the Form of Small Laminations

TABLE XVI

CORRESPONDING GRADES OF SILICON-STEEL

(Graded according to makers' published maximum figures for total loss)

Total Loss in Watts per lb at $B_{max} = 13\,000$, $f = 50$ c.p.s., $d = 0{\cdot}014$ in.	Approx. Silicon Content* (per cent)	Specific Resistance Ohms per cm^3	Richard Thomas and Baldwins, Ltd.	Magnetic & Electric Alloys, Ltd.	Joseph Sankey & Sons, Ltd.	Geo. L. Scott & Co., Ltd.	John Summers & Sons, Ltd.
0·80–0·89	4·3	58–55	Quality 7		Super Stalloy		Lowat 4
0·90–0·99	4·2	58–55	Quality 6	Super Silcor	Extra Special Stalloy	Stantranis No. 3	Lowat 3
1·00–1·04	4·0	56–55	Quality 5	Silcor I	Special Stalloy	Stantranis No. 2	Lowat 2
1·05–1·14	4·0	56–52	Quality 4	Silcor II	Stalloy	Stantranis No. 1	Lowat 1
1·15–1·24	3·5	50–45	Quality 4A		48 Quality		
1·25–1·54	3·0–2·5	45–40		Silcor III	41 Quality 42 Quality		Special Electric
1·55–1·74	1·5	35–30	Quality 3		Medium Resistance	Stantranis B	Intermediate Electric
1·75–1·99	0·75	26–19	Quality 2	Silcor Electric	Special Lohys	Medris	Electric
2·00–2·25	0·2	18–14	Quality 1	Silcor IV	Lohys	Standynis	Armature

Silicon content figures apply to Sankey grades.

low-grade core material (½ per cent silicon, 0·025 in. thick) will then give sufficiently low losses at the flux-densities permissible on magnetizing current grounds.

The magnetizing current (strictly the exciting current) can be calculated from the VA per lb curves of Fig. 170 or the a.c. magnetization curves of Fig. 171. If figures of VA per lb are required for other frequencies within the range 25–60 cycles, it can be assumed that the volt-amperes are proportional to frequency.

Lamination Details

The principal dimensions, weights of a stack of stampings with a square centre limb, and the magnetic length, are given for the "No-waste" series, the 15/16-in. centre limb group, and for other large sizes in Tables XVII–XIX. The lamination numbers used by different makers are listed in Table XX. Many types can be obtained with holes at the corners, the hole centres being on the intersection of the centre lines of the side limbs and the yokes. The existence of holes is denoted in all cases by the addition of the letter *A* after the number. Some of the "No-waste" types are also available with two

TABLE XVII

NO-WASTE SERIES

Centre Limb (in.)	Overall (in.)		Window (in.)		Weight for Sq. Stack* (lb)	Magnetic Length (in.)
	Width	Height	Width	Height		
1/2	1 1/2	1 1/4	1/4	3/4	0·195	3·00
9/16	1 11/16	1 13/32	9/32	27/32	0·277	3·38
5/8	1 7/8	1 9/16	5/16	15/16	0·380	3·75
11/16	2 1/16	1 23/32	11/32	1 1/32	0·506	4·13
3/4	2 1/4	1 7/8	3/8	1 1/8	0·657	4·50
13/16	2 7/16	2 1/32	13/32	1 7/32	0·835	4·88
7/8	2 5/8	2 3/16	7/16	1 5/16	1·04	5·25
1	3	2 1/2	1/2	1 1/2	1·56	6·00
1 1/8	3 3/8	2 13/16	9/16	1 11/16	2·22	6·75
1 1/4	3 3/4	3 1/8	5/8	1 7/8	3·04	7·50
1 3/8	4 1/8	3 7/16	11/16	2 1/16	4·05	8·25
1 1/2	4 1/2	3 3/4	3/4	2 1/4	5·25	9·00
1 3/4	5 1/4	4 3/8	7/8	2 5/8	8·34	10·50
2	6	5	1	3	12·5	12·00

* Weights are given for 2½ per cent silicon steel, 0·020 in. thick, i.e. 0·276 lb per in.^{3} and 94 per cent stacking factor. Figures for 4 per cent 0·014-in. material are approx. 4·7 per cent lower.

notches instead of four holes. These are indicated by the letter *N*. *Weight figures are in all cases given for types without holes or notches.*

Winding Information

Table XXI gives the information normally required when calculating winding resistances and voltage drops. The

TABLE XVIII

$\frac{15}{16}$-IN. CENTRE LIMB SERIES

No. or Ref.	Overall Height (in.)	Window Height (in.)	Weight of Sq. Stack* (lb)	Magnetic Length (in.)
δ	$1\frac{5}{8}$	$\frac{3}{4}$	1·09	5·00
ε	2	$1\frac{1}{8}$	1·26	5·75
(473A)†	$2\frac{1}{16}$	$1\frac{3}{16}$	1·28	5·88
442A	$2\frac{1}{2}$	$1\frac{5}{8}$	1·48	6·75
η	$2\frac{7}{8}$	2	1·64	7·50
404A	$3\frac{3}{16}$	$2\frac{5}{16}$	1·78	8·13
θ	$3\frac{7}{16}$	$2\frac{9}{16}$	1·89	8·63
λ	$3\frac{5}{8}$	$2\frac{3}{4}$	1·97	9·00

TABLE XIX

LARGER LAMINATIONS

(*a*) *T & U Types*

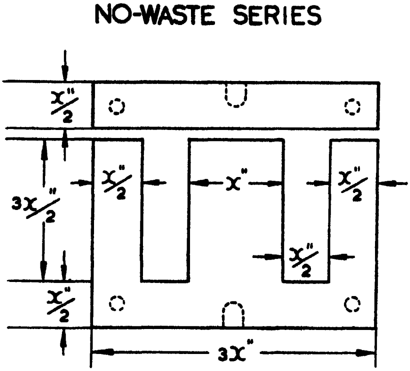
No. or Ref.	Overall (in.)		Centre Limb (in.)	Window (in.)		Weight of Sq. Stack* (lb)	Magnetic Length (in.)
	Width	Height		Width	Height		
475A	4	$3\frac{3}{8}$	1	1	$2\frac{3}{8}$	2·28	8·75
460A	$4\frac{1}{2}$	4	$1\frac{1}{4}$	1	$2\frac{3}{4}$	4·06	10·0
428A	5	$4\frac{1}{4}$	$1\frac{7}{32}$	$1\frac{17}{64}$	3	4·32	11·0
435A	$6\frac{1}{4}$	$5\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$3\frac{3}{4}$	8·04	13·8
437A	$6\frac{3}{4}$	$6\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{5}{8}$	5	13·3	16·8
441A	$8\frac{1}{2}$	$7\frac{1}{4}$	$2\frac{1}{2}$	$1\frac{3}{4}$	$4\frac{3}{4}$	29·2	18·0

(*b*) *E & I Types*

ρ	7	6	2	$1\frac{1}{2}$	4	15·6	15·0
σ	$9\frac{1}{2}$	$9\frac{1}{2}$	3	$1\frac{3}{4}$	$6\frac{1}{2}$	52·5	22·5
τ	$9\frac{1}{2}$	11	3	$1\frac{3}{4}$	8	59·6	25·5

* Weights are given for $2\frac{1}{2}$ per cent silicon steel, 0·020 in. thick, i.e. 0·276 lb per in.^{3} and 94 per cent stacking factor. Figures for 4 per cent 0·014-in. material are approx. 4·7 per cent lower.

† Not included in later R.C.S.C. preferred lists.



CENTRE LIMB = x''
OVERALL WIDTH = $3x''$
" HEIGHT = $\frac{5}{2}x''$
WINDOW WIDTH = $\frac{x}{2}''$
" HEIGHT = $\frac{3}{2}x''$
SURFACE AREA * = $6x^2$ SQ. IN.
CROSS SECTION (SQ. STACK) = x^2 SQ. IN.
VOLUME = $6x^3$ CU. IN.
MAGNETIC LENGTH = $6x''$

*ASSUMING 100% STACKING FACTOR

Fig. 172

Dimensions of "No-waste" Laminations in Terms of Width of Centre Limb.

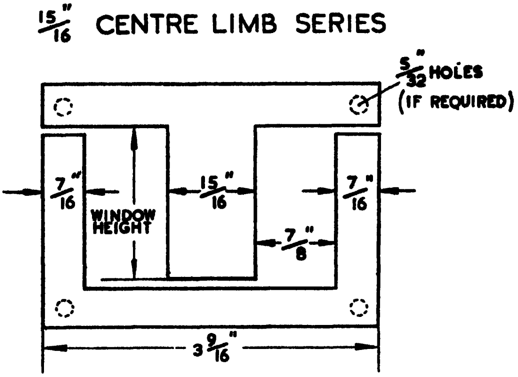


Fig. 173

Dimensions of $\frac{15}{16}$-in. Centre Limb Laminations. The Only Variable is the Height.

TABLE XX

DESCRIPTION	NO. OR REF.	BALDWINS	M & E A AND T.C.M.	ROLA	SANKEY	SCOTT
Long and short E's	450	—	187	—	—	—
T & U	421	—	21	—	—	—
E	α	—	79	—	—	—
E	431	—	31	—	141	—
E & I	423	83	40	—	23	177
"	439	—	39	—	—	207
"	β	—	104	—	—	—
"	496	—	36	—	—	—
"	470	—	70	—	—	—
T & U	γ	—	68	—	68	—
"	403A	—	12A	—	—	104A
"	401A	42C	101A	—	60A	116
"	440A	52	59A	—	40A	84A
$\frac{15}{16}$ Centre Limb T & U	δ	—	97	—	97	79A
"	ε	9A	130A	-	30A	34A
"	(473A)	—	93A	—	73	—
"	442A	194	24A	—	221	42A
"	η	—	189A	—	89	—
"	404A	88	4A	-	4A	4A
"	θ	—	—	—	122	—
"	λ	—	—	—	16	—
T & U	475A	34	75A	—	75A	26A
"	460A	57	60A	-	33A	60A
"	428A	55	28A	—	28A	25A
"	435A	54	235A	—	35A	45A
"	437A	117	137A	—	37A	78A
"	441A	109	41A	—	41A	213A
E & I	ρ	—	126	—	—	—
"	σ	—	171	—	—	—
"	τ	69	122	—	167	—
$\frac{9}{16}$ No waste E & I	—	—	18	1	—	212
$\frac{5}{8}$ "	—	392	145	—	—	145
$\frac{11}{16}$ "	474	386	75	2	74	130
$\frac{3}{4}$ "	417	217	35	—	70	120
$\frac{13}{16}$ "	—	—	182(N)	—	—	—
$\frac{7}{8}$ "	—	—	147	—	—	—
1 "	429A	103	29(A)(N)	4	111	43(A)(N)
$1\frac{1}{8}$ "	—	—	—	—	158	—
$1\frac{1}{4}$	—	420	78(N)	6	133	174
$1\frac{3}{8}$ "	—	362	152(N)	7	—	—
$1\frac{1}{2}$ "	—	—	120(N)	8	149	173(N)

selection of a suitable wire for a given current is facilitated by the use of Table XXII, whilst the space occupied by a layer- or bobbin-winding can be quickly estimated by reference to Table XXIII, which shows turns per in. (single layer) and turns per in.^{2} for wires with different coverings. All the data in this section are derived from information kindly supplied by the London Electric Wire Co., and Smiths, Ltd.

TABLE XXI

BARE COPPER WIRE DATA

SWG	DIAMETER (in.)	CROSS-SECTION (in.^{2})	RESISTANCE (Ohms per 1 000 yd)	WEIGHT (lb per 1 000 yd)
14	0·080	0·005027	4·776	58·13
15	0·072	0·004072	5·897	47·09
16	0·064	0·003217	7·463	37·20
17	0·056	0·002463	9·747	28·48
18	0·048	0·0018096	13·267	20·93
19	0·040	0·0012566	19·105	14·533
20	0·036	0·0010179	23·59	11·772
21	0·032	0·0008042	29·85	9·301
22	0·028	0·0006158	38·99	7·121
23	0·024	0·0004524	53·07	5·232
24	0·022	0·0003801	63·16	4·396
25	0·020	0·0003142	76·42	3·633
26	0·018	0·0002545	94·35	2·943
27	0·0164	0·0002112	113·65	2·443
28	0·0148	0·00017203	139·55	1·9895
29	0·0136	0·00014527	165·27	1·6800
30	0·0124	0·00012076	198·80	1·3966
31	0·0116	0·00010568	227·2	1·2222
32	0·0108	0·00009161	262·1	1·0594
33	0·0100	0·00007854	305·7	0·9083
34	0·0092	0·00006648	361·2	0·7688
35	0·0084	0·00005542	433·2	0·6409
36	0·0076	0·00004536	529·2	0·5246
37	0·0068	0·00003632	661·1	0·4200
38	0·0060	0·00002827	849·1	0·3270
39	0·0052	0·00002124	1130·5	0·2456
40	0·0048	0·000018096	1326·7	0·2093
41	0·0044	0·000015205	1578·9	0·17585
42	0·0040	0·000012566	1910·5	0·14533
43	0·0036	0·000010179	2359	0·11772
44	0·0032	0·000008042	2985	0·09301
45	0·0028	0·000006158	3899	0·07121
46	0·0024	0·000004524	5307	0·05232
47	0·0020	0·000003142	7642	0·03633
48	0·0016	0·000002011	11941	0·02325

TABLE XXII

CURRENT-CARRYING CAPACITY OF COPPER WIRES AND STRIPS

SWG No.	Current Rating in Amperes at				
	1 600 amp per in.²	1 800 amp per in.²	2 000 amp per in.²	2 200 amp per in.²	2 400 amp per in.²
14	8·05	9·05	10·05	11·07	12·07
15	6·52	7·32	8·14	8·95	9·76
16	5·14	5·78	6·43	7·07	7·70
17	3·94	4·44	4·93	5·42	5·91
18	2·89	3·26	3·62	3·97	4·33
19	2·01	2·26	2·51	2·76	3·01
20	1·64	1·83	2·04	2·24	2·44
21	1·28	1·45	1·61	1·77	1·93
22	0·99	1·11	1·23	1·36	1·48
23	0·72	0·81	0·90	0·99	1·09
24	0·61	0·68	0·76	0·84	0·91
25	0·50	0·57	0·63	0·69	0·75
26	0·41	0·46	0·51	0·56	0·61
27	0·34	0·38	0·42	0·46	0·51
28	0·27	0·31	0·34	0·38	0·41
29	0·23	0·26	0·29	0·32	0·35
30	0·193	0·22	0·24	0·27	0·29
31	0·169	0·190	0·21	0·23	0·25
32	0·147	0·165	0·183	0·20	0·22
33	0·126	0·141	0·157	0·173	0·188
34	0·106	0·120	0·133	0·146	0·159
35	0·089	0·100	0·111	0·122	0·133
36	0·073	0·082	0·091	0·100	0·109
37	0·058	0·065	0·073	0·080	0·087
38	0·045	0·051	0·057	0·062	0·068
39	0·034	0·038	0·042	0·047	0·051
40	0·029	0·033	0·036	0·040	0·043
41	0·024	0·027	0·030	0·033	0·037
42	0·020	0·023	0·025	0·028	0·030
43	0·0162	0·0182	0·020	0·022	0·024
44	0·0128	0·0144	0·0160	0·0176	0·0192
45	0·0099	0·0111	0·0124	0·0136	0·0149
46	0·0072	0·0081	0·0090	0·0099	0·0108
47	0·0050	0·0056	0·0062	0·0068	0·0074
48	0·0032	0·0036	0·0040	0·0044	0·0048

Strip Dimensions (in.)	Current Rating in Amperes at				
	1 000 amp per in.²	1 200 amp per in.²	1 400 amp per in.²	1 600 amp per in.²	1 800 amp per in.²
0·04 × 0·160	6·4	7·7	9·0	10·2	11·5
0·04 × 0·200	8·0	9·6	11·2	12·8	14·4
0·05 × 0·200	10·0	12·0	14·0	16·0	18·0
0·05 × 0·250	12·5	15·0	17·5	20·0	22·5
0·06 × 0·240	14·4	17·3	20·2	23·0	—
0·06 × 0·360	21·6	25·9	30·2	34·6	—
0·08 × 0·320	25·6	30·7	35·8	—	—
0·10 × 0·300	30·0	36·0	42·0	—	—
0·10 × 0·400	40·0	48·0	56·0	—	—

TABLE XXIII

SWG No.	Turns Per Inch				Turns per In.^{2}			
	Enam.	E. & S.S.	D.S.C.	D.C.C.	Enam.	E. & S.S.	D.S.C.	D.C.C.
14	10·8	—	—	10·5	117	—	—	110
15	13·2	—	—	11·8	174	—	—	139
16	14·8	14·1	14·5	13·0	219	199	210	169
17	16·9	16·0	16·5	14·5	287	256	272	210
18	19·7	18·9	19·4	16·8	388	357	376	282
19	23·5	22·3	23·0	19·4	552	497	529	376
20	26·0	24·6	25·3	21·0	676	605	640	441
21	29·2	27·5	28·3	23·0	853	756	801	529
22	33·0	31·2	31·9	25·4	1 089	973	1 018	645
23	38·3	36·1	36·9	28·4	1 467	1 303	1 362	807
24	41·6	39·5	37·8	31·0	1 730	1 560	1 429	961
25	45·5	43·4	43·1	33·1	2 070	1 884	1 858	1 096
26	50·2	48·1	48·3	35·4	2 520	2 313	2 333	1 253
27	55·1	52·3	52·3	37·6	3 036	2 735	2 735	1 414
28	61·0	57·4	57·4	38·6	3 721	3 295	3 295	1 490
29	66·0	61·7	61·7	40·5	4 356	3 807	3 807	1 640
30	72·5	67·1	66·6	44·4	5 256	4 502	4 436	1 971
31	77·5	71·4	70·4	46·0	6 006	5 098	4 956	2 116
32	82·7	76·3	74·6	47·8	6 839	5 822	5 565	2 285
33	89·3	81·3	79·3	49·7	7 974	6 610	6 288	2 470
34	97·0	87·7	84·7	51·7	9 409	7 691	7 174	2 673
35	105	94·3	90·9	57·1	11 025	8 892	8 263	3 260
36	116	102	97·9	59·9	13 456	10 404	9 584	3 588
37	128	112	104	63·7	16 384	12 544	10 816	4 058
38	145	125	113	67·7	21 025	15 625	12 769	4 583
39	164	143	125	70·9	26 896	20 449	15 625	5 027
40	178	151	131	75·1	31 684	22 801	17 161	5 640
41	192	163	149	—	36 864	26 569	22 201	—
42	208	175	158	—	43 264	30 625	24 964	—
43	227	192	169	—	51 529	36 864	28 561	—
44	256	208	181	—	65 536	43 264	32 761	—
45	286	227	196	—	81 796	51 529	38 416	—
46	333	256	212	—	110 889	65 536	44 944	—
47	385	285	232	—	148 225	81 225	53 824	—

Rectifier Loads

Two of Schade's curves showing the a.c. voltage and r.m.s. current in the secondary windings of transformers supplying capacitor filter circuits are reproduced in Figs. 174 and 175. In Fig. 174 the d.c. output voltage is shown relative to the peak of the applied a.c. voltage of each half-winding, for various proportions of internal circuit resistance to load resistance. It should be noted that the internal resistance figure must include not only the half-secondary winding resistance itself but also the primary resistance referred to the half-secondary and the rectifier equivalent resistance.

Fig. 175 gives the corresponding r.m.s. current value in the secondary (or half-secondary) in terms of the direct current

FULL-WAVE RECTIFIER CIRCUIT (WITH CONDENSER FILTER)

OUTPUT VOLTAGE ($V_{D.C.}$) AS PERCENTAGE OF A.C PEAK VOLTAGE (E_{max})
(From Schade, Proc. I.R.E. July, 1943)

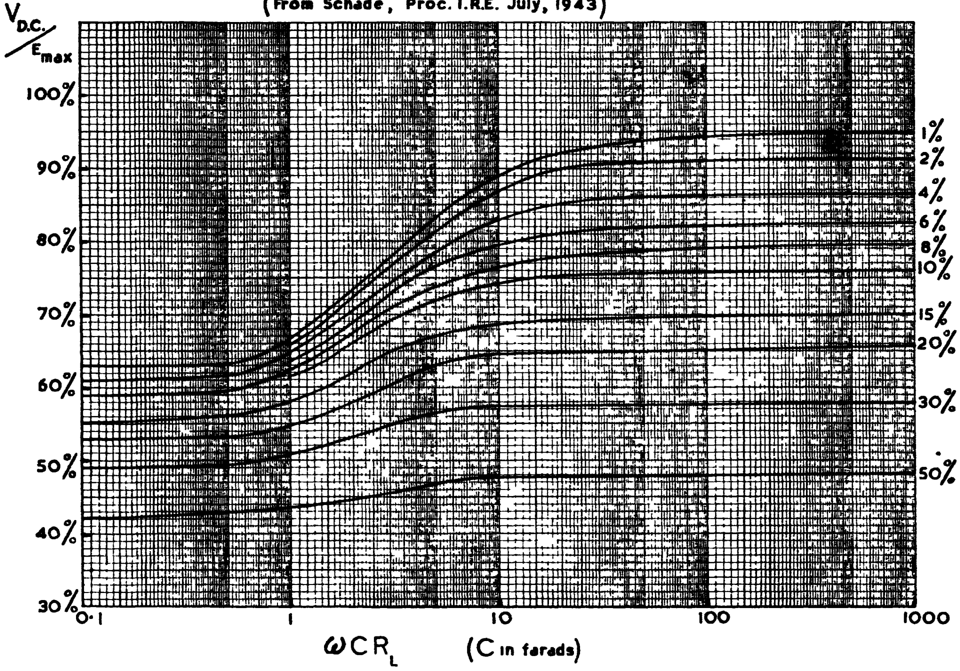
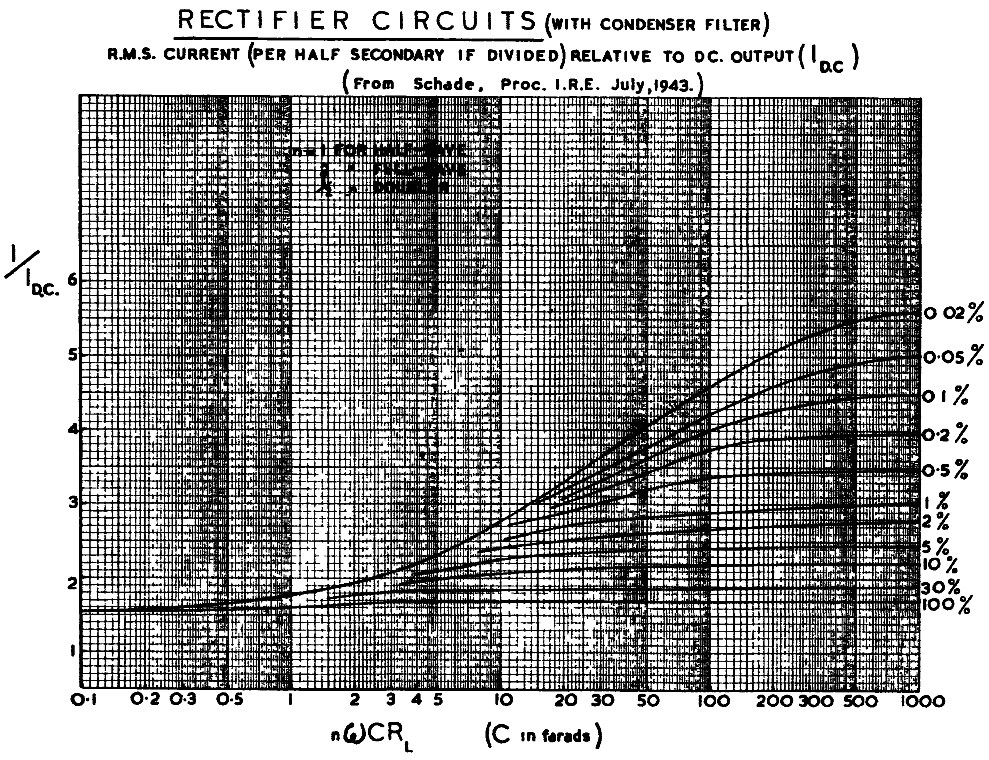


FIG. 174

Schade


RECTIFIER CIRCUITS (WITH CONDENSER FILTER)

R.M.S. CURRENT (PER HALF SECONDARY IF DIVIDED) RELATIVE TO DC. OUTPUT ($I_{D.C}$)

(From Schade, Proc. I.R.E. July, 1943.)

FIG. 175

Schade

through the load resistance. Here again the value depends on the ratio of internal resistance to load resistance.

2. AUDIO-TRANSFORMERS

Much of the information given in Section 1 is also required for audio-frequency design work. An additional list of smaller laminations is given in Table XXIV, however. Weights are given in this case for Mumetal; owing to the absence of scale on the stampings a higher stacking factor of 0·95 has been assumed.

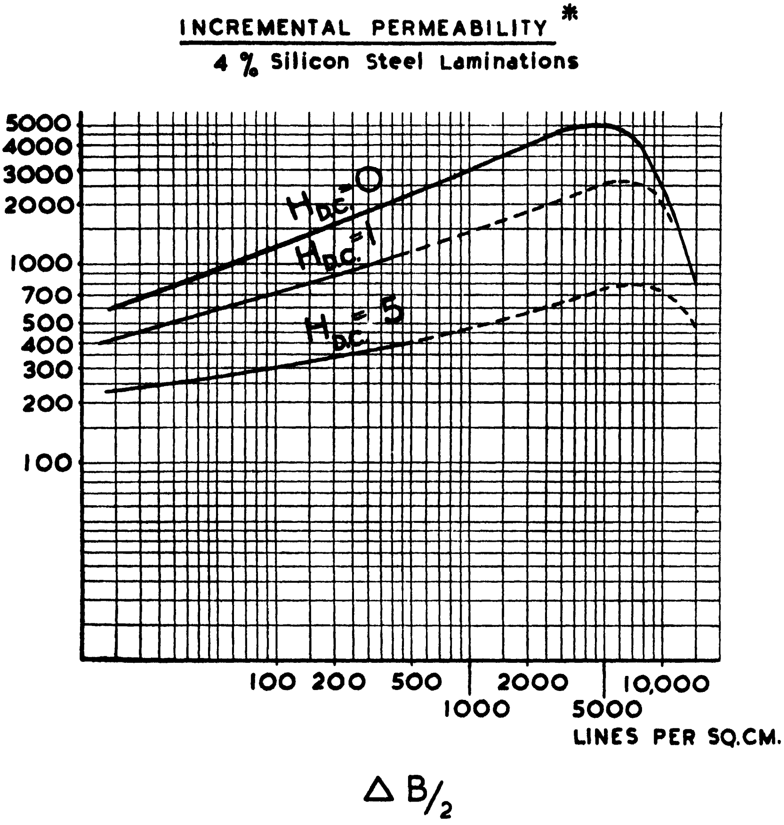
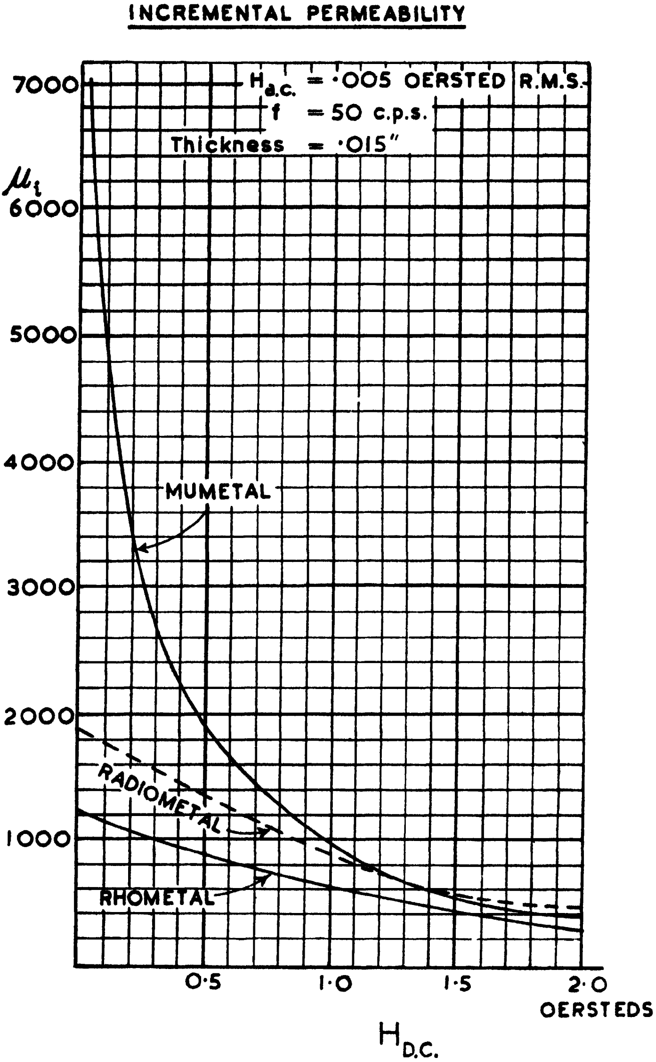


FIG. 176 *Jos. Sankey, Ltd.*

Incremental Permeability of 4 per cent Silicon Steel at Different Values of Polarizing Force ($H_{D.C.}$) and Flux-density.



T.C.M., Ltd.

FIG. 177
Incremental Permeability of Nickel-alloys.

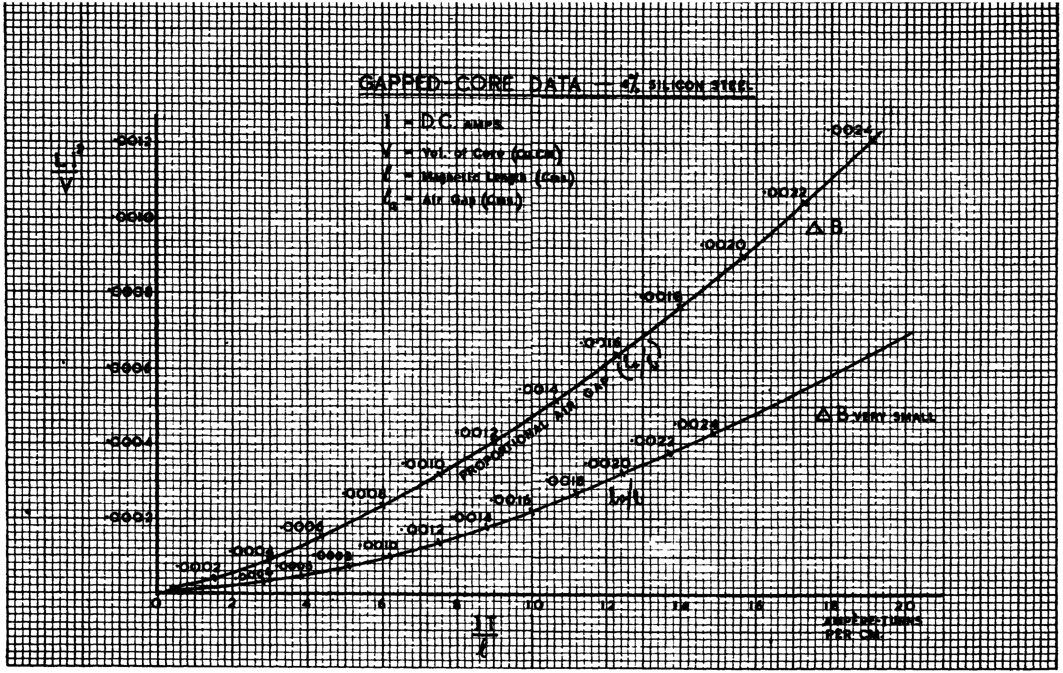


Fig. 178 *Scroggie, Wireless World*

LI²/V Curves for 4 per cent Silicon Steel.

TABLE XXIV

NICKEL-IRON LAMINATIONS FOR A.F. TRANSFORMERS

(In most cases also available in silicon iron)

No. or Ref.	Overall Width (in.)	Overall Height (in.)	Centre Limb (in.)	Window Width (in.)	Window Height (in.)	Weight for Sq. Stack* (lb)	Magnetic Length (in.)
450	1	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0·038	2·00
421	$1\frac{11}{32}$	$1\frac{1}{32}$	$\frac{7}{16}$	$\frac{17}{64}$	$\frac{21}{32}$	0·138	2·63
α	$1\frac{3}{8}$	$1\frac{3}{16}$	$\frac{1}{4}$	$\frac{13}{32}$	$\frac{11}{16}$	0·66	2·97
431	$1\frac{1}{2}$	1	$\frac{3}{16}$	$\frac{15}{32}$	$\frac{5}{8}$	0·42	2·84
423	$1\frac{5}{8}$	$1\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	0·154	2·94
439	$1\frac{5}{8}$	$1\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{8}$	0·179	3·44
β	$1\frac{5}{8}$	$1\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$1\frac{1}{8}$	0·20	3·94
496	$1\frac{27}{32}$	$1\frac{1}{2}$	$\frac{7}{16}$	$\frac{29}{64}$	1	0·25	3·88
470	$2\frac{1}{16}$	$1\frac{13}{16}$	$\frac{1}{2}$	$\frac{15}{32}$	$1\frac{3}{16}$	0·40	4·50
γ	$2\frac{7}{32}$	$1\frac{27}{32}$	$\frac{19}{32}$	$\frac{33}{64}$	$1\frac{1}{4}$	0·50	4·72
403A	$2\frac{5}{8}$	$1\frac{21}{32}$	$\frac{5}{8}$	$\frac{11}{16}$	$1\frac{1}{32}$	0·55	4·69
401A	3	$1\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$1\frac{1}{8}$	0·89	5·25
440A	$3\frac{1}{4}$	$2\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{5}{8}$	1·52	6·50

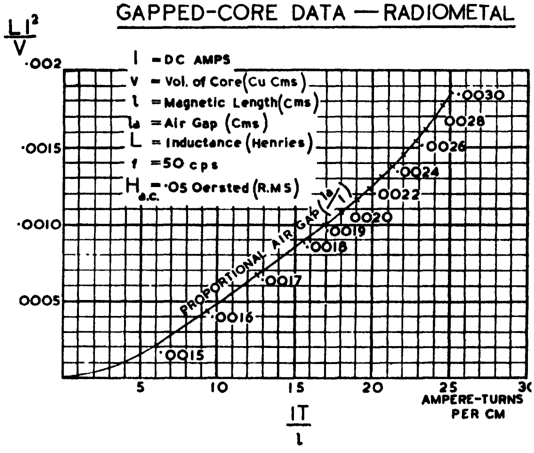


FIG. 179 *T.C.M., Ltd.*

LI^2/V Curves for Radiometal.

To assist calculation of the inductances of transformers carrying a d.c. component in their windings, Figs. 176 and 177 are values of incremental permeabilities for 4 per cent silicon steel and for various nickel-alloys. When the d.c. component is large enough to make gapping advantageous, curves of LI^2/V given in Figs. 178 and 179 will be found more useful.

---

* Weight given for Mu-metal 0·015 in. thick, assuming 95 per cent stacking factor. Figures for 4 per cent Silicon Steel, 0·014 in. thick (91 per cent stacking factor) are 18 per cent lower.

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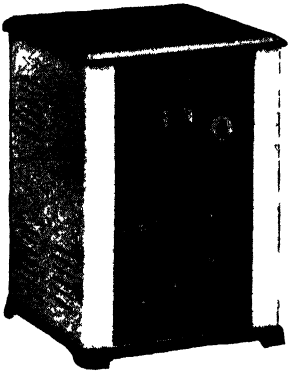

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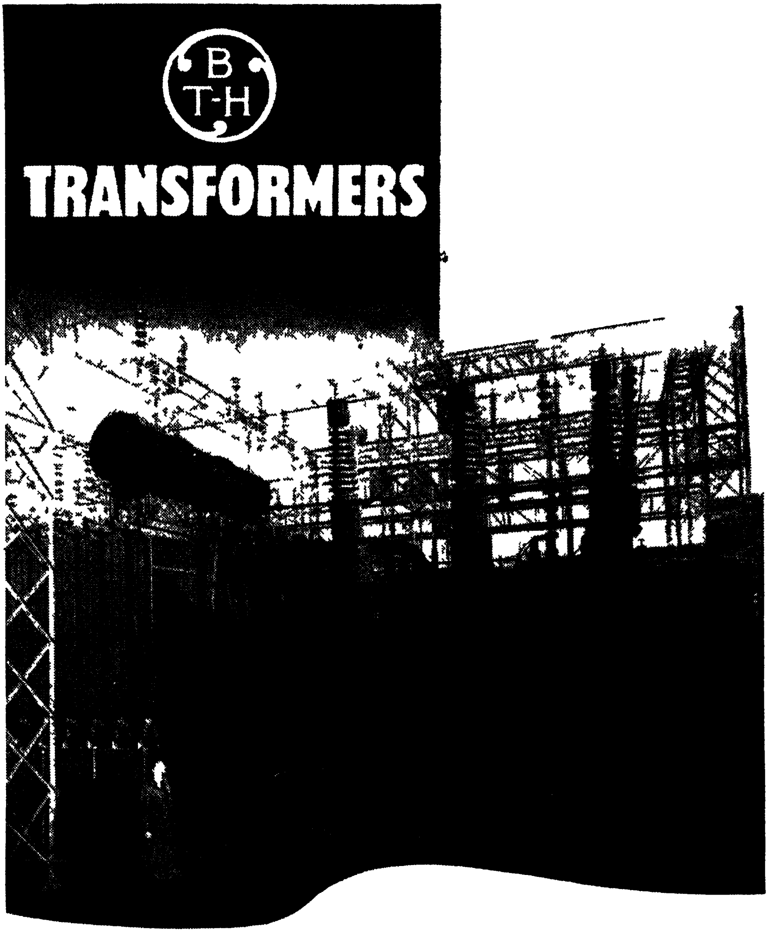

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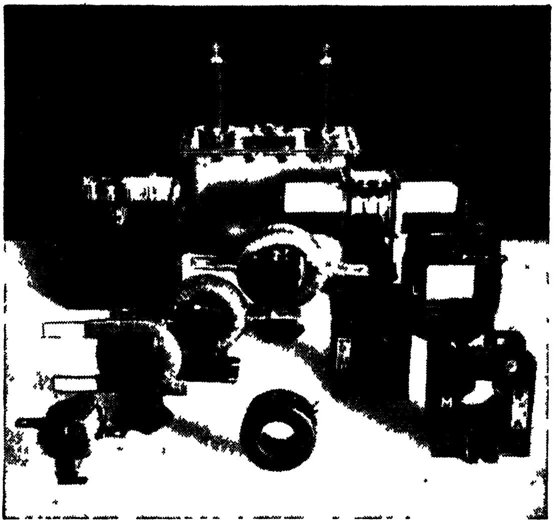

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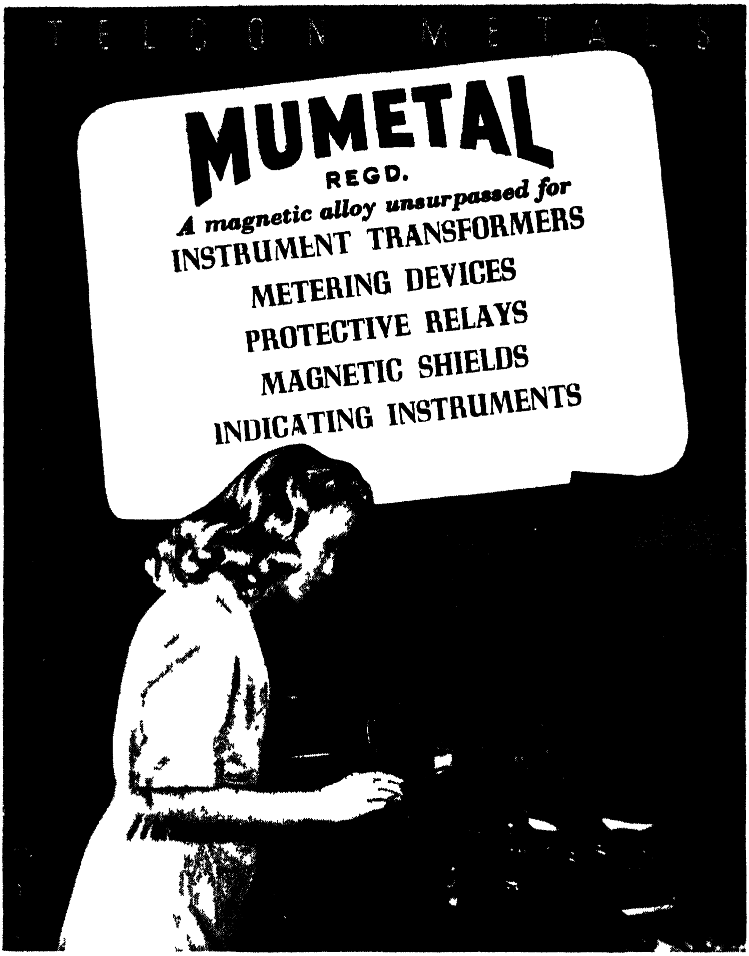

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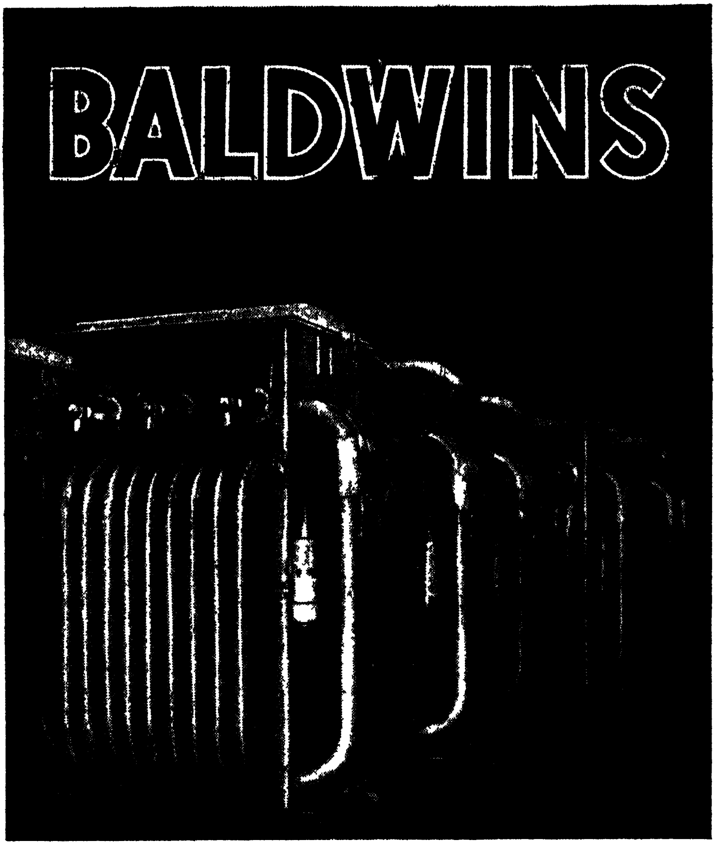

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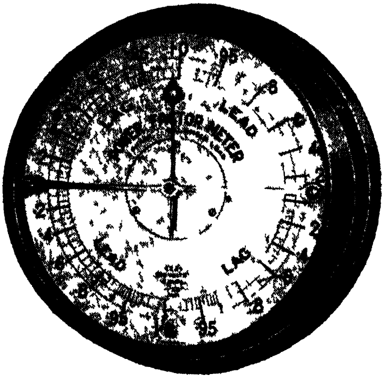

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